Q=5) In order to solve (1)

$h(x) = a(x) + b$ for one dimension

we can write the ~~der~~ ~~Sep.~~ feature vector as a matrix

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \\ 1 & x_m \end{bmatrix}$$

where column containing 1's is added to ~~Keep~~ act as coefficient value for b

Hence the target weight vector becomes $\begin{bmatrix} b \\ a \end{bmatrix} = \underline{W}$

Based on discussion in class, the error can be written as

$$E = \begin{bmatrix} h_w(x_1) - y_1 \\ \cdots \\ h_w(x_m) - y_m \end{bmatrix} = \begin{bmatrix} x_1 w \\ \cdots \\ x_m w \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = Xw - y$$

where X & W are defined above & Y is $\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$

Based on formula derived in class, we know that optimal weight vector $W = (X^T X)^{-1} X^T Y$.

$$X^T = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_m \end{bmatrix}$$ (transpose of matrix X above)

$$X^T X = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_m \end{bmatrix} \cdot \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_m \end{bmatrix} = \begin{bmatrix} m & \sum_{i=1}^{m} x_i \\ \sum_{i=1}^{m} x_i & \sum_{i=1}^{m} x_i^2 \end{bmatrix}$$

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$$X^T Y = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_m \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} y_i \\ \sum_{i=1}^{m} x_i y_i \end{bmatrix}$$

$$(X^T X)^{-1} = \frac{1}{\left[m\left(\sum_{i=1}^{m} x_i^2\right)\right] - \left[\left(\sum_{i=1}^{m} x_i\right)\left(\sum_{i=1}^{m} x_i\right)\right]} \begin{bmatrix} \sum_{i=1}^{m} x_i^2 & -\sum_{i=1}^{m} x_i \\ -\sum_{i=1}^{m} x_i & m \end{bmatrix}$$

basedon $X^T X$ computed above.

$$\therefore W = \begin{bmatrix} b \\ a \end{bmatrix} = (X^T X)^{-1} \cdot (X^T Y)$$

$$= \frac{1}{\left[m\left(\sum_{i=1}^{m} x_i^2\right)\right] - \left[\left(\sum_{i=1}^{m} x_i\right)\left(\sum_{i=1}^{m} x_i\right)\right]} \begin{bmatrix} \sum_{i=1}^{m} x_i^2 & -\sum_{i=1}^{m} x_i \\ -\sum_{i=1}^{m} x_i & m \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{m} y_i \\ \sum_{i=1}^{m} x_i y_i \end{bmatrix}$$

$$\begin{bmatrix} b \\ a \end{bmatrix} = \frac{1}{\left[m\left(\sum_{i=1}^{m} x_i^2\right)\right] - \left[\left(\sum_{i=1}^{m} x_i\right)\left(\sum_{i=1}^{m} x_i\right)\right]} \begin{bmatrix} \sum_{i=1}^{m} y_i \sum_{i=1}^{m} x_i^2 - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} x_i y_i \\ m \sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i \end{bmatrix}$$

$$\therefore b = \frac{\sum_{i=1}^{m} y_i \sum_{i=1}^{m} x_i^2 - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} x_i y_i}{\left(m \sum_{i=1}^{m} x_i^2\right) - \left(\sum_{i=1}^{m} x_i\right)^2}$$

$$\& \quad a = \frac{m \sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{\left(m \sum_{i=1}^{m} x_i^2\right) - \left(\sum_{i=1}^{m} x_i\right)^2}.$$