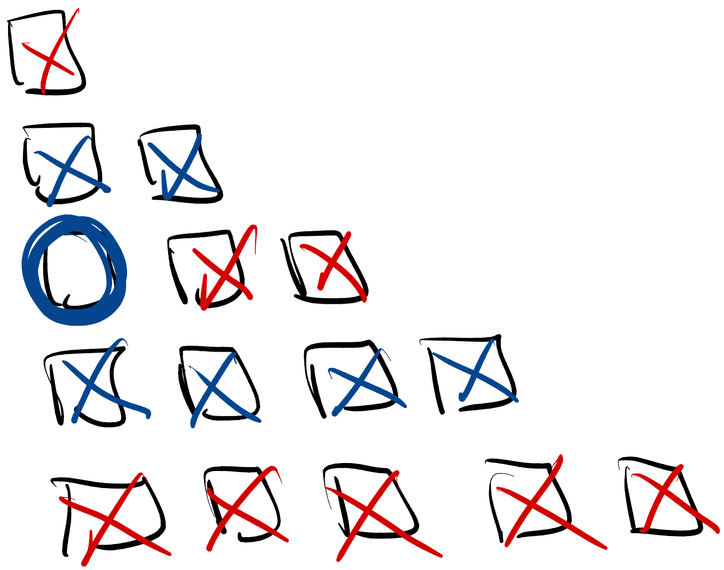
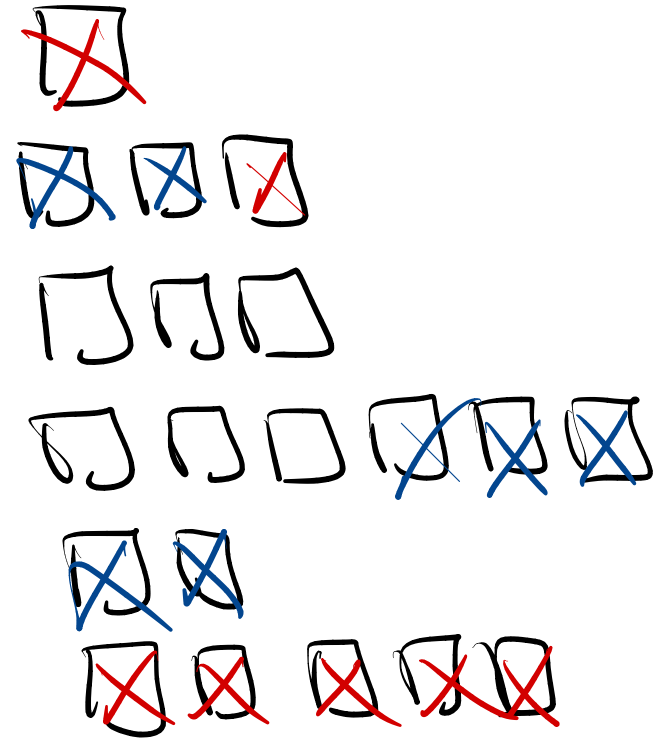
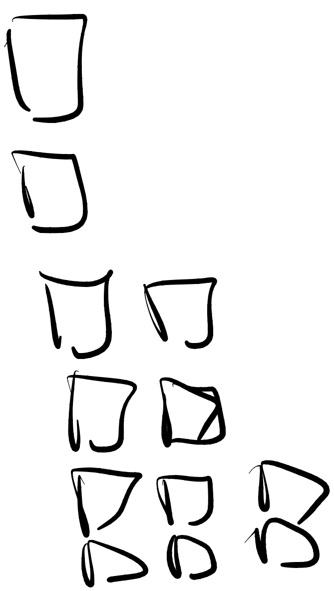




hint 1

Mirror board



Strategy player B:

maintain "mirror" invariant

binary epresentation

$n = 3$ bits

Unsigned representation (positives)

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

Range $[0 : 7]$

$\underline{0}\ \underline{0}\ \underline{0}$ min = 0

$\underline{1}\ \underline{1}\ \underline{1}$ max = 7

$5 = 4 + 1 = 2^2 + 2^0$

$= \boxed{\underline{1}\ \underline{0}\ \underline{1}}$

$2^2 \quad 2^1 \quad 2^0$

$235_{(10)} = \boxed{2} \cdot 10^2 + \boxed{3}\, 10^1 + \boxed{5}$

$= \boxed{1} \cdot 2^7 + \boxed{1}\, 2^6 + \boxed{1}\, 2^5 + \boxed{0} \cdot 2^4 + \boxed{1}\, 2^3 + \boxed{0}\, 2^2 + \boxed{1}\, 2^1 + \boxed{1}\, 2^0$

128 +
64
32
8

100000
010000
01000
0100
10
1

111111 max (on 6 bits)

Unsigned 6 bits

Range $[0 : 2^6-1]$

$= [0 : 63]$

$x = 2$

$x^a \cdot x^b = x^{a+b}$

Geometric progression

Th $x^0 + x^1 + x^2 + \cdots x^{n-1} = \frac{\boxed{x^n - 1}}{x-1}$ if $x \neq 1$

$x=2$: $2^0 + 2^1 + 2^2 + \cdots 2^{n-1} = 2^n - 1$

<u>proof</u> $(x^0 + x^1 + x^2 \cdots + x^{n-1})(\boxed{x} - \boxed{1}) =$

$= \cancel{x^1} + \cancel{x^2} + \cancel{x^3} + \cdots + \cancel{x^{n-1}} + x^n$

$- x^0 - \cancel{x^1} - \cancel{x^2} - \cancel{x^3} \cdots - \cancel{x^{n-1}}$

$= x^n - 1$

$x = 10 \qquad 10^1 + 10^2 + 10^3 + \dots + 10^{n-1} = \frac{10^n - 1}{9}$

$9 \cdot 10^1 + 9 \cdot 10^2 + 9 \cdot 10^3 + \dots 9 \cdot 10^{n-1} = 10^n - 1$

max

9

90

900

9000

$9 \dots 0 \qquad 9 \cdot 10^{n-1}$

$999 \dots 9$ = max Range in base 10 $[0 : 10^n - 1]$

$10^n - 1$

# Binary Search (concept)

6 bits Unsigned Range [0 : 63]

| 0 | 1 | 0 | 1 | 1 | 0 |
|---|---|---|---|---|---|
| $b_5$ | $b_4$ | $b_3$ | $b_2$ | $b_1$ | $b_0$ |

val = 22

---

1) m = middle of range

$= \frac{0+63}{2} \simeq 32$

is val < 32 $\boxed{\text{Yes!}}$

$\Rightarrow$ move Low Range = [0 : 31]

is $b_5 = 0$? $\boxed{\text{Yes}}$

Range = 0 _ _ _ _ _

---

2) $m = \frac{0+31}{2} \simeq 16$

is val < 16? NO

$\Rightarrow$ move high Range = [16 : 31]

is $b_4 = 0$? NO

Range = 01 _ _ _ _

---

3) $m = \frac{16+31}{2} \simeq 24$

is val < 24? Yes!

$\Rightarrow$ move Low Range = [16 : 23]

is $b_3 = 0$?

Range 010 _ _ _

---

4) $m = \frac{16+23}{2} \simeq 20$

is Val < 20? NO!

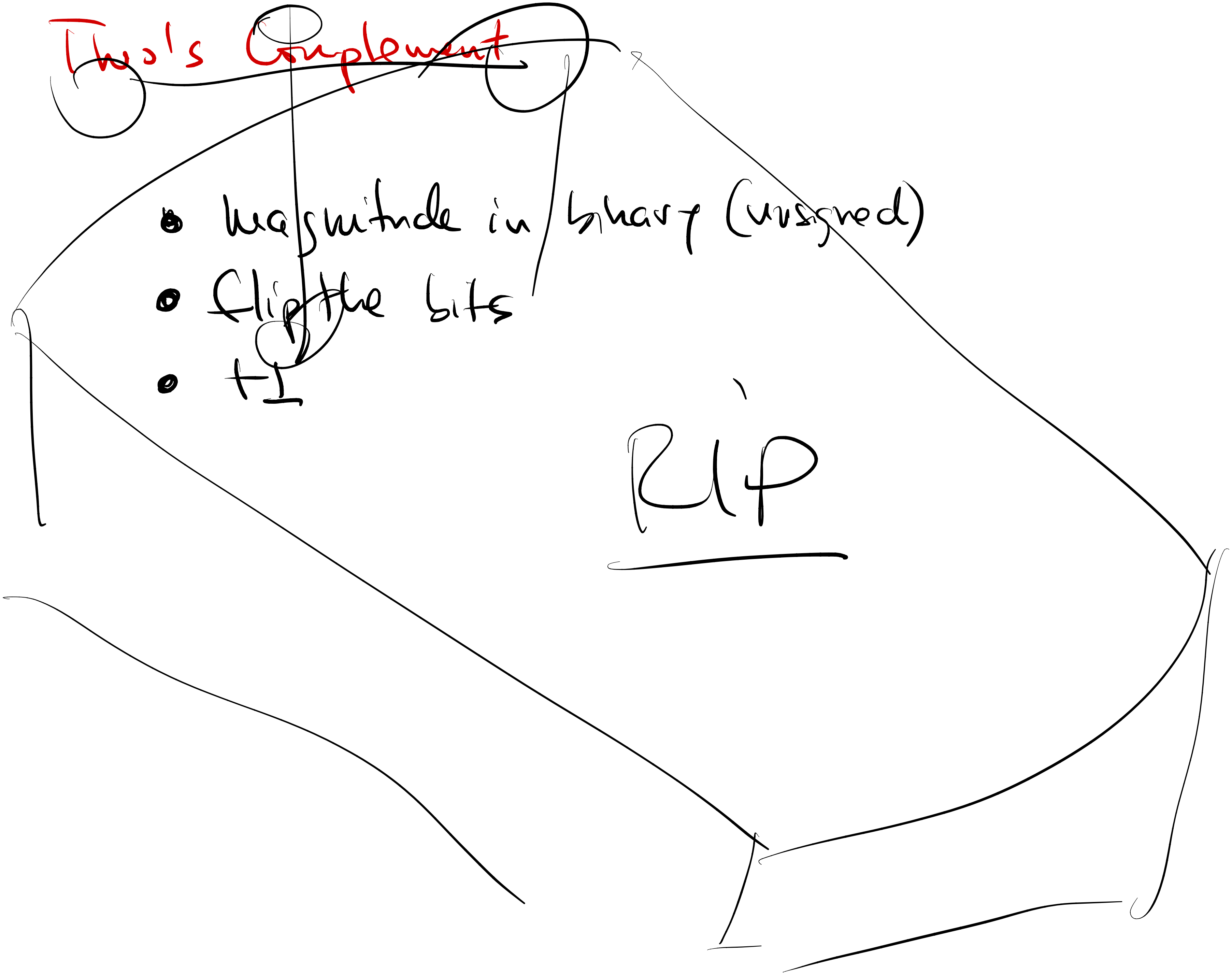
move high Range = [20 : 23]

is $b_2 = 0$? No

Range 0101 _ _

# Two's Complement

- magnitude in binary (unsigned)
- flip the bits
- +1



Poor's man two's complement 4 bits.

$$-5 = ? \quad \boxed{1 \quad 0 \quad 1 \quad 1}$$

$$+5 = \underline{0} \quad \underline{1} \quad \underline{0} \quad \underline{1}$$

---

$$+0 = 0 \quad 0 \quad 0 \quad 0$$

3bits unsigned R=[0:7]

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 7 |
| 1 | 1 | 0 | 6 |
| 1 | 0 | 1 | 5 |
| 1 | 0 | 0 | 4 |
| 0 | 1 | 1 | 3 |
| 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |

+1

3bits R=[

Signed two's complement

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | -1 |
| 1 | 1 | 0 | -2 |
| 1 | 0 | 1 | -3 |
| 1 | 0 | 0 | -4 |

+1

+1

sign 1 = negative
0 = positive

Two's complem 6 bits __ __ __ __ __ $R = [-32 : 31]$

negative

$-29 = 1 \quad \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{1}$

$-2^5 \qquad\qquad 2^1 \quad 2^0$

$-32$

$-12 \quad \underline{1} \quad \underline{1} \quad \underline{0} \quad \underline{1} \quad \underline{0} \quad \underline{0}$

$-32 \qquad 2^4 \qquad 2^2$

$-2^5$

$-32 + 20 = -12$

20

formally 2's complement DEF K+1 bits

$\underline{b_k}\, \underline{b_{k-1}}\, \underline{b_{k-2}} \ldots \underline{b_1}\, \underline{b_0} =$

$= -b_k \cdot 2^k + b_{k-1} \cdot 2^{k-1} + \ldots + b_1 \cdot 2^1 + b_0 \cdot 2^0$

$\underline{1}\,\underline{1}\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,\underline{1}$ $\underset{\text{unsigned}}{=}$ $2^6 + 2^5 + 2^3 + 2^2 + 2^0$

$2^6$ $\quad$ $2^2$

$= 64 + 32 + 8 + 4 + 1 =$

$= 109$ ?

$1101101$ $\quad$ 2's compl 7 bits

$-2^6 + 2^5 + 2^3 + 2^2 + 2^0$

$= -64 + 32 + 8 + 4 + 1$

$= -19$

Bool Rule

$+19 = \underline{0}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{1}\ \underline{1}$

flip $\quad 1\ \boxed{1\ 0\ 1\ 1\ 0\ 0}$

$+1 \quad 1\ 1\ 0\ 1\ 1\ 0\ 1$ ✓