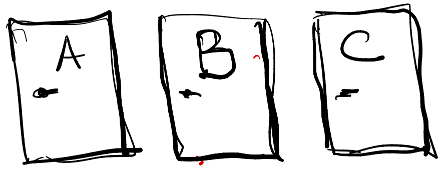
Monty Hall Puzzle



3 Doors: 2 empty
1 with treasure

- You choose one at random initially (A)
- Monty opens a diff door (B) without treasure at random (if many)
- Do you stick to initial door (A), or switch to the other door (C)?

intuition: why would C be more likely than A? A → 1/3 chance

- Monty does not give info on door A. But it does (impl) on door C C → 2/3 chance

R.V R = door with the treasure ($\Omega(R) = \{A, B, C\}$ initially uniform)

M = door that Monty opens: not the one picked, not the one with treasure

Say initial pick is $\boxed{A}$ $\Rightarrow$ $\Omega(M) = \{B, C\}$ OBSERVE M=B

before M=R.v

after M=B var

$$P(R=A \mid \underbrace{M=B}_{\text{given}}) = \frac{P(M=B \mid R=A)\,\overbrace{P(R=A)}^{\text{init prior}}}{\underbrace{P(M=B)}_{\text{across all choices by player}}} = \frac{1/2 \cdot 1/3}{1/2} = \boxed{1/3}$$

$$P(R=B \mid M=B) = \frac{P(M=B \mid R=B) \cdot P(R=B)}{P(M=B)} = \frac{0 \cdot 1/3}{1/2} = 0$$

0 as Monty never opens the door with treasure

$$P(R=C \mid M=B) = \frac{P(M=B \mid R=C) \cdot P(R=C)}{P(M=B)} = \frac{1 \cdot 1/3}{1/2} = \boxed{\frac{2}{3}}$$

$= 1 - P(R=A \mid M=B)$

⟹ better to switch to door C, prob treasure = $\frac{2}{3}$

$$P(M=B) = P(M=B \mid R=A) \cdot P(R=A) + \qquad \tfrac{1}{2} \cdot \tfrac{1}{3}$$

$$+ P(M=B \mid R=B) \cdot P(R=B) \qquad 0$$

$$+ P(M=C \mid R=C) \cdot P(R=C) \qquad 1 \cdot \tfrac{1}{3}$$

$$\tfrac{1}{6} + \tfrac{1}{3} = \tfrac{1}{2}$$