Deadline: October 23 at 8pm Eastern

CS1802 Recitation 6

Fall 2020

October 20 - 23, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 6pm Eastern timezone. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that final deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

Question 1.

In the game of craps, you roll two six-sided dice. If the sum of the two outcomes is 7 or 11, you win! If the sum of the two outcomes is 2, 3, or 12, you lose. :(Otherwise, it's a draw. The first time Laney played this game in a casino she got really annoyed because apparently there are 3 ways to lose and only two ways to win. So unfair! Right??

What is the probability of:

(a) winning?

(b) losing?

(c) drawing?

What do you get if you add up the three numbers obtained in (a), (b), (c)?

Question 2.

Suppose we define a *weighted* six-sided die to be rigged such that the outcome of rolling a 5 is twice as likely as any other outcome. All the remaining outcomes are equally likely.

(a) What is the probability of rolling a 5 on a weighted die? What is the probability of rolling any other number?

(b) Suppose you roll two weighted dice. Die no. 1 comes up five. What is the probability that their sum is greater than 7, given this new information?

(c) Suppose you roll a weighted die 3 times. What is the probability of seeing exactly...

- 3 fives?
- 2 fives?
- 1 five?
- 0 fives?

What can you say about the sum of those four numbers?

(d) Let X be the random variable assigned to the number of fives that we see. What is the expected value of X if the die is rolled 3 times?

Question 3.

Please watch this important video before starting this question:

https://www.youtube.com/watch?v=naQeNNaZYoA

Jimmy and my boy Neil Patrick Harris started out with a carton of 12 eggs. 8 were boiled, 4 were raw. Note that the initial probability of getting a raw egg is 1/3, but that changes once they start cracking them on their faces.

Suppose a single person (could be Jimmy or NPH) cracks 3 eggs. What is the expected number of raw eggs that get smashed on that person's face?

Question 4.

Weatherman Bill learned in his meteorology class that, in a particular town, there are on average 26 thunderstorms in a year (on 26 different days). It rains on 9 out of 10 thunderstormy days, so Bill wonders whether rain provides good evidence for a storm. On the other hand, there are many *false positives*: without a thunderstorm, the chance of rain in the town is still 20%.

Bill is very nervous about botching his first forecast on live TV. Will there be a thunderstorm? Help him:

(a) Determine the probability that it rains on *any* day, stormy or not.

(b) Use the result from (a) to compute the probability for a thunderstorm occurring *on a rainy day*.

Question 5.

Suppose Laney has bought a 3-pack of earrings from Target. The pack has a pair of red ones, a pair of blue ones, and a pair of yellow ones. She doesn't bother to organize them, but just makes a pile of loose earrings in a tiny bowl.[1]

(a) If she picks two earrings at random out of the bowl, what is the probability that they match?

(b) If she picks four earrings at random out of the bowl, what is the probability that there are *two* matching pairs?

(c) Suppose Laney has two piercings in her left ear and two in her right ear, let's call them $L1, L2, R1, R2$. If she picks four earrings at random out of the bowl, and then puts two earrings in each ear, what is the probability that she ends up with corresponding pairs in each ear (i.e., the earring in $L1$ is the same color as the earring in $R1$, and the same for $L2, R2$)?

[1] Based on a true story.

Questions to take home (optional)

Question 6.

This problem will be discussed about week later, before the miderm. Try it now if you want, before you see the answer.
We do balls into bins with m indistinguishable balls and k bins $(B_1, B_2, ..., B_k)$.

(a) What is the expected number of balls in a bin?

(b) ★ What is the expected number of empty bins ?

(c) ★ ("coupon collector problem") Suppose we fix k the number of bins, but not the number of balls; instead we throw m balls until there is no empty bin (m is a random variable here). What is the expected number of balls that are needed $E[m]$?