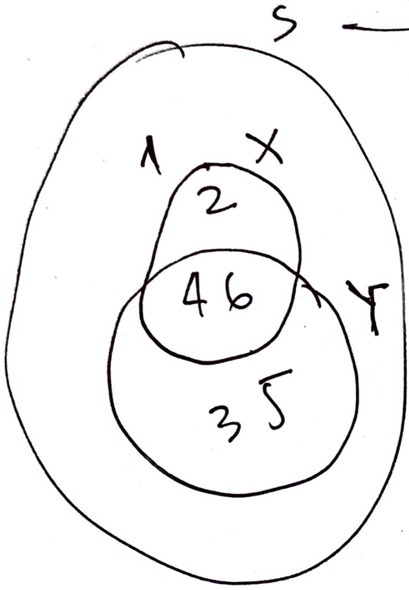
CONDITIONAL PROB EXAMPLE 1



X R.variable binary $p(X=TRUE) = 3/6$

X = "even" $p(X=FALSE) = 3/6$

Y = R.variable binary $p(Y=TRUE) = 4/6$

Y = "≥ 3" $p(Y=FALSE) = 2/6$

Joint R.variable (X,Y) has four possible values:

(T,T); (T,F); (F,T); (F,F)

set {4,6}   set {2}   {3,5}   {1}

Joint table

	X=T	X=F	Y marginal
Y=T	$P(X,Y) = 2/6$	$P(\neg X, Y) = 2/6$	$P(Y=T) = 4/6$
Y=F	$P(X, \neg Y) = 1/6$	$P(\neg X, \neg Y) = 1/6$	$P(Y=F) = 2/6$
X marginal	$P(X=T) = 3/6$	$P(X=F) = 3/6$	

$P\begin{pmatrix} X=T \text{ and} \\ Y=T \end{pmatrix}$

$P(X=F, Y=T)$

prob $(X=T, Y=F)$

CONDITIONAL PROB $P(X=T \mid Y=T) =$ probability that $X=T$ given that $Y=T$.

"given that $Y=T$" means we <u>restrict the space</u> to cases $Y=T$ that is {3,4,5,6}. within this restricted space, $P(X=T) = 2/4$

$$P(X=T \mid Y=T) = \frac{P(X=T \wedge Y=T)}{P(Y=T)}$$

$\to$ both true

$\to$ restrict space to $Y=T$

in general for any values : $P(X) \cdot P(Y \mid X) = P(X,Y) = P(Y) \cdot P(X \mid Y)$

X happens, then Y happens given X

X and Y

Y happen, then X happens given Y

$$\Rightarrow P(X \mid Y) = \frac{P(Y \mid X) \cdot P(X)}{P(Y)} \quad \text{Bayes Theorem.}$$

CONDITIONAL PROB EXAMPLE 2

- random variables X, Y and density functions
- conditional probability $\mathbf{P}[X|Y]$
- joint probability $\mathbf{P}[X,Y] = \mathbf{P}[X|Y] \cdot \mathbf{P}[Y]$
- Bayes rule $\mathbf{P}[X|Y] \cdot \mathbf{P}[Y] = \mathbf{P}[Y|X] \cdot \mathbf{P}[X]$
- independence $\mathbf{P}[X,Y] = \mathbf{P}[X] \cdot \mathbf{P}[Y]$
- marginalization $\mathbf{P}[X] = \sum_{Y=y} \mathbf{P}[X|Y=y] \cdot \mathbf{P}[Y=y]$

prob(square ∧ red)

JOINT TABLE p(s,c)

	red	blue	green	
square	0.25	0.10	0.21	0.56
round	0.17	0.04	0.23	0.44
	0.42	0.14	0.44	

prob(round, red) = prob(round ∧ red)

Set of objects with 2 attributes
- shape ∈ {□, ○}
- color ∈ {Red, Blue, Green}

⇓

2 random variables "Shape" "Color"
2 values 3 values

marginal P(s) P(c)

partition rule for marginals
- $p(\text{red}) = p(\text{red} \wedge \text{square}) + p(\text{red} \wedge \text{round})$
  $= 0.25 + 0.17$
- $p(\text{round}) = p(\text{round} \wedge \text{Red}) + p(\text{round} \wedge \text{Blue}) + p(\text{round} \wedge \text{Gr}$
  $= .17 + .04 + .23$

Bayes: $p(s|c) \cdot p(c) = p(c|s) \cdot p(s)$
$= p(s,c)$

CONDITIONAL PROBABILITY

$$P(\text{shape}=\square \mid \text{color}=\text{red}) = P(\square|\text{red}) = \frac{P(\square \wedge \text{red})}{P(\text{red})} = \frac{0.25}{0.42}$$

why? it is the probability of square within restricted set of COLOR = red = the subset of red objects

|restricted space to color red| = 0.42 = p(red)

out of that space 0.25 are squares
0.17 are rounds.

So $P(\square|\text{red})$ is 0.25 out of 0.42.

in general $P(X|Y)$ is different distribution of X given value of Y

$$\underset{c=\text{red}}{P(s|c)} = \begin{cases} 0.25/0.42 & \text{for } \square \\ 0.17/0.42 & \text{for } \circ \end{cases} \qquad \Big\| \qquad \underset{c=\text{Blue}}{P(s|c)} = \begin{cases} 0.10/0.14 & \text{for } \square \\ 0.04/0.14 & \text{for } \circ \end{cases}$$