Q-8) Proof :- $\nabla_A tr(ABA^TC) = \underline{CAB + C^TAB^T}$

To prove ~~ea~~ bove equivalence, we can use below linear algebra equations

a function $H(x,y)$ has derivative with respect to a specific point as below.

$$\frac{\partial H(x,y)}{\partial z} = \frac{\partial H}{\partial x}(x,y) \cdot \frac{dx}{dz} + \frac{\partial H}{\partial y}(x,y) \cdot \frac{dy}{dz}. \quad \text{---} (1)$$

from the given ~~equ~~ equation, trace function could be re-written as

$$\underline{tr((AB)(A^TC))} \quad \text{---} (2)$$

from eq (1) & (2) above. $\nabla_A tr(ABA^TC)$ can be re-written as

$$\nabla_A tr(ABA^TC) = \left(\nabla_{AB} tr((AB)A^TC)\right) \cdot \nabla_A tr(AB) + \nabla_A tr((AB)A^TC)$$

Here $x = AB$ & $y = A$

using trace ~~properties~~ derivative properties discussed in class

$$\nabla_A tr(AB) = B^T$$

Similarly $\nabla_{AB} tr((AB)A^TC) = (A^TC)^T = C^TA$

also ~~$\nabla_y H(x,y)$~~ $\frac{\partial H(x,y)}{\partial y} = \nabla_y H(x,y) = C \frac{dy}{dz} = (AB)$

Combining all above equations, we get

$\nabla_A tr(ABA^TC) = C^TA \cdot B^T + C \cdot AB$ which can be written as

$= CAB + C^TAB^T \quad (Q.E.D)$