Deadline: October 2 at 8pm eastern

CS1802 Honors Recitation 3

Fall 2020

September 29 - October 2, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

Question 1.

We define the Powerset of a set S, $\mathcal{P}(S)$, to be a collection of all the subsets of S, including the empty set.

(a) Let $S = \{2, 5\}$. What is $\mathcal{P}(S)$?

(b) Let $|S| = 4$. What is $|\mathcal{P}(S)|$?

(c) Let $\mathcal{P}(S) = \{\{\}, \{c\}, \{d\}, \{h\}, \{c, d\}, \{c, h\}, \{d, h\}, \{c, d, h\}\}$. What is S?

(d) True or False? If $S = \{1, 2, 3\}$, then $\{1, 2\} \in \mathcal{P}(S)$

(e) True or False? If $S = \{1, 2, 3\}$, then $\{1, 2\} \subseteq \mathcal{P}(S)$

(f) True or False? If $S = \{1, 2, 3\}$, then $\{\{1, 2\}\} \subseteq \mathcal{P}(S)$

Question 2.

Let $A = \{2, 4, 6, 8\}$, $B = \{2, 6, 10, 14\}$, $C = \{14, 16\}$ and the universal set $U = \{2, 4, 6, 8, 10, 12, 14, 16\}$. Write the following in list notation:

(a) $\overline{A} \cap \overline{B}$

(b) $A - (B \cup C)$

(c) $(\overline{B} \cap C) \cup (C - A)$

Question 3.

Let $S = \{2, 4, 6, 8, 10, 12\}$. Write the following in list notation:

(a) $\{x | x \in S \wedge x + 6 \in S\}$

(b) $\{x | x + 6 \in S\}$

Question 4.

Using the set equivalence laws, show that $\overline{(\overline{A} \cup \overline{B})} \cap A = \overline{A} \cup B$

Question 5.

Show that, for arbitrary sets A, B, and C, $((A \cap B) \cup (A \cap \overline{C})) \subseteq A$. For this proof, assume an arbirary element $x \in (A \cap B) \cup (A \cap \overline{C})$ and use the set equivalence laws and logical definitions to show that $x \in A$.

Question 6.

There are 408 students at Woodstock Union High School. The foreign language department offers French, Latin, and Spanish (actual enrollment and language offerings at Laney's actual high school, in Woodstock, VT).

Draw a Venn Diagram representing the sets described below:

- There are 408 students total in the school (this should be your Universal set).
- 8 students study all three languages.
- 248 students study Spanish.
- 96 students study French.
- 106 students study Latin.
- There are twice as many students who study both French and Spanish (but not Latin) as who study both French and Latin (but not Spanish).
- The group of students who study both French and Spanish (but not Latin) is exactly the same size as the group made up of students who study both Latin and Spanish (but not French).
- 54 students do not study any foreign language.

Optional Fun Bonus Question (if you have time): Go back to problem 5, and show the same relationship with a set membership table, similar to a truth table: For an arbitrary element x, an entry of 1 indicates that x is a member of the set, and an entry of 0 means x is not a member of the set.

Questions to take home (optional)

More Basic Set Operations

1. When is $A \triangle B = \emptyset$?
   Recap : Symmetric Difference of sets A, B is $A \triangle B = (A \cup B) - (A \cap B)$
2. Do sets $A \neq B$ exist such that $A \times B = B \times A$?
3. Are there sets A and B such that $A \times B = \{(1, 2)\}$?
4. Are there sets A and B such that $A \times B = \{(1, 2), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$?
5. True or False: $(A - C) - B = A - (C - B)$
6. True or False: $(A - C) \cup (A - B) = A - (B \cup C)$
7. True or False:
   $(A \cap (B \cup C)) - ((A \cup C) \cap (D \cup C) \cap (B \cup C)) = ((A \cup B) \cup (C \cap A) \cup (A \cap B)) - (((A \cup D) \cup B) \cup C)$

Venn Diagrams True or false:

The oldest chess player among mathematicians and the oldest mathematician amongst chess players can be two different people.

The best chess player among mathematicians and the best mathematician among chess players can be two different people.

★ How many positive integers less than 111 are divisible by 11 or 2 or 5?
Hint: inclusion-exclusion of three sets, each conting multiples up to 111.

★: Cutting parts from a large sheet

Can one cut three 3x3 squares and six 2x3 rectangles from a sheet of paper 8x8 square?

Can one cut four 3x3 squares and eight 2x4 rectangles from a sheet of paper 10x10 square?

Can one cut four 4x4 squares and and six 2x4 rectangles from a sheet of paper 11x11 square?