Threshold ECDSA from ECDSA assumptions: *the multiparty case*

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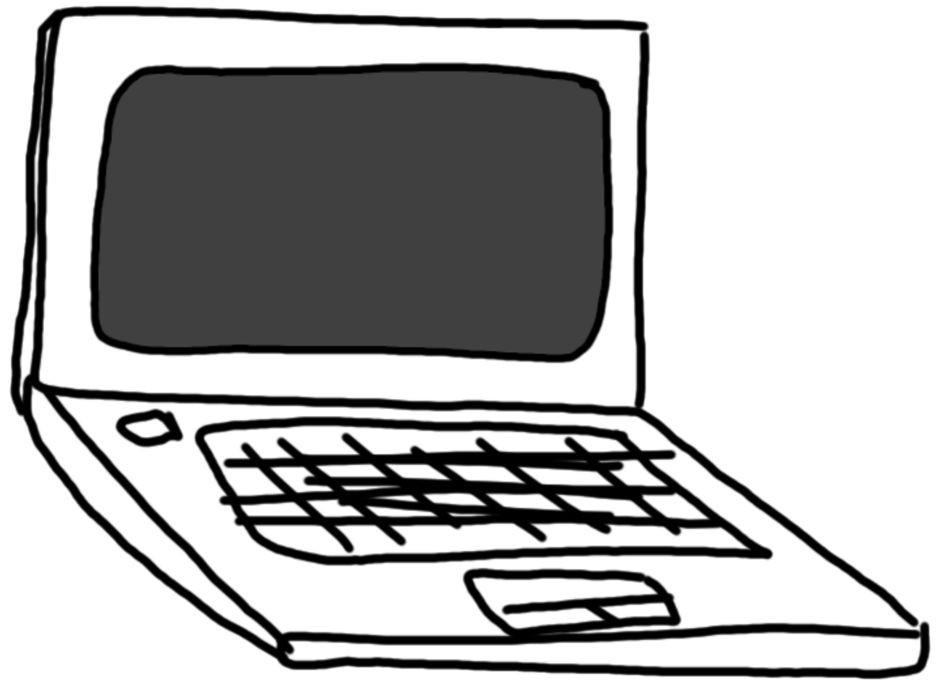
Northeastern University

Traditional Signature

pk



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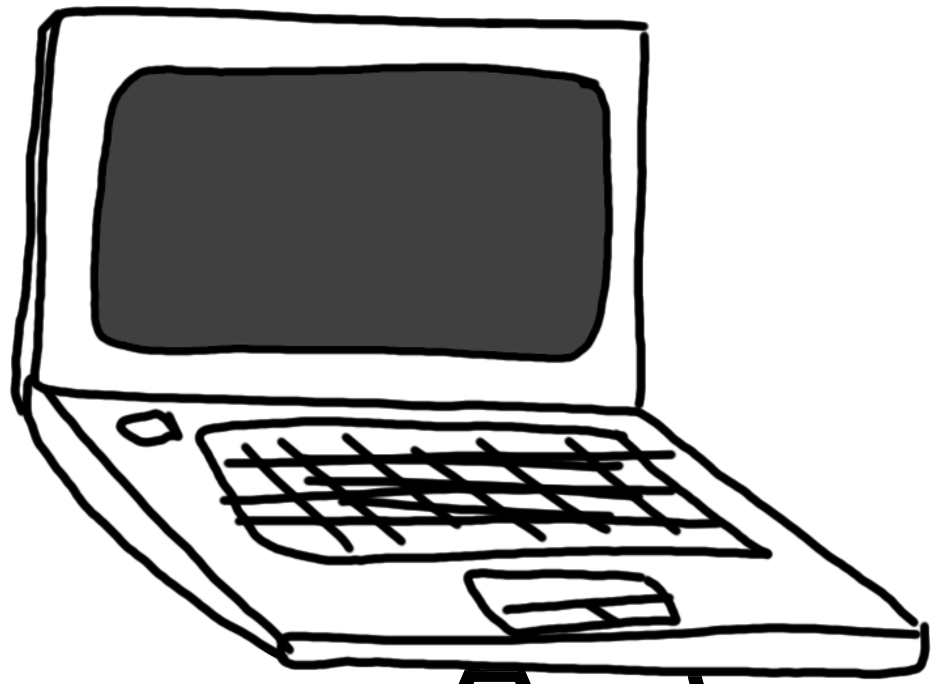
$$\{\mathsf{sk}_\mathsf{A}, \mathsf{sk}_\mathsf{B}, \mathsf{sk}_C\} \leftarrow \mathrm{Share}(\mathsf{sk})$$

pk



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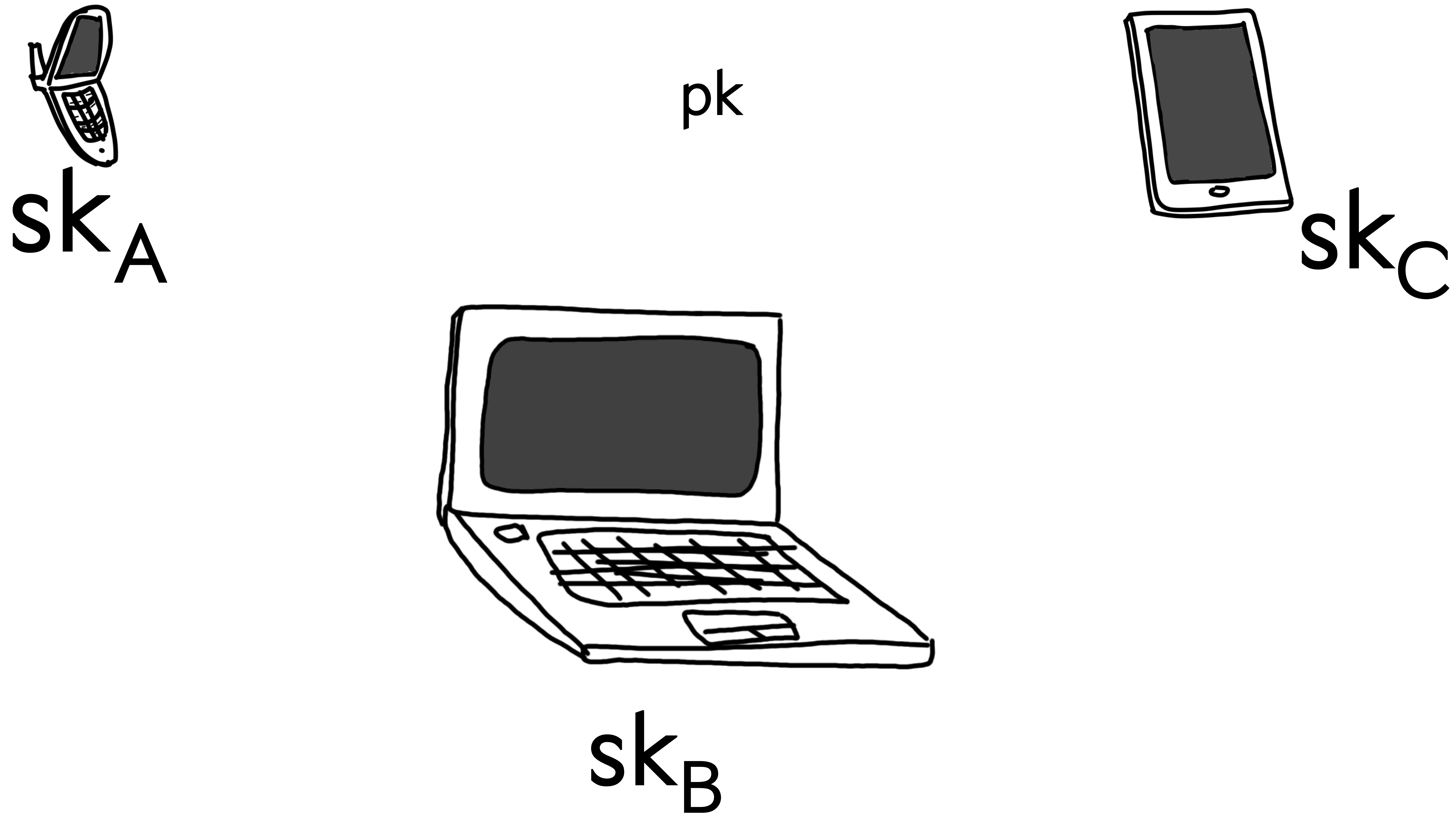

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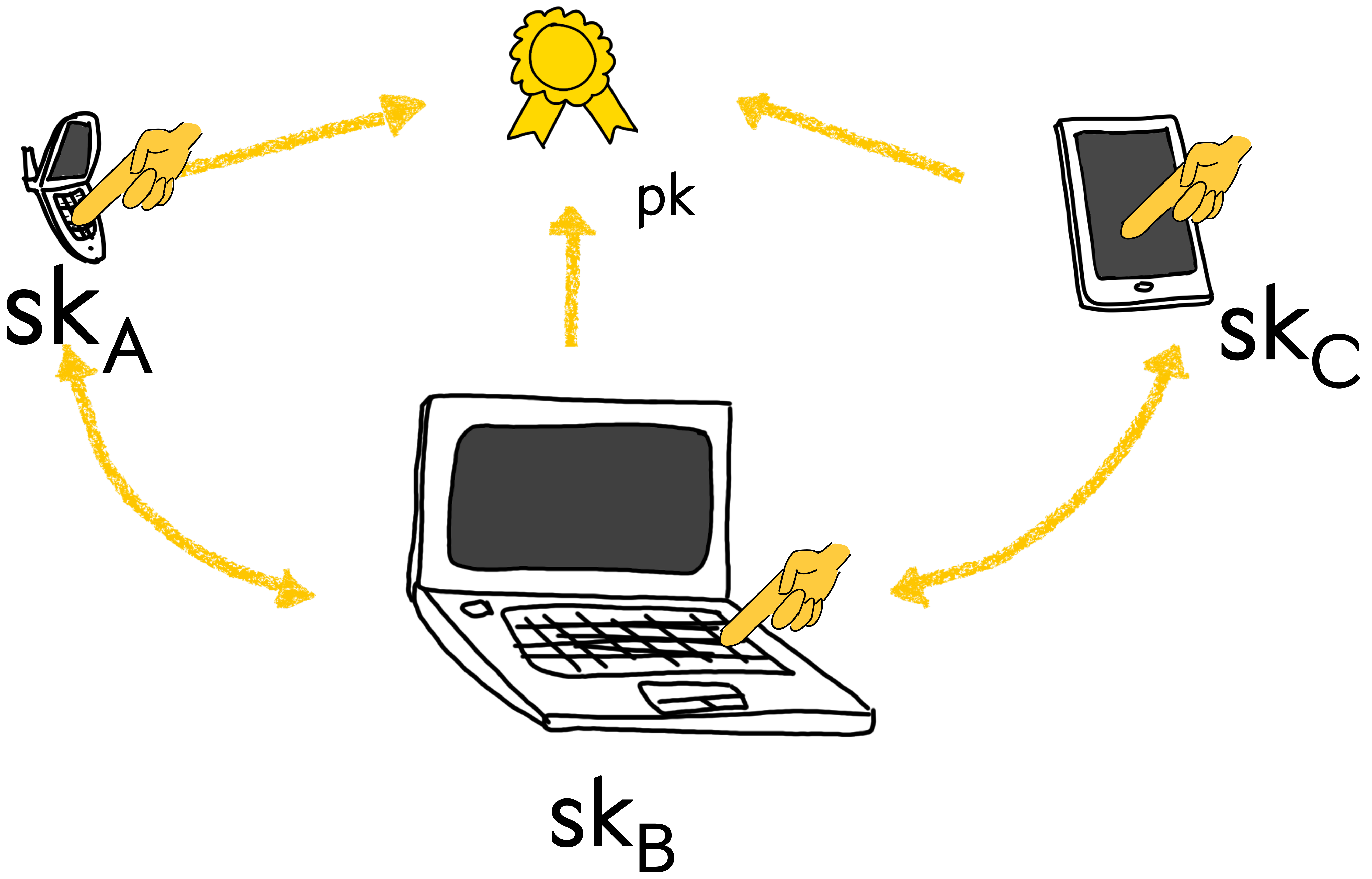

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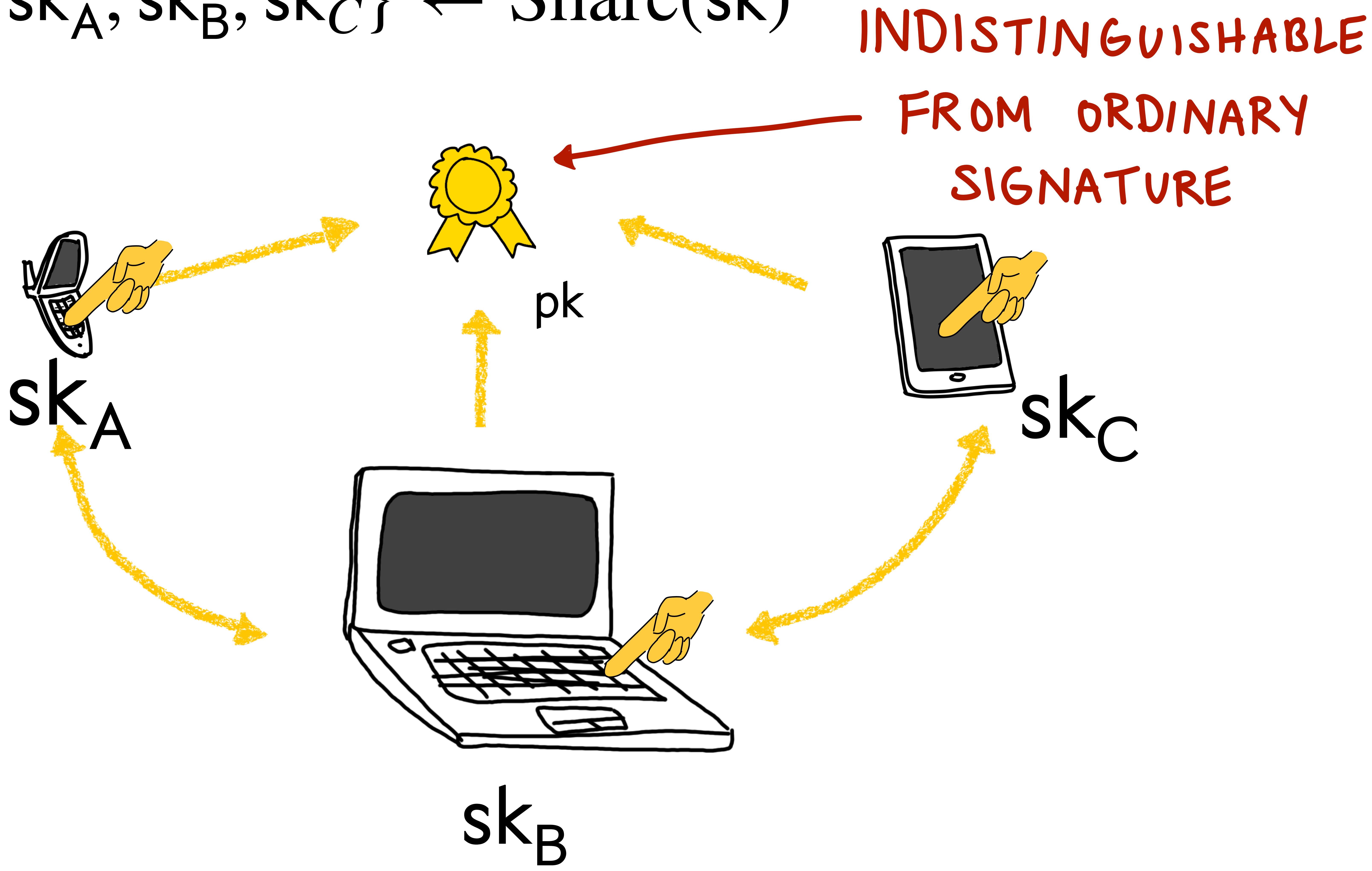
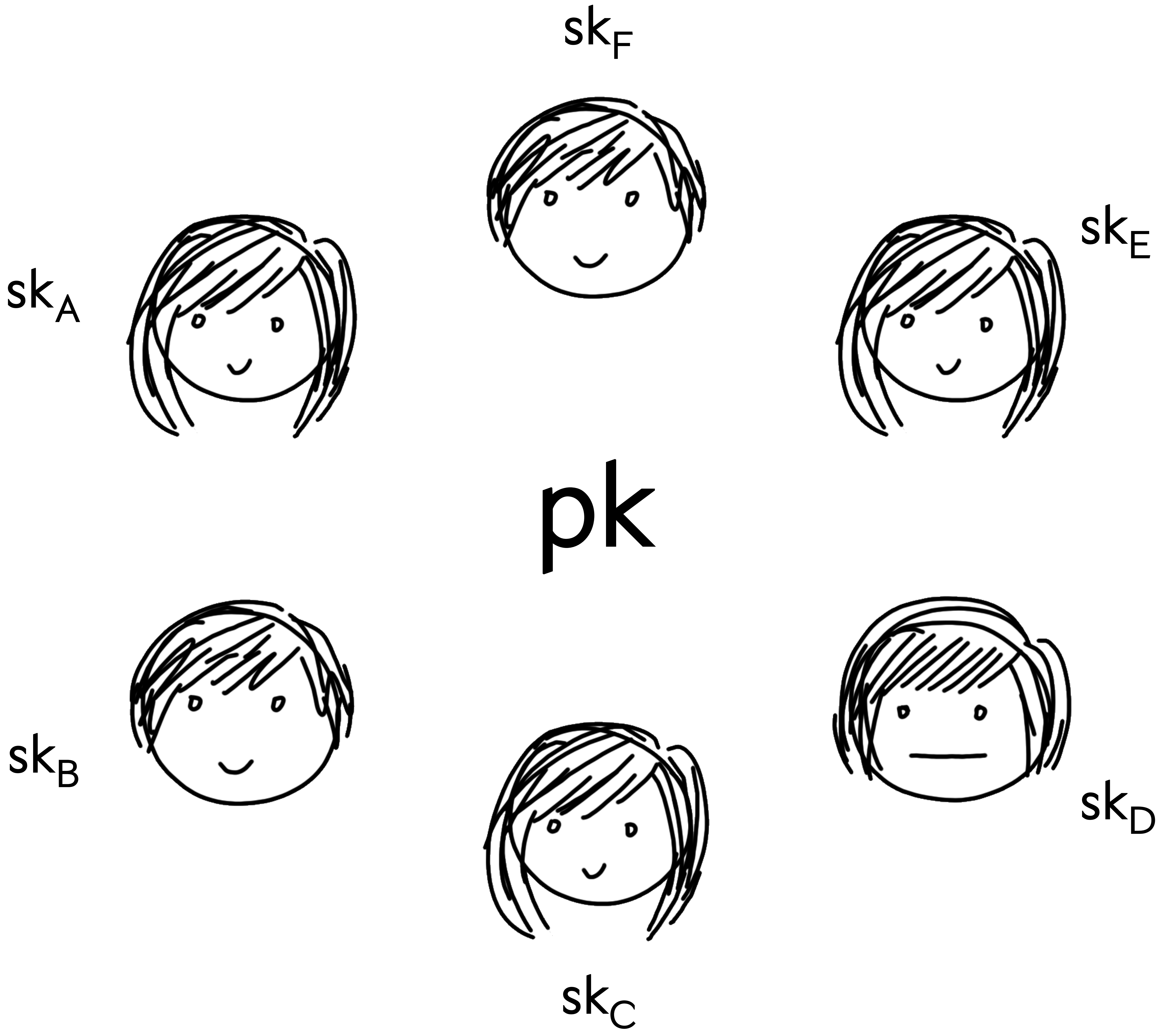


3-of-n Signature Scheme



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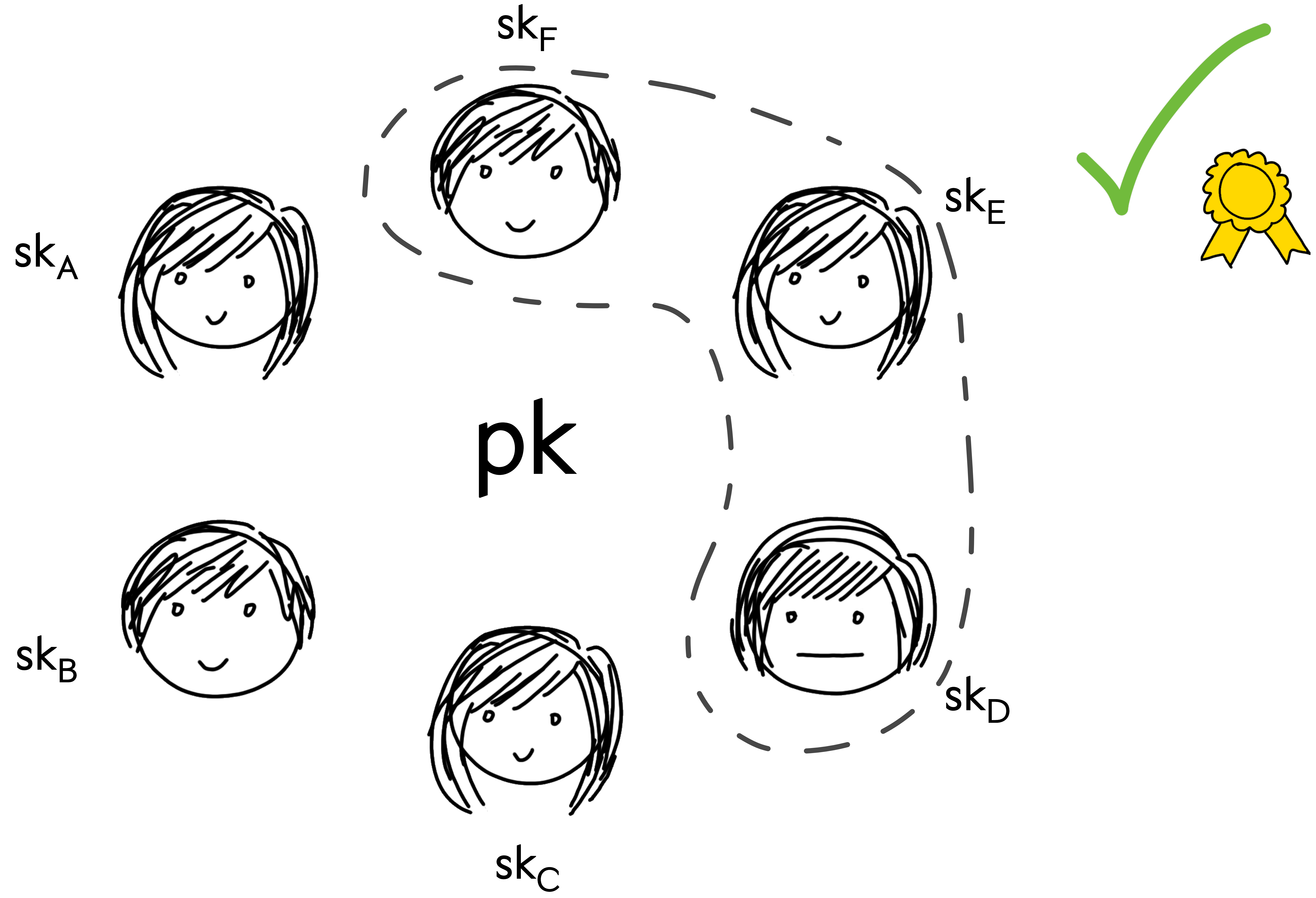

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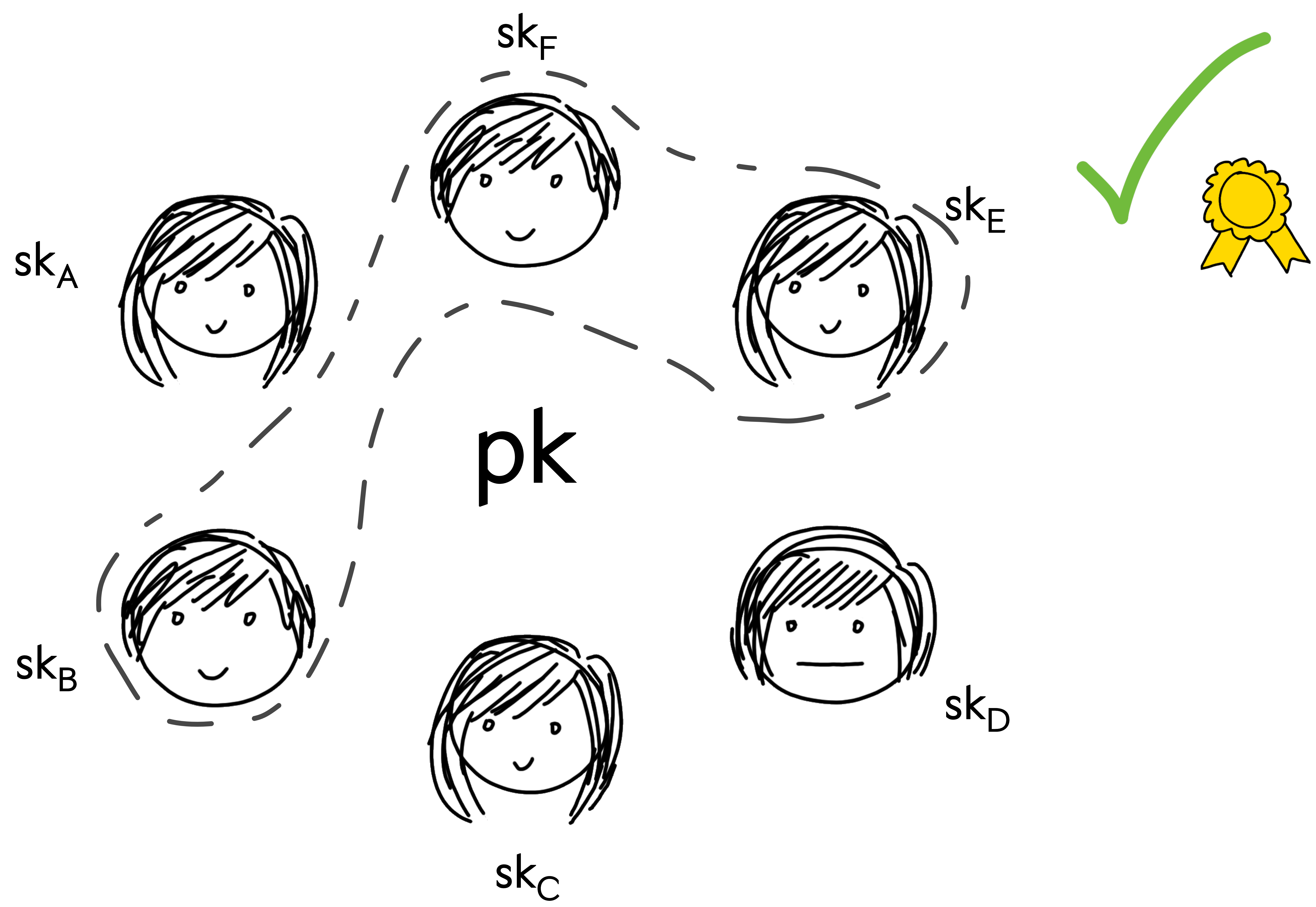

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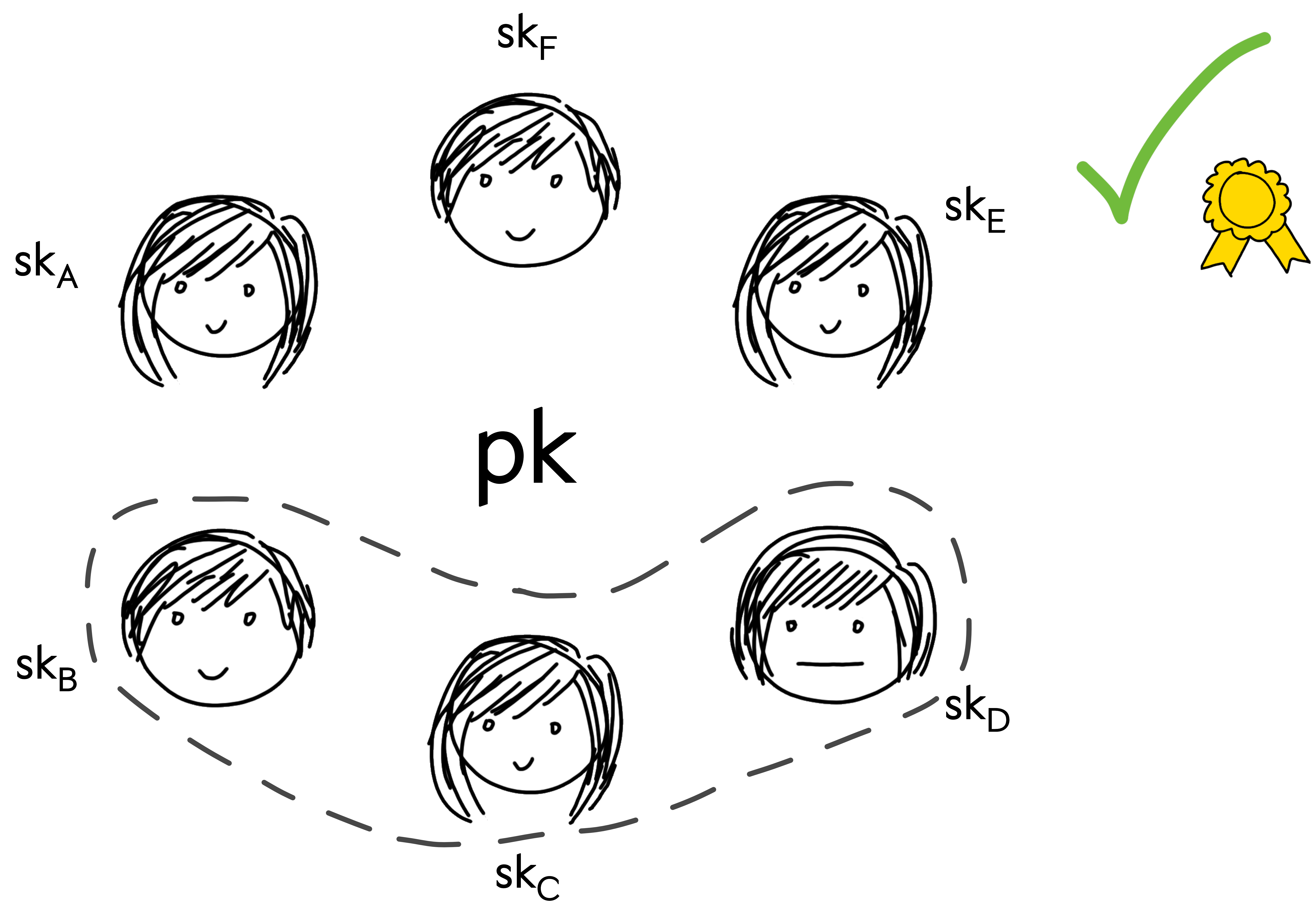

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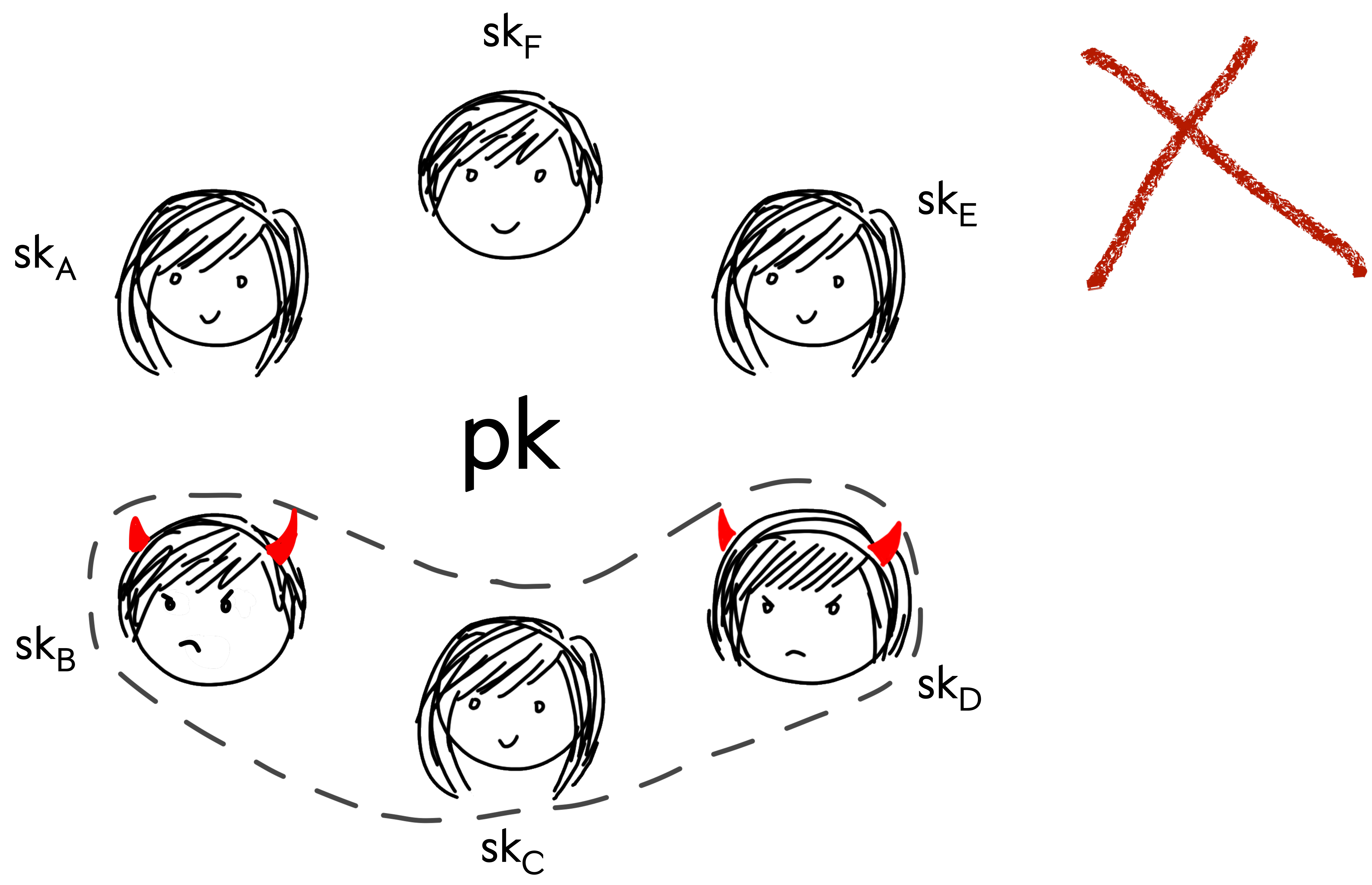

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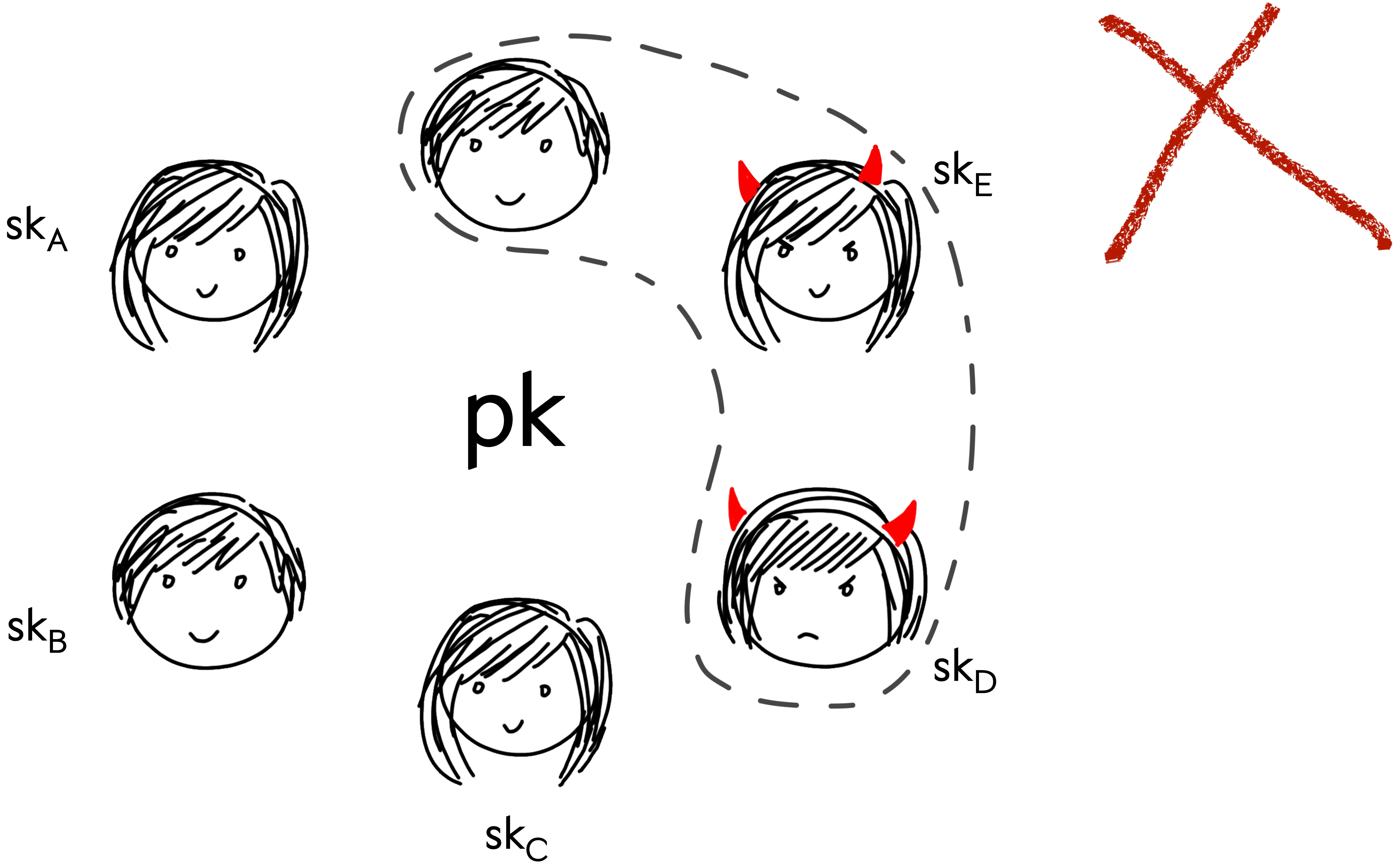

Full Threshold

- Scheme can be instantiated with any $t <= n$
- Adversary corrupts up to t-1 parties

ECDSA

- **E**lliptic **C**urve **D**igital **S**ignature **A**lgorithm
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Notation

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Elliptic curve parameters $G \quad q$

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Elliptic curve parameters G q

Secret values sk k

Notation

Elliptic curve parameters	G	q
Secret values	sk	k
Public values	pk	R

ECDSA Recap

$$R = k \cdot G$$

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$$sign(m, \mathsf{sk}, k) =$$

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$$sign(m, \mathsf{sk}, k) = H(m)$$

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$$sign(m, \mathsf{sk}, k) = H(m) + \mathsf{sk} \cdot r_x$$

ECDSA Recap

x-coordinate of ***R***

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ECDSA Recap

x-coordinate of *R*

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x-coordinate of ***R***

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Non-linearity makes ‘thresholdization’ difficult

Threshold ECDSA

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- **Our work last year** [DKLs18]: 2-of-n ECDSA under native assumptions
- **This work**: Full-Threshold ECDSA under native assumptions

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  - **Pros:**
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    - **Native assumptions** (**CDH** in the same curve)
  - **Con:** Higher bandwidth (**100s of KB/party**)

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  - Subverting checks implies solving **CDH** in the same curve

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- Using OT-MUL is very light on computation, but more demanding of bandwidth than alternative approaches; we argue this is not an issue for most applications
- Our wall clock times (even WAN) are an **order of magnitude** better than the next best concurrent work

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- Security with abort

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- **Setup:** MUL setup, VSS for [sk]
- **Signing:**
  1. Get candidate shares [k], [$1/k$], and $R=k \cdot G$
  2. Compute [sk/k] = MUL([$1/k$], [sk])
  3. Check relations in exponent
  4. Reconstruct sig = [$1/k$]$\cdot H(m)$+[sk/k]

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- **MUL setup**: Pairwise among parties (128 OTs)
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  - Every party Shamir-shares a random secret
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  - Verify in the exponent that parties' shares are on the same polynomial

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- **One approach** (implemented):
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  - Multiplicative to additive shares: log(t)+c rounds
- **Alternative**: [Bar-Ilan&Beaver '89] approach yields constant round protocol (work in progress)

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- **Signing**:

1. Get candidate shares [k], [$1/k$], and $R=k\cdot G$
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Can’t use Diffie-Hellman Exchange for *R*

Check in Exponent

- There are **three** relations that have to be verified

$$[k] \qquad \left[\frac{1}{k}\right] \qquad \left[\frac{\mathsf{sk}}{k}\right]$$

Check in Exponent

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$$[k] \qquad \left[\frac{1}{k}\right] \qquad \left[\frac{\mathsf{sk}}{k}\right]$$

- **Technique**: Each equation is verified in the exponent, using ‘auxiliary’ information that’s already available
- **Cost**: 5 exponentiations, 5 group elements per party independent of party count, and no ZK proofs

Check in Exponent

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- **Task:** verify relationship between [k] and [$1/k$]

Check in Exponent

- **Task:** verify relationship between [*k*] and [1/*k*]

- **Idea**: verify $\left[\frac{1}{k}\right][k] = 1$ by verifying $\left[\frac{1}{k}\right][k] \cdot G = G$

Check in Exponent

Attempt at a solution:

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Public R

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Broadcast $\Gamma_i = \left[\frac{1}{k}\right]_i \cdot R$

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Verify $\sum_{i \in [n]} \Gamma_i = G$

Check in Exponent

Attempt at a solution:

Adversary's contribution

Honest Party's contribution

Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\frac{1}{k_A}\frac{1}{k_h}\right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = G$$

Check in Exponent

Attempt at a solution:

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Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\left(\frac{1}{k_{\mathrm{A}}} + \epsilon \right) \frac{1}{k_h} \right]_i \cdot R$$

Verify

Check in Exponent

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Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\left(\frac{1}{k_A} + \epsilon \right) \frac{1}{k_h} \right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = G + \epsilon k_A \cdot G$$

Check in Exponent

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$$R = k_A k_h \cdot G$$

Broadcast

$$\Gamma_i = \left[\left(\frac{1}{k_A} + \epsilon \right) \frac{1}{k_h} \right]_i \cdot R$$

Verify

$$\sum_{i \in [n]} \Gamma_i = G + \underline{\epsilon k_A \cdot G}$$

Easy for Adv. to offset

Idea: Randomize Target

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Idea: Randomize Target

- Currently we expect $\sum \Gamma_i$ to hit a fixed target *G*
- **Idea**: randomize the multiplication so target is unpredictable
- Compute $\left[\frac{\phi}{k}\right]$ instead of $\left[\frac{1}{k}\right]$
- Reveal ϕ only after *every* other value is committed

Check in Exponent

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Broadcast $$\Gamma_i = \left[\frac{1}{k_A}\frac{1}{k_h}\right]_i \cdot R$$

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Verify $$\sum_{i \in [n]} \Gamma_i = \phi_A \phi_h \cdot G$$

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Attempt at a solution:

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Broadcast $$\Gamma_i = \left[\frac{\phi_A}{k_A} \frac{\phi_h}{k_h} \right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = \Phi$$

Check in Exponent

Adversary's contribution

Honest Party's contribution

Attempt at a solution:

Public

$$R = k_A k_h \cdot G$$

Broadcast

$$\Gamma_i = \left[\left(\frac{\phi_A}{k_A} + \epsilon \right) \frac{\phi_h}{k_h} \right]_i \cdot R$$

Verify

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Verify

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Attempt at a solution:

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Honest Party's contribution

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Broadcast $\Gamma_i = \left[\left(\frac{\phi_A}{k_A} + \epsilon \right) \frac{\phi_h}{k_h} \right]_i \cdot R$

Verify $\sum_{i \in [n]} \Gamma_i = \Phi + \epsilon \underline{\phi_h} k_A \cdot G$

Completely unpredictable

Check in Exponent

There are **three** relations that have to be verified

$$\overset{R}{[k]} \qquad \left[\frac{1}{k}\right] \qquad \left[\frac{\mathsf{sk}}{k}\right]$$

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$$\begin{tikzcd}[k] \arrow[r, bend left, dash, "R"] & \left[\frac{1}{k}\right] \arrow[r, bend left, dash, "\mathsf{pk}"] & \left[\frac{\mathsf{sk}}{k}\right]\end{tikzcd}$$

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$$
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There are **three** relations that have to be verified

Each costs, per party:

$$
\begin{tikzcd}
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Each costs, per party:

-2 exponentiations

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There are **three** relations that have to be verified

Each costs, per party:

-2 exponentiations

-2 field elements

$$\begin{tikzcd}[column sep=huge]
{[k]} \arrow[r, bend left, dash, "R"] \arrow[rr, bend right=40, dash, "{R, \mathrm{pk}}"'] & \left[\dfrac{1}{k}\right] \arrow[r, bend left, dash, "\mathrm{pk}"] & \left[\dfrac{\mathrm{sk}}{k}\right]
\end{tikzcd}$$

Check in Exponent

There are **three** relations that have to be verified

Each costs, per party:

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Two broadcast rounds

R pk

$$[k] \qquad \left[\frac{1}{k}\right] \qquad \left[\frac{\mathsf{sk}}{k}\right]$$

R, pk

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Broadcast linear combination of shares

Dominant Costs

Setup

Signing

Dominant Costs

	Rounds
Setup	
Signing	

Dominant Costs

	Rounds	Public Key
Setup		
Signing		

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup			
Signing			

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup			
Signing			

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5		
Signing			

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5	$520n$	
Signing			

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5	520*n*	21*n* KB
Signing			

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5	520n	21n KB
Signing	log(t)+6		

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5	520n	21n KB
Signing	log(t)+6	5	

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5	520*n*	21*n* KB
Signing	log(*t*)+6	5	<100*t* KB

Dominant Costs

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Journal version (in progress): **8 round signing**

(à la [Bar-Ilan Beaver 89])

Benchmarks

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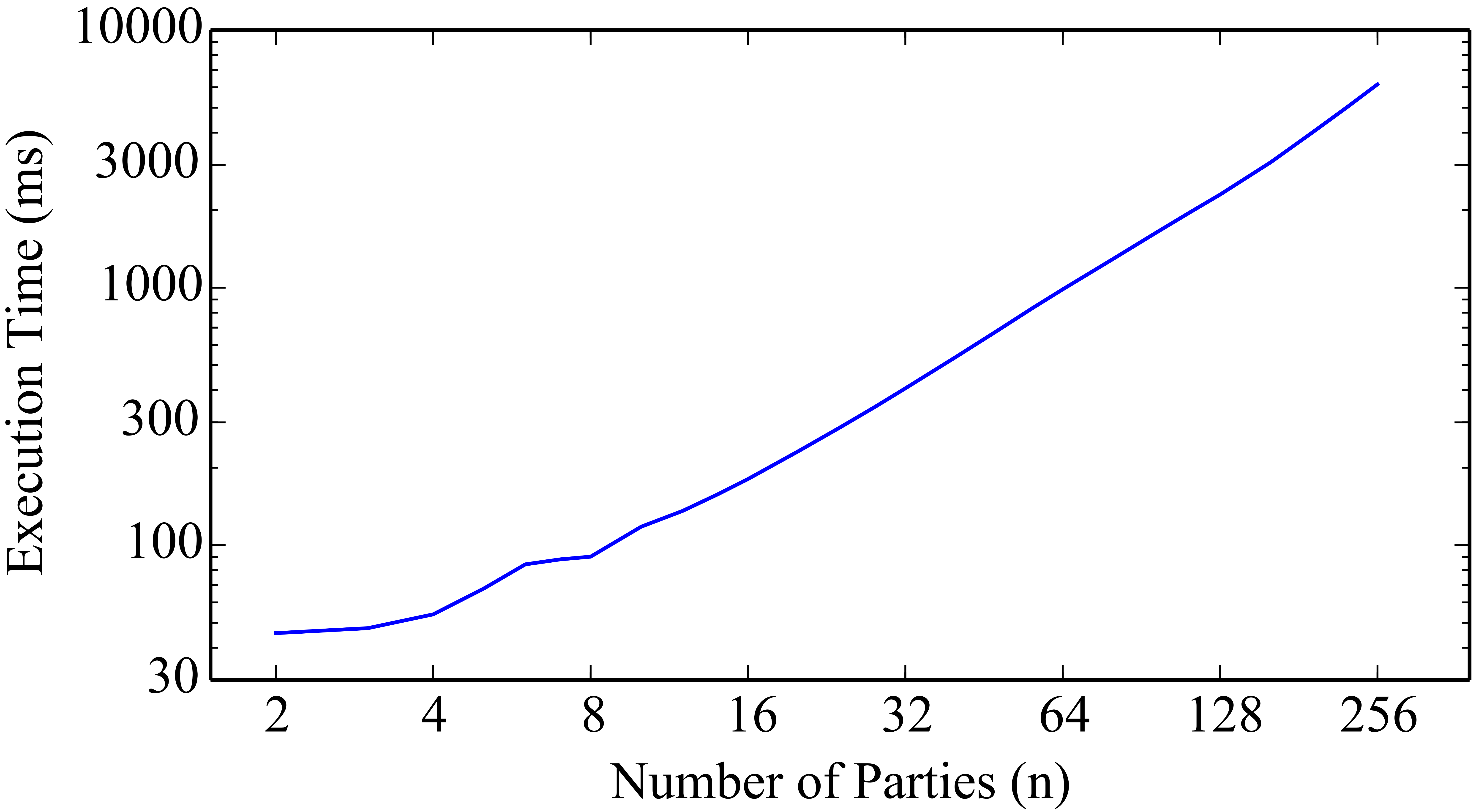
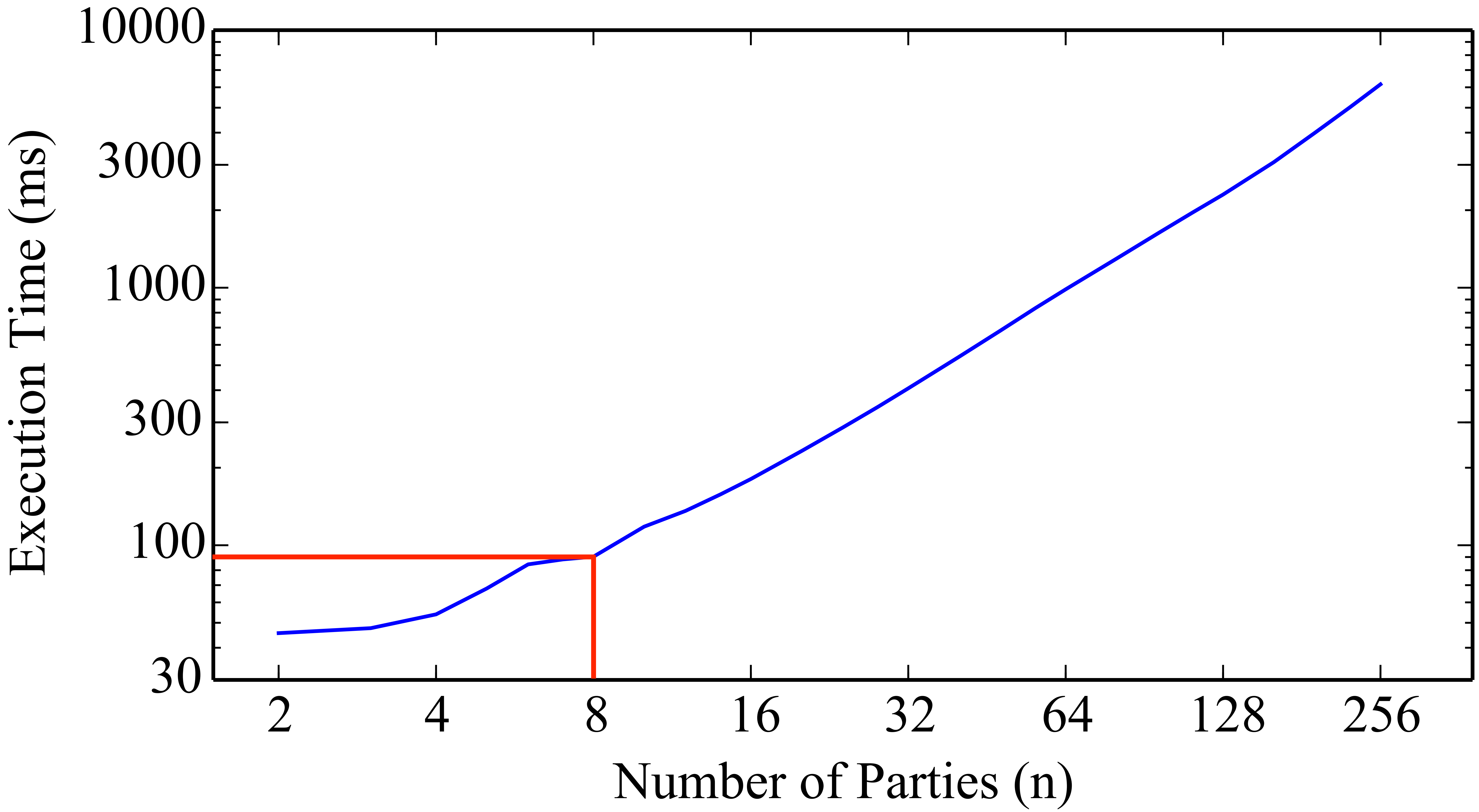
- **Low Power Friendliness**: Raspberry Pi (~93ms for 3-of-3)

LAN Setup



Broadcast PoK (DLog), **Pairwise**: 128 OTs

LAN Setup



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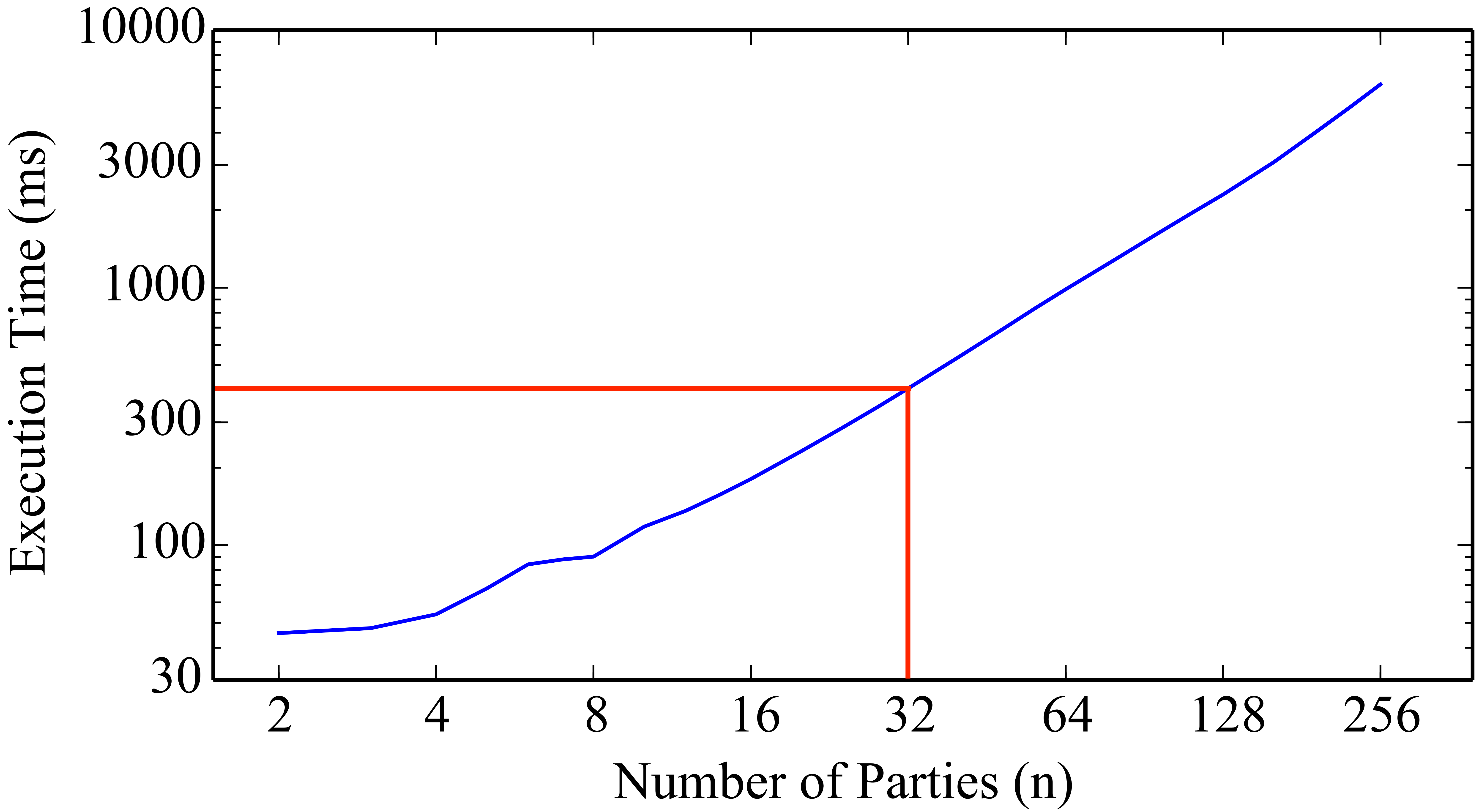

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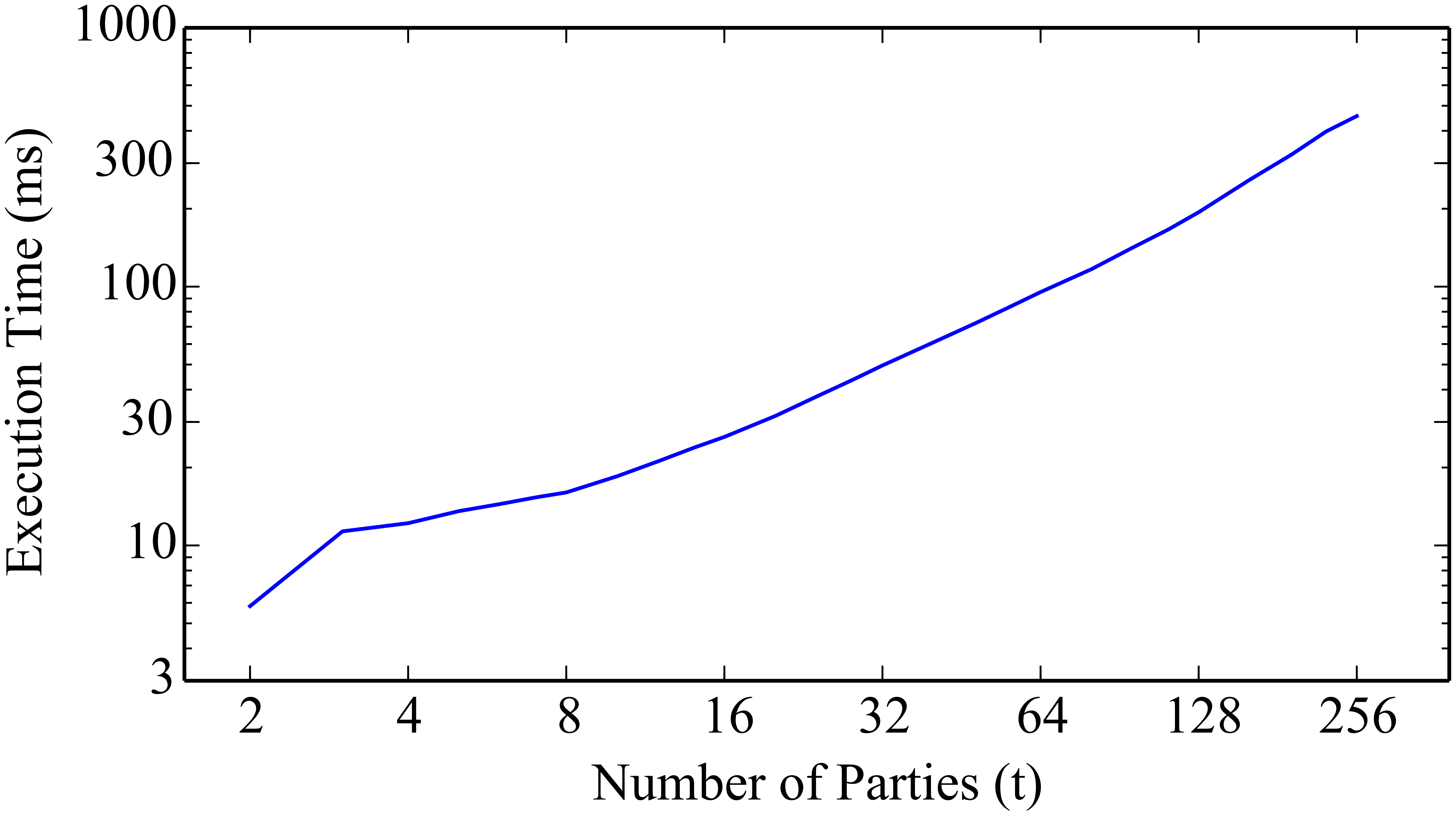
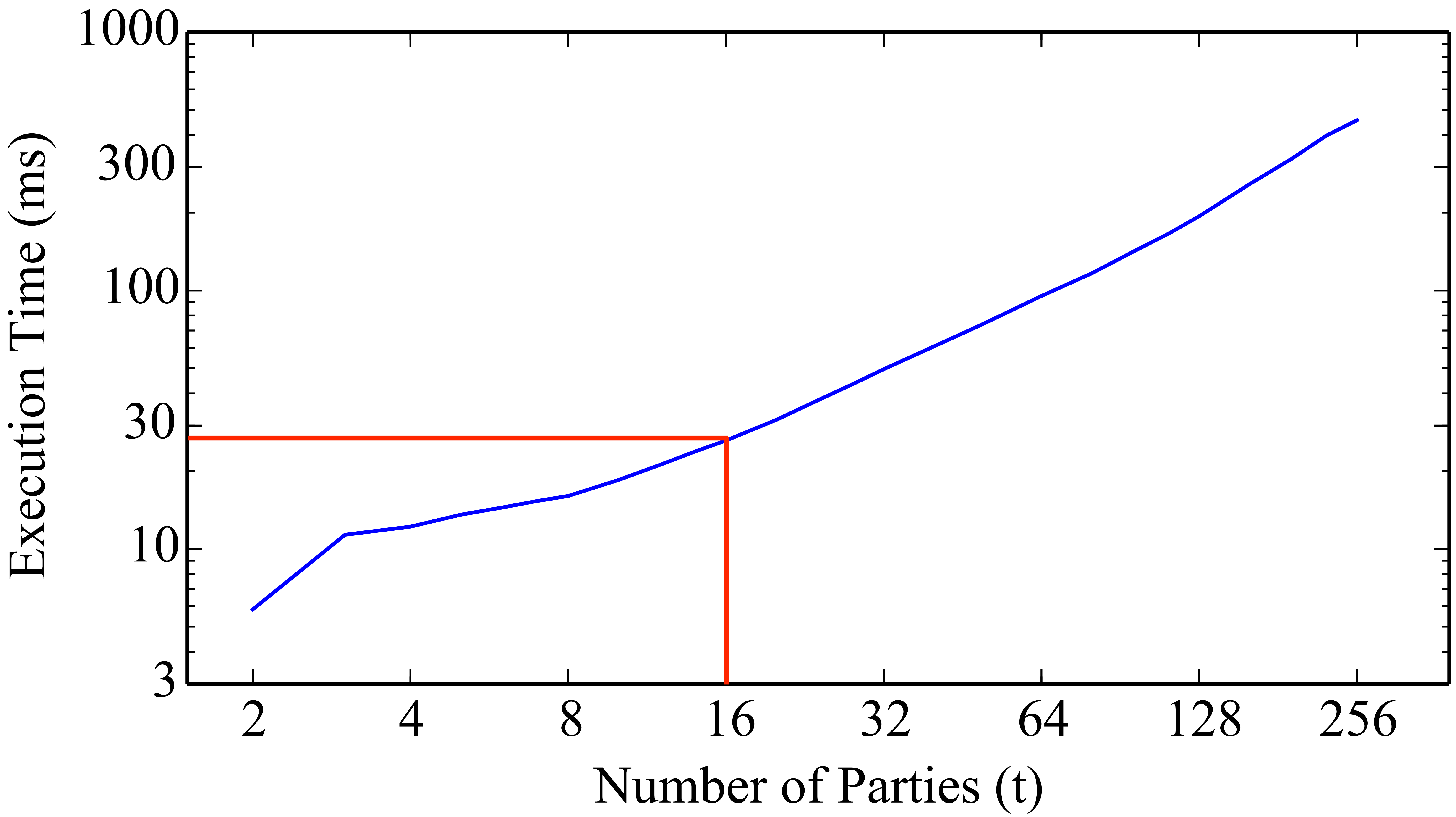


1000
300
100
30
10
3
Execution Time (ms)
2
4
8
16
32
64
128
256
Number of Parties (t)

LAN Signing



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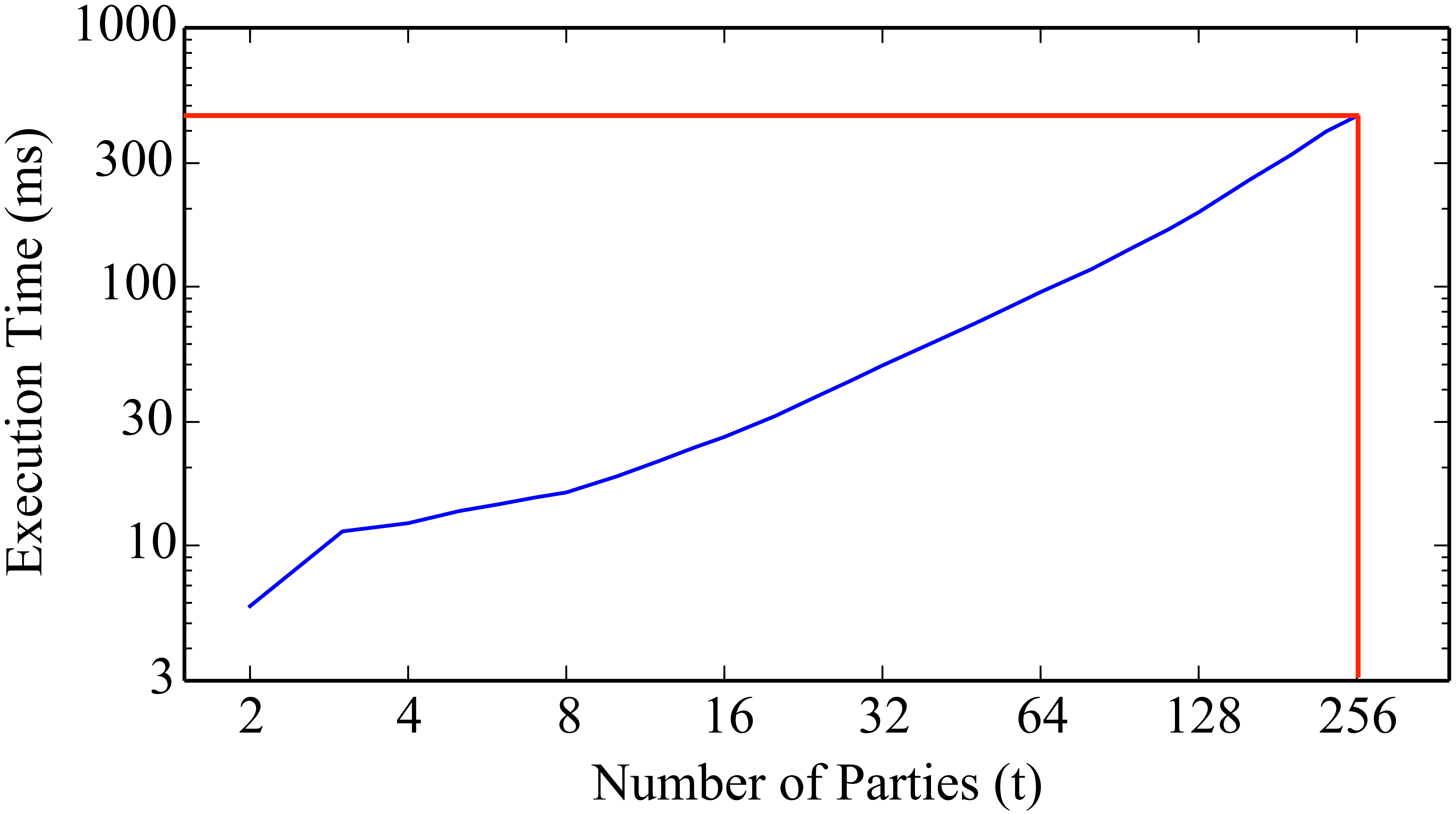

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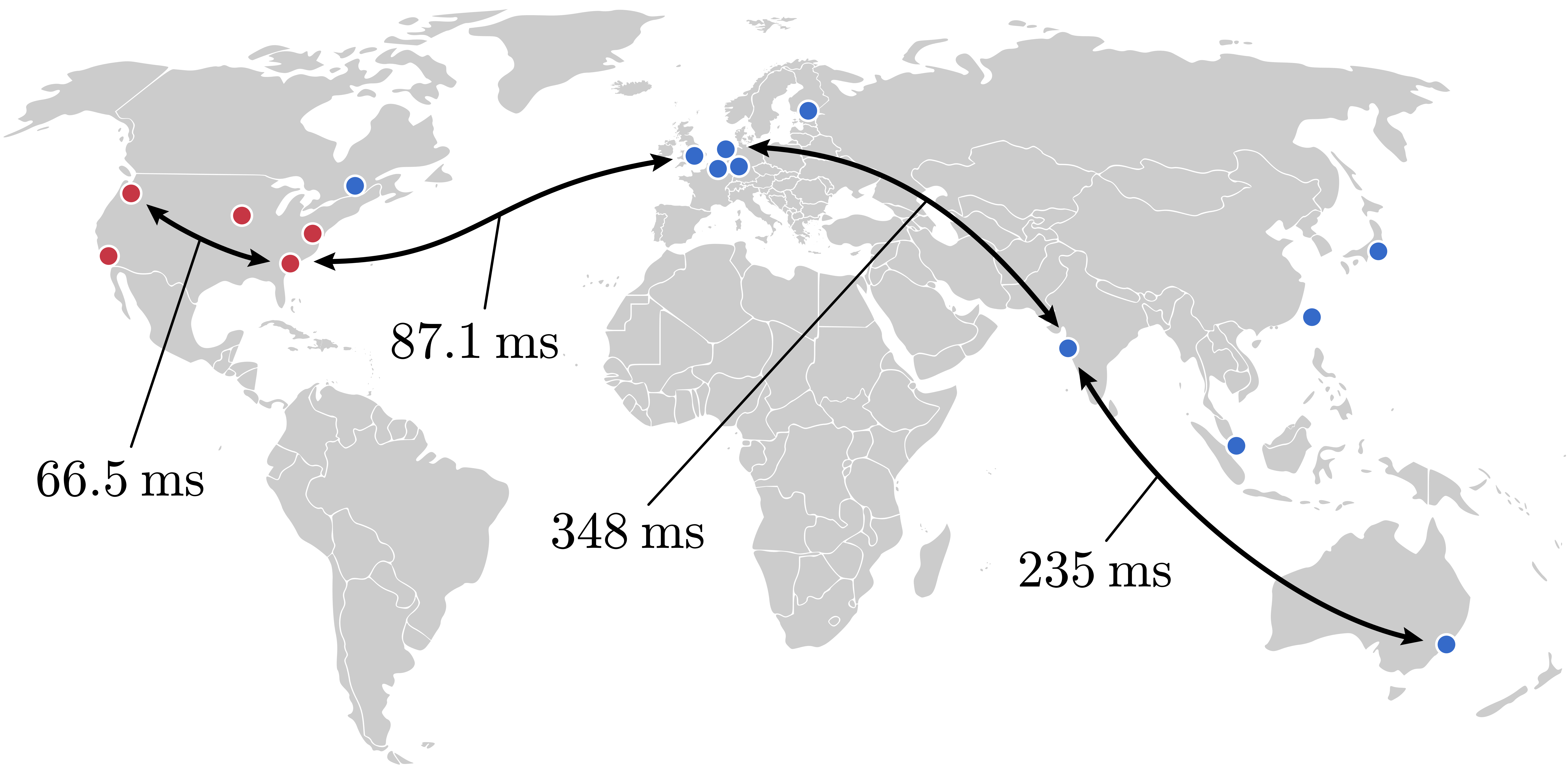
WAN Nodes



WAN Benchmarks

All time values in milliseconds

Parties/Zones	Signing Rounds	Signing Time	Setup Time
5/1	9	13.6	67.9
5/5	9	288	328
16/1	10	26.3	181
16/16	10	3045	1676
40/1	12	60.8	539
40/5	12	592	743
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Comparison

All time figures in milliseconds

Protocol	Signing $t = 2$	Signing $t = 20$	Setup $n = 2$	Setup $n = 20$
This Work	**9.5**	**31.6**	**45.6**	**232**
GG18	77	509	–	–
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Note: Our figures are wall-clock times; includes network costs

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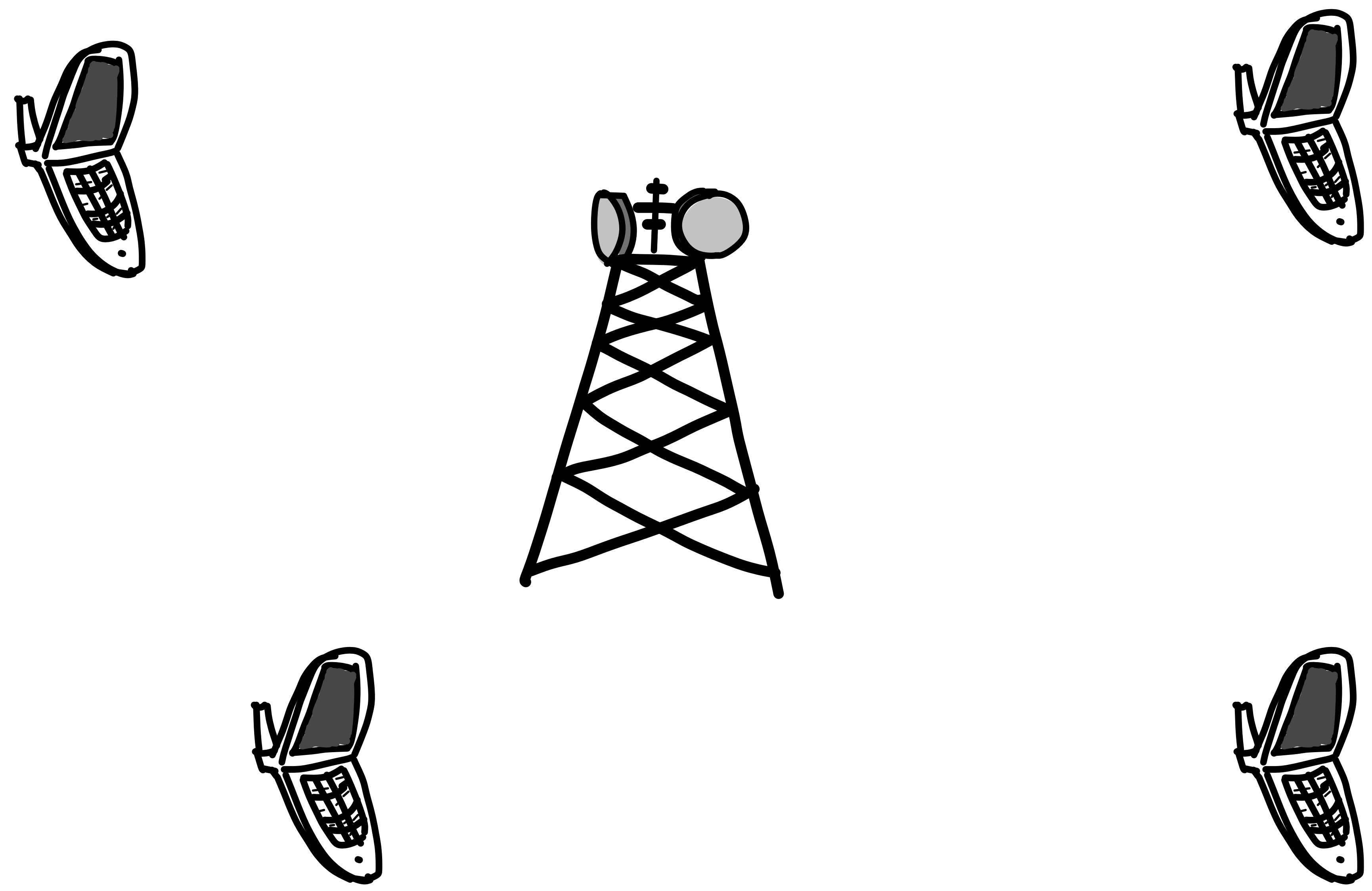

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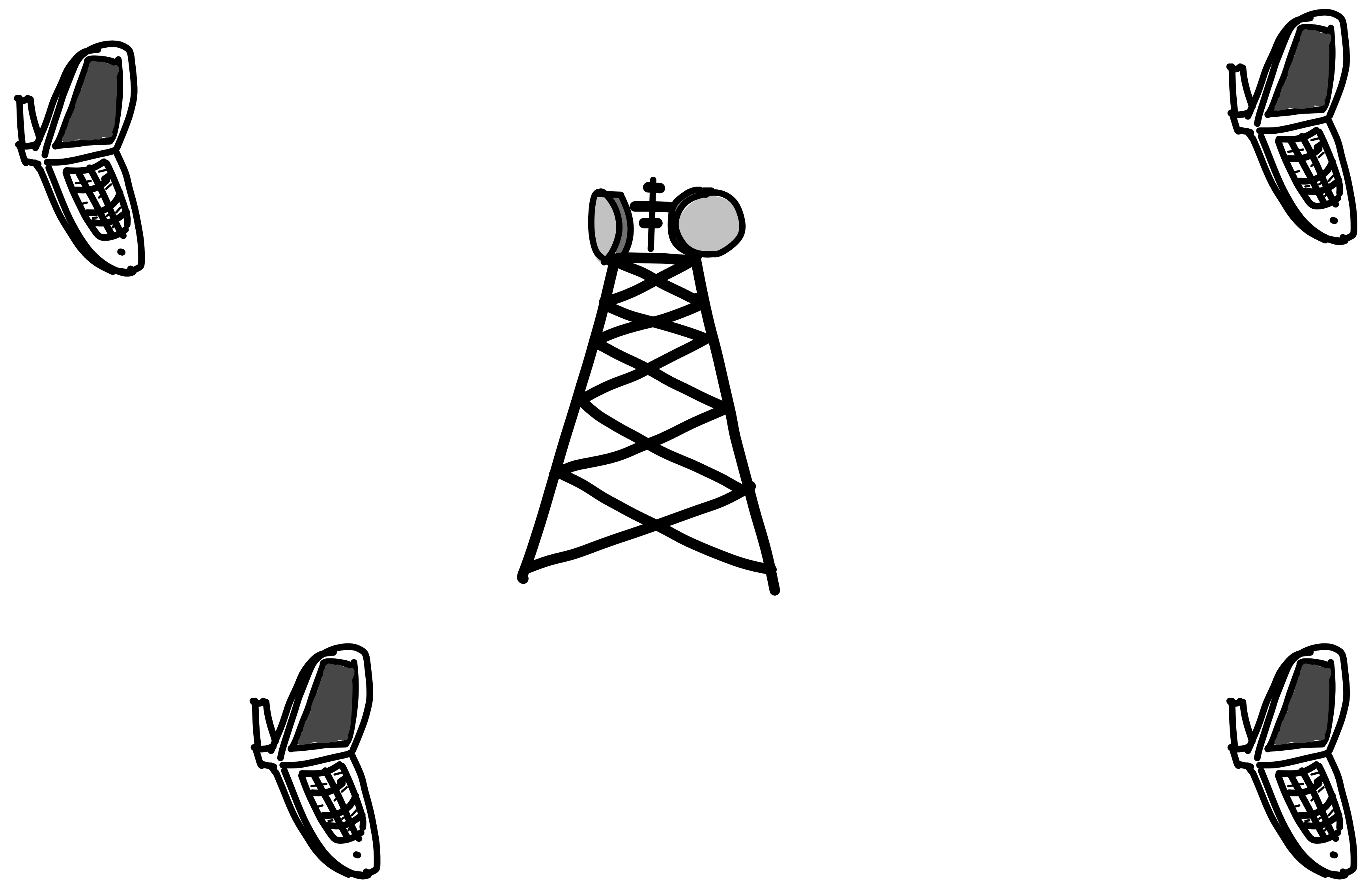

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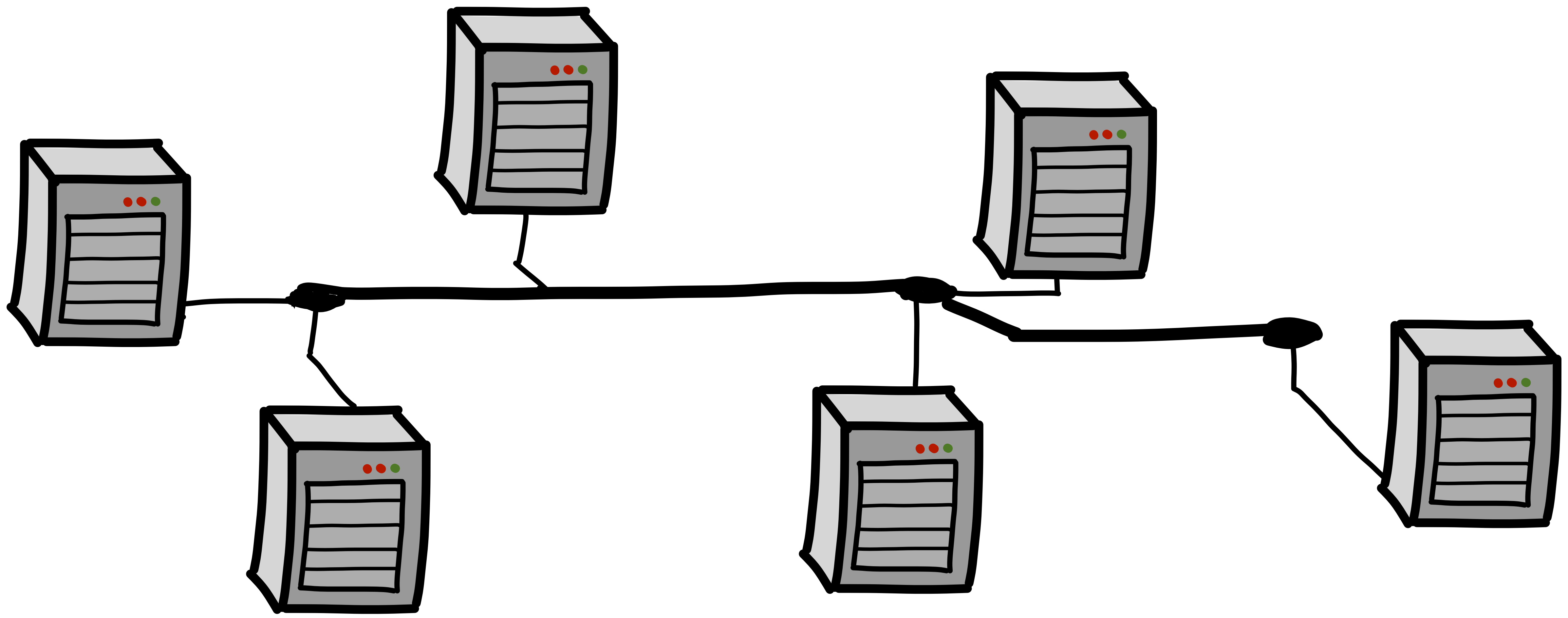

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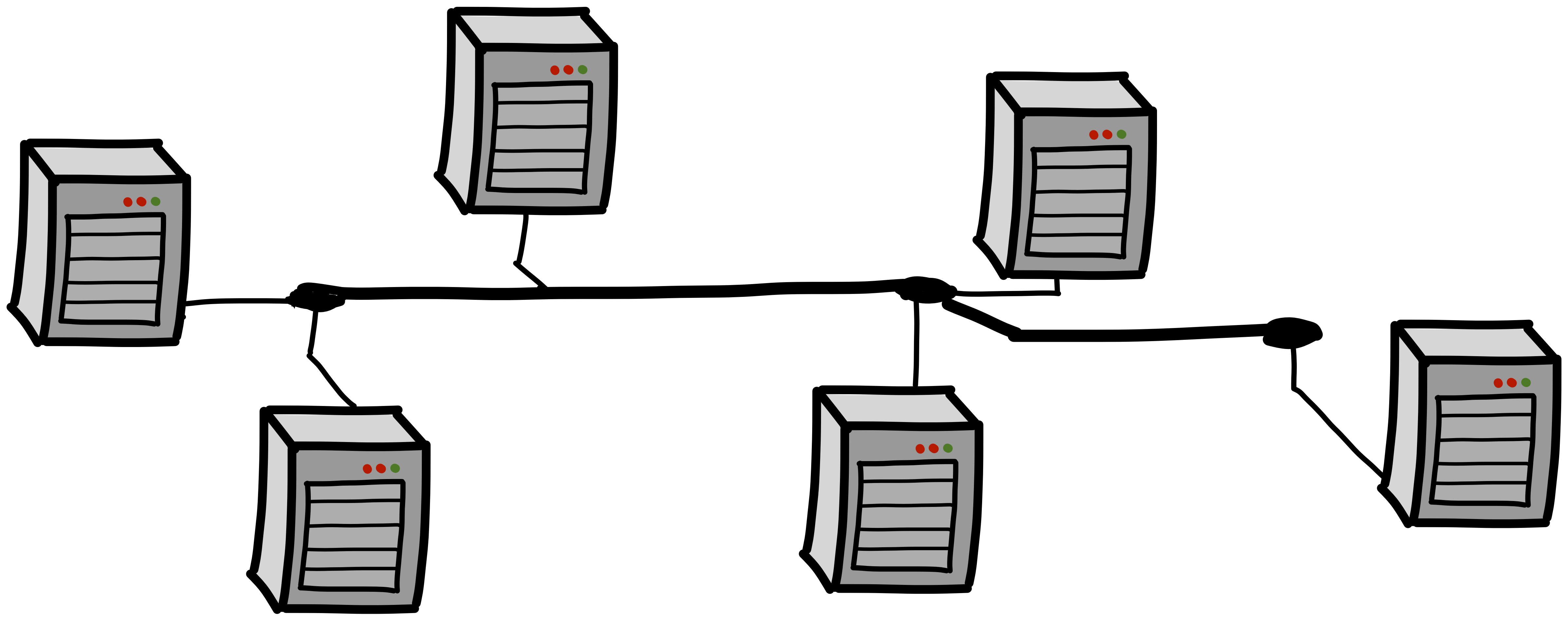

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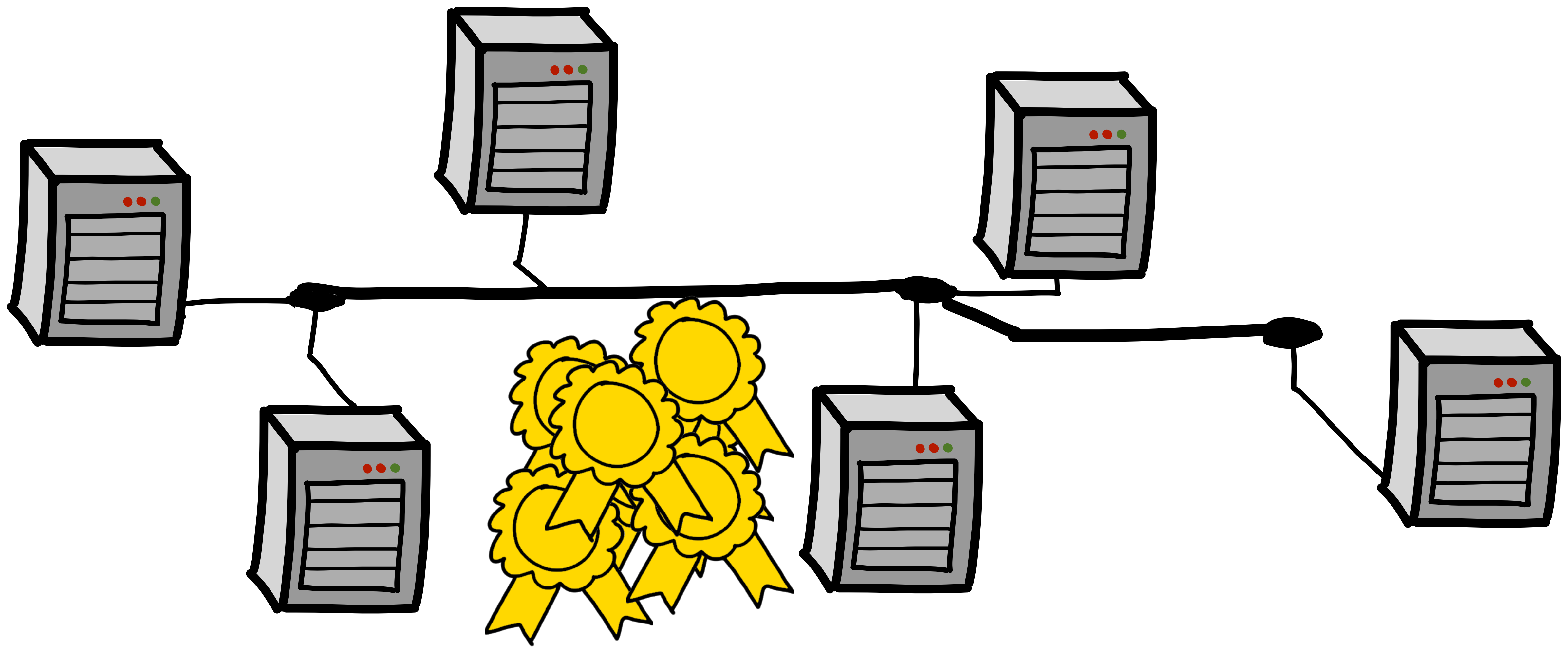

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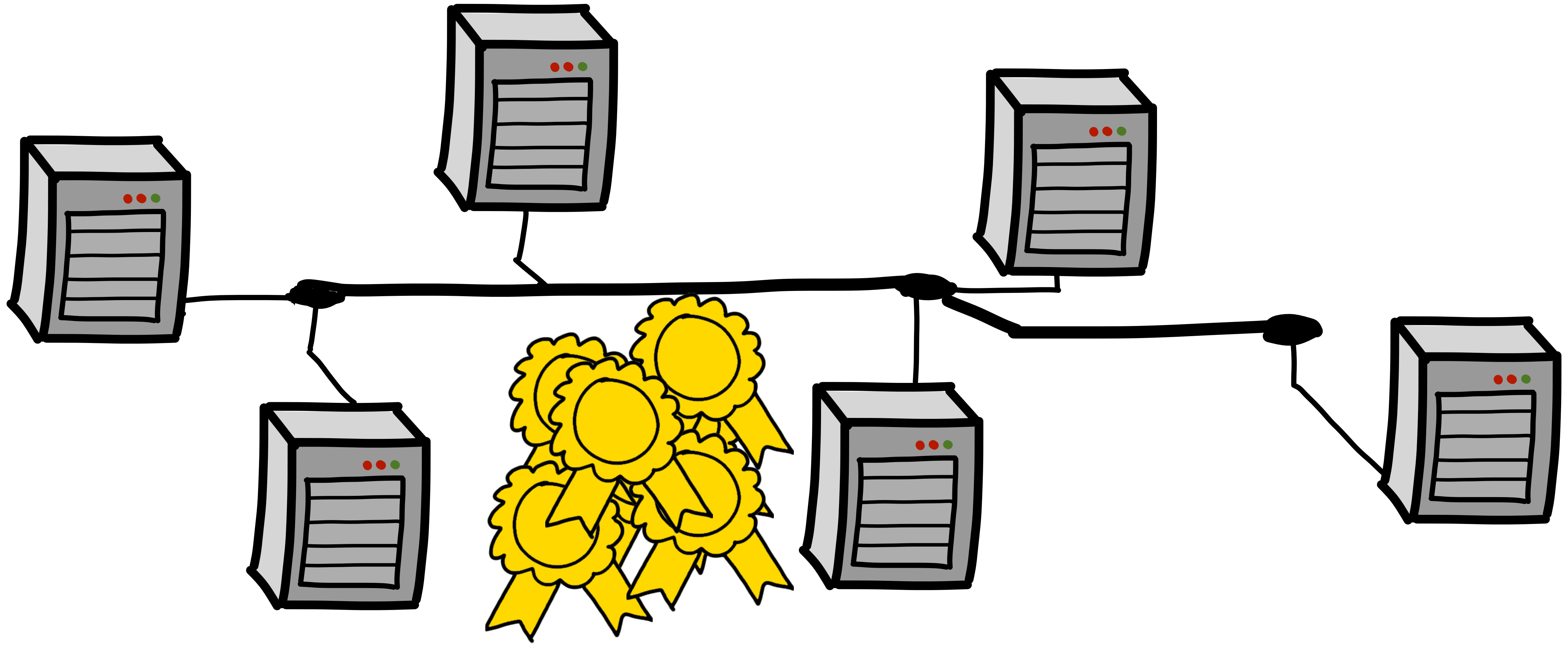

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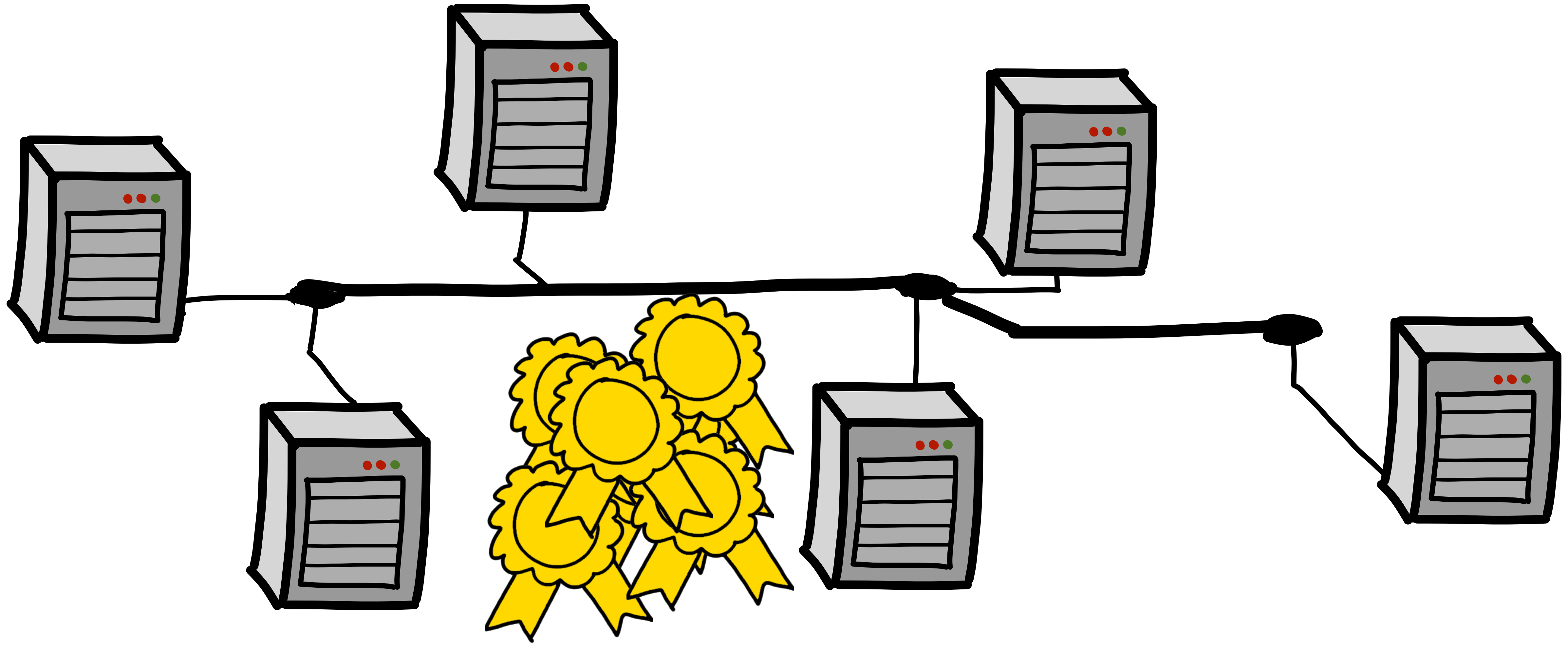

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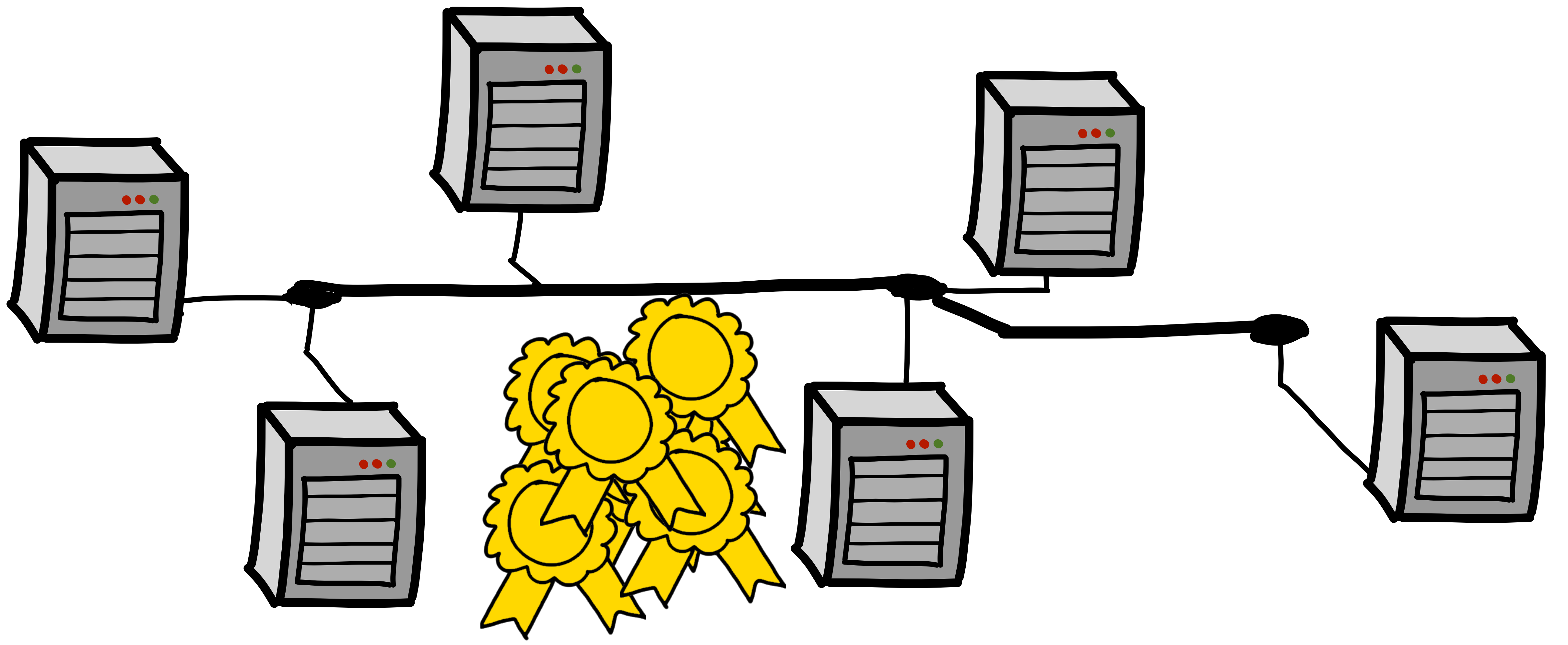

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- Both settings need <500Mb bandwidth

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Thank you!

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