

Q=6) (a) A piecewise linear machine can be used to classify (1.22) points to specific classes. In order to classify a point x into one of the ~~percept~~ available " C " classes. C different equations of below type are developed:-

$$g_i(x) = w_i^T x + w_i^0 \text{ where } i \text{ is one of the classes}$$

& w^T is the weight vector

In order to classify a point, all these equations are solved with respect to x & g_i which has the greatest value is determined. & the point x is classified as belonging to the class/region pertaining to g_i which has this greatest value.

(b) Decision regions of a linear machine can be non-convex & connected at multiple points. For two points to be non-convex, they should meet below criteria

$x_1 \in R_1$ & $x_2 \in R_2$ i.e. x_1 & x_2 points should belong to two different regions.

but $g_{i=R_1}(x_1) = g_{j=R_2}(x_2)$ i.e. the discriminant functions for R_1 & R_2 for x_1 & x_2 should have same value.

$$g_{i=R_1}(x_1) = w_{R_1}^T x_1 + w_{R_1}^0$$

$$g_{j=R_2}(x_2) = w_{R_2}^T x_2 + w_{R_2}^0$$

to be non-convex $w_{R_1}^T x_1 + w_{R_1}^0 = w_{R_2}^T x_2 + w_{R_2}^0$

~~Any~~ Above equation could be satisfied with multiple (2 of 2)
 values of $w_{R1}^+, w_{R1}^0, w_{R2}^+, w_{R2}^0$ & X_1 & X_2 .

Any ~~for~~ set of values which satisfy above equation will intersect (as a line or hyperplane).

Hence piecewise linear machine's decision regions could be convex or connected at multiple points.

(C) for one dimension, $g_{ij}(x)$ will be ~~set of~~ lines represented with below equations:-

$$g_i(x_1) = g_j(x_1) \Rightarrow \begin{array}{l} \underline{x_1 = 2} \\ g_i(2) = g_j(2) \\ \Rightarrow \boxed{w_i^+ 2 + w_i^0 = 2w_j^+ + w_j^0} \quad (1) \end{array}$$

$$\begin{array}{l} \underline{x_2 = 1} \\ g_i(1) = g_j(1) \\ \Rightarrow \boxed{w_i^+ 1 + w_i^0 = w_j^+ 1 + w_j^0} \quad (2) \end{array}$$

from (1) $2(w_i^+ - w_j^+) + (w_i^0 - w_j^0) = 0$

from (2) $(w_i^+ - w_j^+) + (w_i^0 - w_j^0) = 0$

Above equations form a line equation & will intersect each other at $(w_i^0 - w_j^0)$ point. The plot is shown below

