CS1800
Discrete Structures
Fall 2017

Lecture 29
11/13/17

Last time

- Algorithmic Analysis
  - search
  - sort

Today

- Finish mergesort

- Analysis
  - sequences
  - series

Next time

- Series

Sorting

Insertion Sort : $1+2+3+4+5+\cdots+n = \frac{n(n+1)}{2} = \frac{n^2}{2}+n/2$

Selection Sort : $n+(n-1)+(n-2)+\cdots+3+2+1 = \frac{n(n+1)}{2}$

Merge Sort :

6 3 7 2 | 4 8 1 5

$\cancel{2}$ 3 6 7 $\qquad$ $\cancel{1}$ 4 5 8

1 2 3 4 5 6 7 8

6 3 7 2 | 4 8 1 5

63 | 72   48 | 15

6 | 3   7 | 2   4 | 8   1 | 5

splitting phase

36   27   48   15

merging

2 3 6 7   1 4 5 8

1 2 3 4 5 6 7 8

$$T(n) = 2 \cdot T(n/2) + n$$

answer:
$T(n) = n \log_2 n + n$

maybe
$T(n) = n^2$?

$$n^2 \stackrel{?}{=} 2 \cdot (n/2)^2 + n$$
$$= 2 \cdot \frac{n^2}{4} + n$$
$$= \frac{n^2}{2} + n \quad \times$$

$n = 1{,}000{,}000 = 10^6$

I. S.	n^2	$\rightarrow$	$10^{12} = 1{,}000{,}000{,}000{,}000$	trillion
S. S.	n^2	$\rightarrow$	10^{12}	
M. S.	$n\log_2 n$	$\rightarrow$	$20{,}000{,}000$	20 million

$>$ factor of 50,000

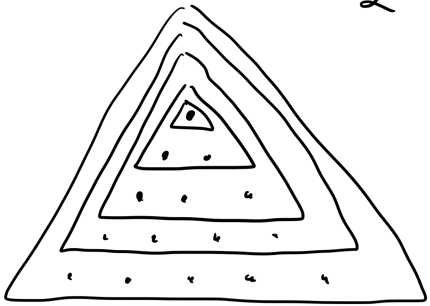
1 sec vs. 13.88 hours

Sequences & Series

- Sequence: $a_n : a_1, a_2, a_3 \ldots$

- Examples
  - $1, 1/2, 1/3, 1/4, 1/5, \ldots$ $\quad a_n = 1/n$
  - $2, 4, 6, 8, 10, \ldots$ $\quad a_n = 2 \cdot n$
  - $7, 8, 9, 10, 11, \ldots$ $\quad a_n = n + 6$
  - $1, 3, 6, 10, 15, 21, \ldots$ $\quad a_n = \dfrac{n(n+1)}{2}$

$\uparrow$ triangular numbers



Common types of sequences: arithmetic, geometric, quadratic (harmonic)

① Arithmetic sequences ← difference between consecutive elements is <u>constant</u>.

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & \dots \\ 4, & 7, & 10, & 13, & 16, & \dots \\ & 3 & 3 & 3 & 3 & \end{array}$$

e.g. 4, 7, 10, 13, 16, ...

a → 4

d → 3

$a = 4$ (starting value)

$d = 3$ (diffence)

$$a_n = a + (n-1)\cdot d \implies a_n = 4 + (n-1)\cdot 3 = 3n + 1$$

a: starting value

d: constant difference

$$a_n = a + (n-1)\cdot d = d\cdot n + (a-d)$$

$a + (n-1)\cdot d$: informative

$d\cdot n + (a-d)$: simplified

E.g.

$-7, -1, 5, 11, 17, 23, 29, \ldots$

$6 \quad 6 \quad 6 \quad 6 \quad 6 \quad \ldots$

$$a_n = -7 + (n-1)\cdot 6$$
$$= 6n - 13$$

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② Geometric Sequences ← constant factor between consecutive elements.

$$a_n = a \cdot r^{n-1}$$

a ↑ starting value

r ↖ constant ratio

e.g.

$1 \quad 2 \quad 3 \quad 4 \quad 5$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$3, 6, 12, 24, 48, \ldots$

$a = 3$

$r = 2$

$$a_n = 3 \cdot 2^{n-1}$$
$$= \frac{3}{2} \cdot 2^n$$

$a = 1$

$r = \frac{1}{2}$

$$a_n = 1 \cdot \left(\frac{1}{2}\right)^{n-1}$$
$$= \left(\frac{1}{2}\right)^{n-1}$$

Quadratic sequences ← second differences are constant

e.g. 1 3 6 10 15 21 ....

2 3 4 5 6 ← first differences

1 1 1 1 ← second diffences

$$a_n = a\cdot n^2 + b\cdot n + c \quad \text{for some } a, b, c$$

$n=1$ $\quad 1 = a\cdot 1^2 + b\cdot 1 + c \Rightarrow a+b+c=1$

$n=2$ $\quad 3 = a\cdot 2^2 + b\cdot 2 + c \Rightarrow 4a+2b+c=3$

$n=3$ $\quad 6 = a\cdot 3^2 + b\cdot 3 + c \Rightarrow 9a+3b+c=6$

$3a+b=2$

$5a+b=3$

$2a=1$

$\boxed{a=\frac{1}{2}}$

$3\cdot\frac{1}{2}+b=2$

$\boxed{b=\frac{1}{2}}$

$\frac{1}{2}+\frac{1}{2}+c=1$

$\boxed{c=0}$

$$\Rightarrow a_n = \frac{1}{2}\cdot n^2 + \frac{1}{2}\cdot n + 0$$

$$= \frac{1}{2}n^2 + \frac{1}{2}n = \frac{1}{2}(n^2+n)$$

$$= \frac{n(n+1)}{2}$$