Deadline: September 25 at 8pm eastern; ★ problems optional

CS1802 Honors Recitation 2

Fall 2020

September 22, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.

- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.

- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

Question 1.

Consider the statements P, Q, R, and S below; because, ok, it could be going better for my Red Sox this year, but at least they're playing!

- P = Jackie Bradley Jr (JBJ) plays center field.

- Q = The Red Sox win.

- R = Chris Sale gets the win.

- S = The Yankees lose.

Translate each of the sentences below into logic statements using conjunction ($\wedge$), disjunction ($\vee$), negation ($\neg$), and/or implication ($\implies$). English can be squishy but logic can't be, so if a sentence seems ambiguous, choose an interpretation that makes the most sense to you and make it crystal-clear in your logic statement.

(a) If JBJ plays center field, then the Red Sox win.

(b) The Yankees lose, but Chris Sale doesn't get the win.

(c) The Yankees lose if the Red Sox don't win or JBJ doesn't play center field.

(d) Chris Sale gets the win whenever the Red Sox win.

Question 2.

Apply the laws of logical equivalence to show that the following compound logic statement is logically equivalent to T. As with all proofs, make your steps small and clear; identify and apply one law at a time.

$$(\neg p \vee \neg q) \vee (p \wedge q)$$

Question 3.

For this problem, the domain is the set of all the characters on the Netflix show Cobra Kai. Consider the following two predicates:

- $johnny(x)$, meaning "Johnny fights x"

- $karate(x)$, meaning "x studies karate"

Using only variables, logic symbols $(\neg, \wedge, \vee, \implies, \exists, \forall)$, and the predicates $johnny()$ and $karate()$, formulate the statements:

(a) Johnny doesn't fight everyone who studies karate.

(b) The only people Johnny fights are people who study karate.

**Bonus question: Creese is really the bad guy, though, right? Discuss.

Question 4.

Interpret each of the following as English sentences, then decide whether they are true. For this problem, our universe is the set of integers $\mathbb{Z}$.

(a) $\forall x \exists y (x + y = 0)$

(b) $\exists y \forall x (x + y = x)$

(c) $\exists x \forall y (x + y = x)$

Question 5.

Sometimes, we might not have the particular logic gate we need available. Luckily, we can replicate the behavior of some logic gates using other gates. Draw a circuit with the same behavior as an AND gate but does not use an AND gate in the circuit. Show that your answer is correct by representing your circuit's behavior in a truth table.

Question 6.

D'Artagnan would like to figure out the relative salaries of three fellow Musketeers using two facts. First, he knows that if Athos is not the highest paid of the three, then Porthos is. Second, he knows that if Porthos is not the lowest paid, then Aramis is paid the most. Determine the relative salaries of his friends by filling in the table below.

Combination	Possible? (Y/N)	Explanation
Porthos > Athos > Aramis		
Porthos > Aramis > Athos		
Athos > Aramis > Porthos		
Athos > Porthos > Aramis		
Aramis > Athos > Porthos		
Aramis > Porthos > Athos		

Question 7.

Quantifiers and Proofs
Interpret the following statements as English sentences, then decide whether those statements are true.

1. $\forall x \exists y : x + y = 0$

2. $\exists y \forall x : x + y = x$

3. $\exists x \forall y : x + y = x$

4. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} : x + y = prime$

5. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} : x + y = prime$ and $y = prime$

6. ★★★★★ $\forall x \in \mathbb{Z}, x$ even, $\exists y \in \mathbb{Z} : x + y = prime$ and $y = prime$

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7. $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} : x > y \Leftrightarrow x^2 > y^2$

8. $\forall x \in \mathbb{R}, x > 0, \forall y \in \mathbb{R}, y > 0 : x > y \Leftrightarrow x^2 > y^2$

9. $\forall x \in \mathbb{R}, x \neq 0, \forall y \in \mathbb{R} : x + y < 0 \Leftrightarrow x \cdot y < 0$

Question 8.

Bitwise Logical Operations

Interpret the following statements as English sentences, then decide whether those statements are true. Variables a, b, c are boolean and variabels A, B, C are boolean sequences (or sequences of bits), for example $A = a_1 a_2 ... a_k = a_1, a_2, ..., a_k$ is a sequence of k bits or k boolean variables. k is fixed (for example k=32).

A, B, C allow boolean operations "bitwise": $A = a_1 a_2 ... a_k$ and $B = b_1 b_2 ... b_k$ then

$A \wedge B = a_1 \wedge b_1, a_2 \wedge b_2, ..., a_k \wedge b_k$

Similarly $A \vee B = a_1 \vee b_1, a_2 \vee b_2, ..., a_k \vee b_k$, etc

1. $\forall a, b, c \in \{T, F\} : a \wedge b = a \wedge c \Rightarrow b = c$

2. ★ $\forall A, B, C \in \{T, F\}^k : A \wedge B = A \wedge C \Rightarrow B = C$

3. ★ $\exists A, \forall B, C \in \{T, F\}^k : A \wedge B = A \wedge C \Rightarrow B = C$

4. $\forall a, b, c \in \{T, F\} : a \vee b = a \vee c \Rightarrow b = c$

5. ★ $\forall A, B, C \in \{T, F\}^k : A \vee B = A \vee C \Rightarrow B = C$

6. ★ $\exists A, \forall B, C \in \{T, F\}^k : A \vee B = A \vee C \Rightarrow B = C$

7. $\forall a, b, c \in \{T, F\} : a \text{ XOR } b = a \text{ XOR } c \Rightarrow b = c$

8. ★ $\forall A, B, C \in \{T, F\}^k : A \text{ XOR } B = A \text{ XOR } C \Rightarrow B = C$

9. ★ That last XOR equation should allow you to prove the strategy for the square-game: each player wants to leave the table such that when the counts are written in binary, the vector-XOR works out in a certain way:

   Prove that if player A leaves a board with XOR(binary rows)=0, then player B cannot do the same

★★ **Prove** that if player B leaves a board with XOR(binary rows)$\neq 0$, then player A has a move to make the board as desired XOR(binary rows)$= 0$

Question 9.

★★ **10 wise men in the village**

10 wise men live in a village. Each man has a color dot on the forehead either R or B not known to him; knowing his color means immediate death. But everyone knows the other men's colors, i.e. a B person sees 5R and 4B. The men dont speak/communicate to each other, but each morning they meet in a circle and they can see if anyone died. They are extremely smart (can infer anything) and know when someone dies its because he must have figured out his color.

For quite a few days this goes unchanged, until one day a stranger passes to the village and remarks "the number of B colors is not 10".

Prove that eventually everyone in the village will figure out his color and die.