

Deadline: Nothing to submit

CS1802 Midterm Review

Fall 2020

November 3-6, 2020

Instructions: This recitation comprises questions from CS1800 midterms from previous years. Your instructor will identify a subset of these questions to work on during your section, and then will review the solutions.

There is no need to submit anything for this recitation.

Topics: Representation of numbers, binary and other bases, two's complement, boolean logic, circuits, logical equivalence, set operations, set equality, cartesian product, counting, pigeonhole principle, combinations, permutations, probabilities.

Question 1.

Convert $1CF_{16}$ to (a) unsigned binary, and (b) decimal.

Question 2.

Using 8-bit two's complement, add the binary representations of 32 and -16. Show your carry operations (though you don't need to show the carry-out).

Question 3.

How many digits are in the octal (base 8) representation of 8^4 ?

Question 4.

Rewrite the expression $a \wedge b \wedge c$ using only \vee (or) and \neg (not) operators.

Question 5.

Use logic rules to simplify the expression $\neg(\neg p \wedge (p \vee \neg q))$. Show each step and specify when you are applying the following rules: (1) De Morgan, (2) Distributive, (3) complement, (4) identity.

Question 6.

Assume the symbol for logical NOR is ∇ . Show that NOR is logically complete by expressing the following using only ∇ .

- (a) $\neg A$
- (b) $A \wedge B$

Question 7.

Assume x and y are integers, and either explain why the following statement is true, or provide a counterexample: $\forall x \exists y, y^2 = x$.

Question 8.

Let S be an infinite set of people and let A be an infinite set of tasks. Let $P(x, y)$ be a predicate that states that person x can perform task y . Translate the following English statement into predicate logic using quantifiers and logic symbols ($\forall, \exists, \implies, \wedge, \vee, \neg$): No matter who you pick from our set of people, there is someone else who can do everything that person can do.

Question 9.

List the elements in the set described:
 $R = \{n \mid k \in \mathbb{Z}^+, n = 2^k \text{ and } 1 < k \leq 5\}$

Question 10.

If $|A| = 10$, $|B| = 6$, and $|A \cap B| = 4$, what is the cardinality of $\mathcal{P}((A \cup B) - (A \cap B))$?

Question 11.

You're putting a basket together of apples, bananas, and oranges. You can't see or feel which type of fruit you're putting into the basket. What is the smallest number of pieces that must be put in the basket in order to guarantee that there are:

- at least 8 apples, or

- at least 6 bananas or
- at least 9 oranges?

Question 12.

Your car has just 66,000 miles and you're wondering: before you hit 67,000, how many different odometer values will have exactly three 6s?

Question 13.

- (a) Dean Brodley wants to gift a laptop or a phone to a Khoury student* and she has a budget of \$1200. There are 28 laptop models and 16 phone models available from Best Buy for \$1200 each. How many ways can she do this?
- (b) President Aoun, on the other hand, wants to gift a laptop and a smartphone to a Khoury student and he has a budget of \$2400. Same situation at Best Buy. How many ways can he do this?

* I mean, this is probably true that she *wants* to. But this is a made-up example not based on reality.

Question 14.

How many positive numbers smaller than 512 are the sum of 4 different powers of 2? For example, $83 = 64 + 16 + 2 + 1$ is such a number.

Question 15.

How many ways are there to merge the sorted lists $(1, 2, 3, 4)$ and (A, B, C, D) into a sequence of 8 symbols where all elements of each list are still in the correct order? (For example, $AB12C34D$ is such a sequence, but $ABDC1234$ is not, only because D and C are out of order.)

Question 16.

If I have a randomly-generated 8-digit bit string, what is the probability that exactly half the bits set to 1? You can leave your answer in the form of a fraction.

Question 17.

You and 9 friends gather to play some pick-up basketball.

- (a) How many ways are there to make two teams of 5, the Celtics and the Aces?
- (b) How many ways are there to make two teams of 5, if the teams have no names? (Two teams are the same if all players are the same.)

Question 18.

Suppose I choose 5 distinct numbers at random from the set $\{1, 2, 3, \dots, 300\}$. What is the probability that the sum of the numbers I've chosen is even?

Question 19.

Suppose I choose 5 distinct numbers at random from the set $\{1, 2, 3, \dots, 301\}$. Is the sum of the numbers more likely to be even or odd?

Question 20.

A jar contains 3 red balls and 7 blue balls.

- (a) If a ball is picked at random, what is the probability that this ball is blue?
- (b) If two balls are taken out of the jar at random, what is the probability that they are of different colors?

Question 21.

What is the term a_3 of the sequence where a_k is given by...

- (a) $2^k + 1$
- (b) $(k + 1)^{k+1}$
- (c) $\lfloor k/2 \rfloor + \lceil k/2 \rceil$

Question 22.

Find the first 5 terms, the first 4 differences, and the first 3 second-differences of the sequence given by $a_n = 2 \cdot n^2 + 3n + 1$.

Question 23.

(This is HW5 Problem 5i, for additional review/practice!)

Trained contact teams have been dispatched to meet the aliens. A team of 41 people discovers a far off planet inhabited by aliens. The team needs to send 3 groups of size 5, 5, and 7 down to the planet. Two groups are the same if they have the same group members. How many possible groups can be created from the crew?