Deadline: November 20 at 8pm eastern

CS1802 Recitation 8

Fall 2020

November 17 - 20, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignemtn, and submit your solution by the Friday deadline.

Question 1.

In the Module 7 videos, Prof. Hescott showed how we can get by in a world with only 3-cent and 5-cent pieces. You can make any number of cents n, as long as $n \geq 8$. To get from n to $n+1$ cents, you can either: (a) subtract one 5-cent piece and add two 3-cent pieces, $n+1 = n+6-5$, or (b) subtract three 3-cent pieces and add two 5-cent pieces, $n+1 = n+10-9$.

Proofs are not required for this problem, but include a brief explanation of how you'd go from n cents to $n+1$ cents. If possible, also specify the smallest value of k, where we can have n cents as long as $n \geq k$.

Can we make any amount of cents in a world where only...

(a) 4-cent pieces and 5-pieces exist?

(b) 2-cent pieces and 5-cent pieces exist?

(c) 5-cent pieces and 10-cent pieces exist?

(d) 4-cent pieces and 10-cent pieces exist?

Question 2.

Use mathematical induction to prove the summation of odd positive integers.

$P(n)$ states that $\sum_{i=1}^{n} 2i - 1 = n^2$. Show the following claim: $\forall n \in \mathbb{Z}^+ P(n)$.

Question 3.

Remember the summation from Recitation 7, $\sum_{i=1}^{n} i^2$? Of course you do! At the time, we had $P(n)$, stating: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Prove by mathematical induction that it's always true, for any value of $n \geq 1$.

Question 4.

Recall that the Fibonnacci sequence is $1, 1, 2, 3, 5, 8, 13, 21, 34, ...$ We recursively define $F_n = F_{n-1} + F_{n-2}$. In HW7, you showed that $F_n < 2^n$. Now, let's make it a legit proof by induction.

$P(n)$ states that $F_n < 2^n$. Show the following claim: $\forall n \in \mathbb{Z}^+ P(n)$. You'll need strong induction for this one. Your inductive hypothesis should be that $P(1), P(2), ..., P(k)$ are true; then show $P(k+1)$ must be true.

Question 5.

Prove that for all positive integers n, 10^n can be written as a sum of two perfect squares.

$P(n)$ states that $10^n = a^2 + b^2$ for some $a, b \in \mathbb{Z}^+$. Show that $\forall n \in \mathbb{Z}^+ P(n)$. You'll need strong induction for this one. Your inductive hypothesis should be that $P(1), P(2), ..., P(k)$ are true.

Question 6.

We've done some proving of things by induction, but we can define things by induction as well. In an inductive definition, we specify the elements in, for example, a set in terms of other elements in the set.

For example: We can provide an inductive definition of the set $\mathbb{Z}^+$ of positive integers.

- Base case: $1 \in \mathbb{Z}^+$
- Inductive step: $x \in \mathbb{Z}^+ \implies x + 1 \in \mathbb{Z}^+$

See how the inductive definition works here? It puts 1 in the set initially. And then, we can say that:

- 2 is in the set because 1 is in the set.
- 3 is in the set because 2 is in the set.
- 4 is in the set because 3 is in the set.
- ...

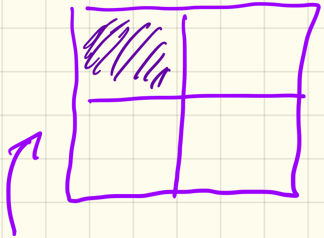
(a) Give an inductive definition of the set $S = \{3, 8, 13, 18, 23, 28, ...\}$

(b) Give an inductive definition of the set $S = \{3, 4, 5, 8, 9, 12, 16, 17, 20, 24, 28, 32, 33, ...\}$. *Hint*: Let $S = A \cup B$, where you define A and B inductively.

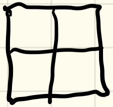
(c) Give an inductive definition of the set $S = \{a, aac, aaacc, aaaaccc, ...\}$

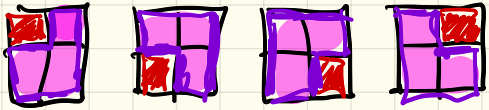
(d) Given an inductive definition of the set $S = \{3, 5, 8, 11, 13, 14, 16, 18, 19, 21, 24, ...\}$

Consider the shape 



for all $n >= 1$ if we have a $2^n \times 2^n$ grid with one square removed we can "pack" (tile) the grid with  shapes.

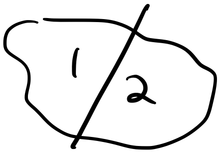
We argue by induction. Consider the base case when $n = 1$. We want to tile the space 

We have 4 cases 

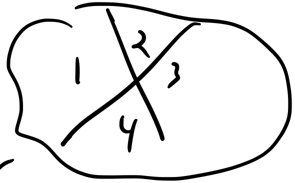
E.g. • n lines in the plane w/ no 2 lines parallel and no 3 lines concurrent.

• how many regions does this divide the plane into?

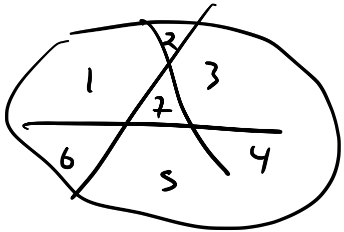
$n=1$:



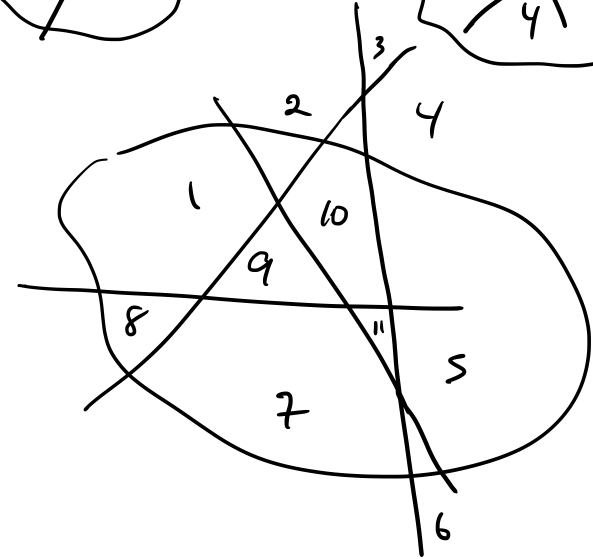
$n=2$:



$n=3$:



$n=4$:



n:	1	2	3	4	...
#:	2	4	7	11	...
		2	3	4	
			1	1	