

Monty Hall Puzzle



3 Doors: 2 empty
1 with treasure

- You choose one at random initially (A)
- Monty opens a diff door (B) without treasure at random (if want)
- Do you stick to initial door (A), or switch to the other door (C)?

intuition: why would C be more likely than A? $A \rightarrow \frac{1}{3}$ chance

- Monty does not give info on door A. But it does (impl) $C \rightarrow \frac{2}{3}$ chance on door C

R.V R = door with the treasure ($\Omega(R) = \{A, B, C\}$ initially uniform)

M = door that Monty opens: not the one picked, not the one with treasure

Say initial pick is $A \Rightarrow \Omega(M) = \{B, C\}$ OBSERVE $M=B$

$$P(R=A | M=B) = \frac{P(M=B | R=A) \cdot P(R=A)}{P(M=B)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

Annotations: "given" above R=A; "init prior" above P(R=A); "before M=R.V" below OBSERVE; "after M=B var" below M=B; "across all choices by player" below P(M=B); "1/2" below 1/2 in numerator; "1/2" below 1/2 in denominator.

$$P(R=B | M=B) = \frac{P(M=B | R=B) \cdot P(R=B)}{P(M=B)} = \frac{0 \cdot \frac{1}{3}}{\frac{1}{2}} = 0$$

Annotations: "0 as Monty never opens the door with treasure" with an arrow pointing to the 0; "1/3" above 1/3 in numerator; "1/2" below 1/2 in denominator.

$$P(R=C | M=B) = \frac{P(M=B | R=C) \cdot P(R=C)}{P(M=B)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Annotations: "1 - P(R=A | M=B)" with an arrow pointing to the equation; "1" above 1 in numerator; "1/3" above 1/3 in numerator; "1/2" below 1/2 in denominator.

$\Rightarrow$ better to switch to door C, prob treasure = $\frac{2}{3}$

$$\begin{array}{rcl}
 P(M=B) & = & P(M=B|R=A) \cdot P(R=A) + \quad \frac{1}{2} \cdot \frac{1}{3} \\
 & + & P(M=B|R=B) \cdot P(R=B) \quad 0 \\
 & + & P(M=B|R=C) \cdot P(R=C) \quad 1 \cdot \frac{1}{3} \\
 \hline
 & & \frac{1}{6} + \frac{1}{3} = \frac{1}{2}
 \end{array}$$