Deadline: September 25 at 8pm eastern

CS1802 Recitation 2

Fall 2020

September 22 - 25, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

Question 1.

Consider the statements P, Q, R, and S below; because, ok, it could be going better for my Red Sox this year, but at least they're playing!

- P = Jackie Bradley Jr (JBJ) plays center field.
- Q = The Red Sox win.
- R = Chris Sale gets the win.
- S = The Yankees lose.

Translate each of the sentences below into logic statements using conjunction ($\wedge$), disjunction ($\vee$), negation ($\neg$), and/or implication ($\implies$). English can be squishy but logic can't be, so if a sentence seems ambiguous, choose an interpretation that makes the most sense to you and make it crystal-clear in your logic statement.

(a) If JBJ plays center field, then the Red Sox win.

(b) The Yankees lose, but Chris Sale doesn't get the win.

(c) The Yankees lose if the Red Sox don't win or JBJ doesn't play center field.

(d) Chris Sale gets the win whenever the Red Sox win.

Solution:

- $P \implies Q$
- $S \wedge \neg R$
- $(\neg Q \vee \neg P) \implies S$
  Another interpretation: $(\neg Q \implies S) \vee \neg P$
- $Q \implies R$

Question 2.

Apply the laws of logical equivalence to show that the following compound logic statement is logically equivalent to T. As with all proofs, make your steps small and clear; identify and apply one law at a time.

$$(\neg p \vee \neg q) \vee (p \wedge q)$$

Solution (option one):

$$(\neg p \vee \neg q) \vee (p \wedge q)$$

$\equiv \neg(p \wedge q) \vee (p \wedge q)$ (DeMorgan's)
$\equiv T$ (Complement)

Solution (option two):

$$(\neg p \vee \neg q) \vee (p \wedge q)$$

$\equiv ((\neg p \vee \neg q) \vee p) \wedge ((\neg p \vee \neg q) \vee q)$ (Distributive)
$\equiv (\neg p \vee \neg q \vee p) \wedge (\neg p \vee (\neg q \vee q))$ (Associative)
$\equiv (\neg p \vee \neg q \vee p) \wedge (\neg p \vee T)$ (Negation)
$\equiv (\neg p \vee \neg q \vee p) \wedge T$ (Annihilator)
$\equiv ((\neg p \vee p) \vee \neg q) \wedge T$ (Associative)
$\equiv (T \vee \neg q) \wedge T$ (Complement)
$\equiv T \wedge T$ (Domination)
$\equiv T$ (Idempotent)

Question 3.

For this problem, the domain is the set of all the characters on the Netflix show Cobra Kai. Consider the following two predicates:

- $johnny(x)$, meaning "Johnny fights x"
- $karate(x)$, meaning "x studies karate"

Using only variables, logic symbols ($\neg, \wedge, \vee, \implies, \exists, \forall$), and the predicates $johnny()$ and $karate()$, formulate the statements:

(a) Johnny doesn't fight everyone who studies karate.

(b) The only people Johnny fights are people who study karate.

**Bonus question: Creese is really the bad guy, though, right? Discuss.

Solution:

- $\exists x, karate(x) \wedge \neg johnny(x)$
  (Aka, there is a karate enthusiast who Johnny doesn't fight)
  Another squishy-English interpretation of the original sentence could be that Johnny doesn't fight ANY person who studies karate, which would be $\forall x, karate(x) \implies \neg johnny(x)$
- $\forall x, johnny(x) \implies karate(x)$
  (Aka, for all people in the Cobra Kai universe, if Johnny fights them, then they study karate.)

Question 4.

Interpret each of the following as English sentences, then decide whether they are true. For this problem, our universe is the set of integers $\mathbb{Z}$.

(a) $\forall x \exists y (x + y = 0)$

(b) $\exists y \forall x (x + y = x)$

(c) $\exists x \forall y (x + y = x)$

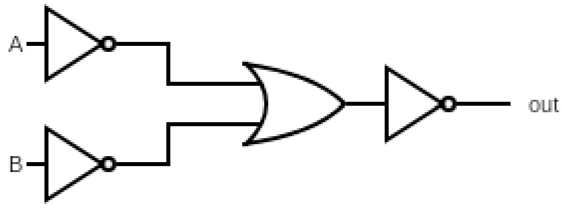
Solution:

- For all integers x, there exists an integer y such that $x + y = 0$. True.
- There exists an integer y such that, for all integers x, $x + y = x$. True, $y = 0$.
- There exists an integer x, such that for all integers y, $x + y = x$. False.

Question 5.

Sometimes, we might not have the particular logic gate we need available. Luckily, we can replicate the behavior of some logic gates using other gates. Draw a circuit with the same behavior as an AND gate but does not use an AND gate in the circuit. Show that your answer is correct by representing your circuit's behavior in a truth table.

Solution:



How do we know it's the same? Here's the truth table:

A	B	$A \wedge B$	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg(\neg A \vee \neg B)$
0	0	0	1	1	1	0
0	1	0	1	0	1	0
1	0	0	0	1	1	0
1	1	1	0	0	0	1

Question 6.

D'Artagnan would like to figure out the relative salaries of three fellow Musketeers using two facts. First, he knows that if Amos is not the highest paid of the three, then Porthos is. Second, he knows that if Porthos is not the lowest paid, then Aramis is paid the most. Determine the relative salaries of his friends by filling in the table below.

Combination	Possible? (Y/N)	Explanation
Porthos > Amos > Aramis		
Porthos > Aramis > Amos		
Amos > Aramis > Porthos		
Amos > Porthos > Aramis		
Aramis > Amos > Porthos		
Aramis > Porthos > Amos		

Solution: The only possible situation is Amos > Aramis > Porthos.

Combination	Possible? (Y/N)	Explanation
Porthos > Amos > Aramis	N	if Porthos is not the lowest paid, then Aramis is paic
Porthos > Aramis > Amos	N	if Porthos is not the lowest paid, then Aramis is paic
Amos > Aramis > Porthos	Y	Only one that meets all the criteria!
Amos > Porthos > Aramis	N	if Porthos is not the lowest paid, then Aramis is paic
Aramis > Amos > Porthos	N	If amos not highest, then porthos is
Aramis > Porthos > Amos	N	If amos not highest, then porthos is11

Question 7.

Quantifiers and Proofs

Interpret the following statements as English sentences, then decide whether those statements are true.

1. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} : x + y = prime$
   TRUE : for any given $x \in \mathbf{Z}$, choose $y = 2 - x$, so $x + y = 2$ prime

2. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} : x + y = prime$ and $y = prime$

FALSE for $y = 7$: there is no $y \in \mathbf{Z}$ such that both y and $y + 7$ are prime

3. $\forall x \in \mathbb{Z}, x \text{ even}, \exists y \in \mathbb{Z} : x + y = prime$ and $y = prime$
   This is a hard question, not clear the answer is known yet. There has been recently significant progress on Golbach Conjecture and related problems
   https://math.stackexchange.com/questions/28247/can-every-even-integer-be-expressed -as-the-difference-of-two-primes

4. $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} : x > y \Leftrightarrow x^2 > y^2$
   FALSE for $x = -1; y = -3$

5. $\forall x \in \mathbb{R}, x > 0, \forall y \in \mathbb{R}, y > 0 : x > y \Leftrightarrow x^2 > y^2$
   TRUE for $x, y > 0$.
   Proof: $x > y \Leftrightarrow x^2 > xy; xy > y^2 \Leftrightarrow x^2 > xy > y^2$

6. $\forall x \in \mathbb{R}, x \neq 0, \forall y \in \mathbb{R} : x + y < 0 \Leftrightarrow x \cdot y < 0$
   FALSE for $x = -1, y = -2$

Question 8.

Bitwise Logical Operations

Interpret the following statements as English sentences, then decide whether those statements are true. Variables a, b, c are boolean and variabels A, B, C are boolean sequences (or sequences of bits), for example $A = a_1a_2...a_k = a_1, a_2, ..., a_k$ is a sequence of k bits or k boolean variables. k is fixed (for example k=32).
A, B, C allow boolean operations "bitwise": $A = a_1a_2...a_k$ and $B = b_1b_2...b_k$ then
$A \wedge B = a_1 \wedge b_1, a_2 \wedge b_2, ..., a_k \wedge b_k$
Similarly $A \vee B = a_1 \vee b_1, a_2 \vee b_2, ..., a_k \vee b_k$, etc

1. $\forall a, b, c \in \{T, F\} : a \wedge b = a \wedge c \Rightarrow b = c$
   FALSE for $a = 0, b = 1, c = 0$

2. ★ $\forall A, B, C \in \{T, F\}^k : A \wedge B = A \wedge C \Rightarrow B = C$
   FALSE for $A = 010, B = 101, C = 001$

3. ★ $\exists A, \forall B, C \in \{T, F\}^k : A \wedge B = A \wedge C \Rightarrow B = C$
   TRUE for $A = 111 : 111 \wedge B = 111 \wedge C \Rightarrow B = C$

4. $\forall a, b, c \in \{T, F\} : a \vee b = a \vee c \Rightarrow b = c$
   FALSE for $a = 1, b = 1, c = 0$

5. ★ $\forall A, B, C \in \{T, F\}^k : A \vee B = A \vee C \Rightarrow B = C$
   FALSE for $A = 010, B = 101, C = 111$

6. ★ $\exists A, \forall B, C \in \{T, F\}^k : A \vee B = A \vee C \Rightarrow B = C$
TRUE

7. $\forall a, b, c \in \{T, F\} : a$ XOR $b = a$ XOR $c \Rightarrow b = c$ TRUE: if bits b and c have the same XOR with bit fixed bit a (either 0 or 1) then they must be equal

8. ★ $\forall A, B, C \in \{T, F\}^k : A$ XOR $B = A$ XOR $C \Rightarrow B = C$ TRUE

9. ★ That last XOR equation should allow you to prove the strategy for the square-game: each player wants to leave the table such that when the counts are written in binary, the vector-XOR works out in a certain way:

   Prove that if player A leaves a board with XOR(binary rows)=0, then player B cannot do the same

★★ **Prove** that if player B leaves a board with XOR(binary rows)$\neq 0$, then player A has a move to make the board as desired XOR(binary rows)$= 0$