Deadline: September 18, 8pm eastern

CS1802 Recitation 1

Fall 2020

September 15-18, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

Question 1.

Write down the 4-bit binary numbers from 0 to 15. In decimal, we can tell if a number is even or odd based on the rightmost digit – multiple of 2?

Cool, must be even. How about with binary – how can we tell, at a glance, whether a binary number represents an odd or an even number?

Solution: You'll notice that the bits in the one's place alternate with each value, and that a 1 in the one's place indicates an odd number, and a 0 in the one's place denotes an even number. Another pattern you might notice: In the 2's place, the bit values alternate every two, in the 4's place they alternate every four, and so on.

Question 2.

Convert the following unsigned numbers to base 10. In other bases, when we run out of digits 0-9, we start using letters of the alphabet, A = 10, B = 11, and so on:

$$101110010_2$$

$$257_8$$

$$2A8_{11}$$

$$EF2_{16}$$

Solution:

$$101110010_2 = 1\cdot2^8+0\cdot2^7+1\cdot2^6+1\cdot2^5+1\cdot2^4+0\cdot2^3+0\cdot2^2+1\cdot2^1+0\cdot2^0 = 370_{10}$$

$$257_8 = 2\cdot8^2+5\cdot8^1+7\cdot8^0 = 175_{10}$$

$$2A8_{11} = 2\cdot11^2+10\cdot11^1+8\cdot11^0 = 360_{10}$$

$$EF2_{16} = 14\cdot16^2+15\cdot16^1+2\cdot16^0 = 3826_{10}$$

Question 3.

(a) Convert the (unsigned) decimal number 109_{10} to binary. Show your work.

(b) Convert the (unsigned) decimal number 63_{10} to binary. Show your work.

Solution: We'll use the division algorithm to solve these.

- $109 \div 2 = 54R1$
- $54 \div 2 = 27R0$
- $27 \div 2 = 13R1$
- $13 \div 2 = 6R1$
- $6 \div 2 = 3R0$
- $3 \div 2 = 1R1$
- $1 \div 2 = 0R1$

Put these together to get $109_{10} = 1101101_2$

- $63 \div 2 = 31R1$
- $31 \div 2 = 15R1$
- $15 \div 2 = 7R1$
- $7 \div 2 = 3R1$
- $3 \div 2 = 1R1$
- $1 \div 2 = 0R1$

Altogether, we get $63_{10} = 111111_2$.

Q: Why do computer scientists celebrate Halloween on Christmas?
A: Because oct-31 is the same as dec-25 hahahahaha!!!!!!

Question 4.

Suppose you have 5 bits to represent binary numbers in two's complement format.

(a) What is the highest number you can represent? Answer with a decimal (base 10) number and then express it in 5-bit two's complement.

(b) What is the lowest number you can represent? Answer with a decimal (base 10) number and then express it in 5-bit two's complement.

(c) How many total values can you represent?

Solution:

- The highest number we can represent is $01111_2 = 15_{10}$.
- The lowest number we can represent is $10000_2 = -16_{10}$.
- We can represent 32 total values (0 through 15 and -1 through -16).

Question 5.

Convert each decimal number below to 7-bit two's complement, perform the arithmetic indicated, and tell us the final answer in 7-bit two's complement.

(a) $13_{10} + 10_{10}$

(b) $5_{10} - 28_{10}$

(c) $-8_{10} - 11_{10}$

Solution:

- $13_{10} = 0001101_2, 10_{10} = 0001010_2$ Add: $0001101 + 0001010 = 0010111$
  Sanity-check: $0010111_2 = 1 + 2 + 4 + 16 = 23_{10}$

- $5_{10} = 0000101_2, 28_{10} = 0011100_2$ But that's a positive 28, so flip the bits: 1100011_2 and add one to get -28: 1100100_2
  Add: $0000101 + 1100100 = 1101001_2$
  Sanity-check: Is $1101001_2 = -23_{10}$? ... Flip the bits: 0010110 and add one: 0010111, which is $1 + 2 + 4 + 16 = 23_{10}$, so the original number was -23... good!!

- $8_{10} = 0001000_2, 11_{10} = 0001011_2$ Turn that 8 to negative, flip the bits 1110111, and add one to get -8: 1111000
  Turn the 11 to negative, flip the bits 1110100, and add one to get -11: 1110101
  Add: $1111000 + 1110101 = 1101101_2$
  Sanity check: Is $1101101_2 = -19_{10}$?...Flip the bits: 0010010 and add one: 0010011, which is $1 + 2 + 16 = 19_{10}$, so the original number was -9!

Question 6.

Prince Humperdinck has been presented with 128 bottles of wine, but one of them has been poisoned(!) with iocane powder. 7 members of the Brute Squad volunteer to sacrifice themselves to find the poisoned bottle. The poison has absolutely no effect on the drinker until exactly 24 hours later, when you instantly drop dead.

In other words, the Prince has 24 hours hours to discover a poison that takes 24 hours to take effect. He doesn't mind if Brute Squad members die. How can he use the 7 to find the poisoned bottle?

Solution: Here is the Brute Squad, there are seven members: A, B, C, D, E, F and G. There are 128 bottles of wine, let's number them 0, 1, 2, ..., 127. With seven members, we know that if we represent anything binary (0/1, on/off, drink from a bottle don't drink, die from poison/don't die from poison) we can represent $2^7 = 128$ values.
Let's use a one to indicate that a prisoner drank from a corresponding bottle, and the value of the resulting decimal number to identify the bottle.

- 0 0 0 0 0 0 0 No one drinks from any bottles
- 0 0 0 0 0 0 1 G drinks from bottle 1, no one else drinks bottle 1
- 0 0 0 0 0 1 0 F drinks from bottle 2
- 0 0 0 0 0 1 1 G and F drink from bottle 3
- 0 0 0 0 1 0 0 E drinks from bottle 4
- 0 0 0 0 1 0 1 E and G drink from bottle 5
- 0 0 0 0 1 1 0 E and F drink from bottle 6.
- 0 0 0 0 1 1 1 E, F, and F drink from bottle 7
- ...
- 1 1 1 1 1 1 1 Everyone drinks from bottle 127

Now that everyone has drunk, we wait 24 hours and see who dies.

- If no one dies, bottle 0 was poisoned
- If only G dies, bottle 1 was poisoned
- If only F dies, bottle 2 was poisoned
- If F and G die, bottle 3 was poisoned
- If only E dies, bottle 4 was poisoned
- If E and G die, bottle 5 was poisoned
- If E and F die, bottle 6 was poisoned
- If E, F, and G die, bottle 7 was poisoned
- ...
- If everyone dies, bottle 127 was poisoned