Progressions / series

$$1+2+3+\dots+n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

arithmetic

quadratic

$$1^2+2^2+3^2+\dots+n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$\frac{1}{1-x}$

geometric

base $x>0$ fixed

if $x<1$: $x^0+x^1+x^2+\dots+x^n+\dots = \underline{\text{finite}}$

$x>1$

$$x^0+x^1+x^2+\dots+x^n = \frac{x^{n+1}-1}{x-1} \quad \text{if } x \neq 1$$

$x=1$: $1^0+1^1+1^2+\dots+1^n = n+1$ $\Theta(n)$ if $x=1$

Harmonic

$$H_n = \frac{1}{1}+\frac{1}{2}+\dots+\frac{1}{n} \simeq \ln(n)+\text{constant} \quad \Theta(\ln(n))$$

Geom series

base x

- $$\boxed{1 + x + x^2 + x^3 + \dots + x^{n-1} = \frac{x^n - 1}{x - 1} \quad \text{if } x \neq 1}$$
- $n = 1 + x + x^2 + \dots + x^{n-1}$ if $x = 1$

proof

$$S = 1 + \cancel{x} + \cancel{x^2} \dots + \cancel{x^{n-1}}$$

$$S \cdot x = \cancel{x} + \cancel{x^2} + x^3 + \dots + \cancel{x^{n-1}} + x^n$$

$$S(x-1) = Sx - S = x^n - 1$$

$$S = \frac{x^n - 1}{x - 1}$$

ind step

$$1+x+x^2+\cdots+x^{n-1}=\frac{x^n-1}{x-1} \Rightarrow 1+x+x^2+\cdots+x^n=\frac{x^{n+1}-1}{x-1}$$

proof

$$1+x+x^2+\cdots+x^n = 1+x+x^2+\cdots+x^{n-1}+x^n$$

$$=\frac{x^n-1}{x-1}+x^n$$

$$=\frac{\cancel{x^n}-1+x^{n+1}-\cancel{x^n}}{x-1}$$

$$=\frac{x^{n+1}-1}{x-1} \checkmark$$

$T(n) = 4T\left(\frac{n}{2}\right) + n$

$T(n) = 4T(n/2) + \Theta(n)$

Divide & conquer

pb(n) RT

solve 4 subproblems of size $n/2$

non-rec load
- before solving subpb — split / decisions
- after solving subpb (combine)

ITERATIONS
- algebra or tree

$k=1 \quad T(n) = n + 4T(n/2)$

$k=2 \quad = n + 4\left[\frac{n}{2} + 4T(n/4)\right] = n + 2n + 4^2 T\left(\frac{n}{4}\right)$

$k=3 \quad = n + 2n + 4^2\left[\frac{n}{4} + 4T\left(\frac{n}{8}\right)\right] = n + 2n + 4n + 4^3 T\left(\frac{n}{2^3}\right)$

$k=4 \quad = n + 2n + 4n + 4^3\left[\frac{n}{2^3} + 4T\left(\frac{n}{2^4}\right)\right] = n + 2n + 4n + 8n + 4^4 T\left(\frac{n}{2^4}\right)$

General k

$= n + 2n + 4n + \cdots 2^{k-1} n + 4^k T\left(\frac{n}{2^k}\right)$

$$= n\left(\underbrace{2^0+2^1+2^2+\cdots+2^{k-1}}\right)+4^k T\left(\frac{n}{2^k}\right)$$

$$= n\,\frac{2^k-1}{2-1} + 4^k T\left(\frac{n}{2^k}\right)$$

Last k we want $T\left(\frac{n}{2^k}\right) = \text{base} \simeq T(1)$

$$\frac{n}{2^k} \simeq 1 \iff k \simeq \log(n)$$

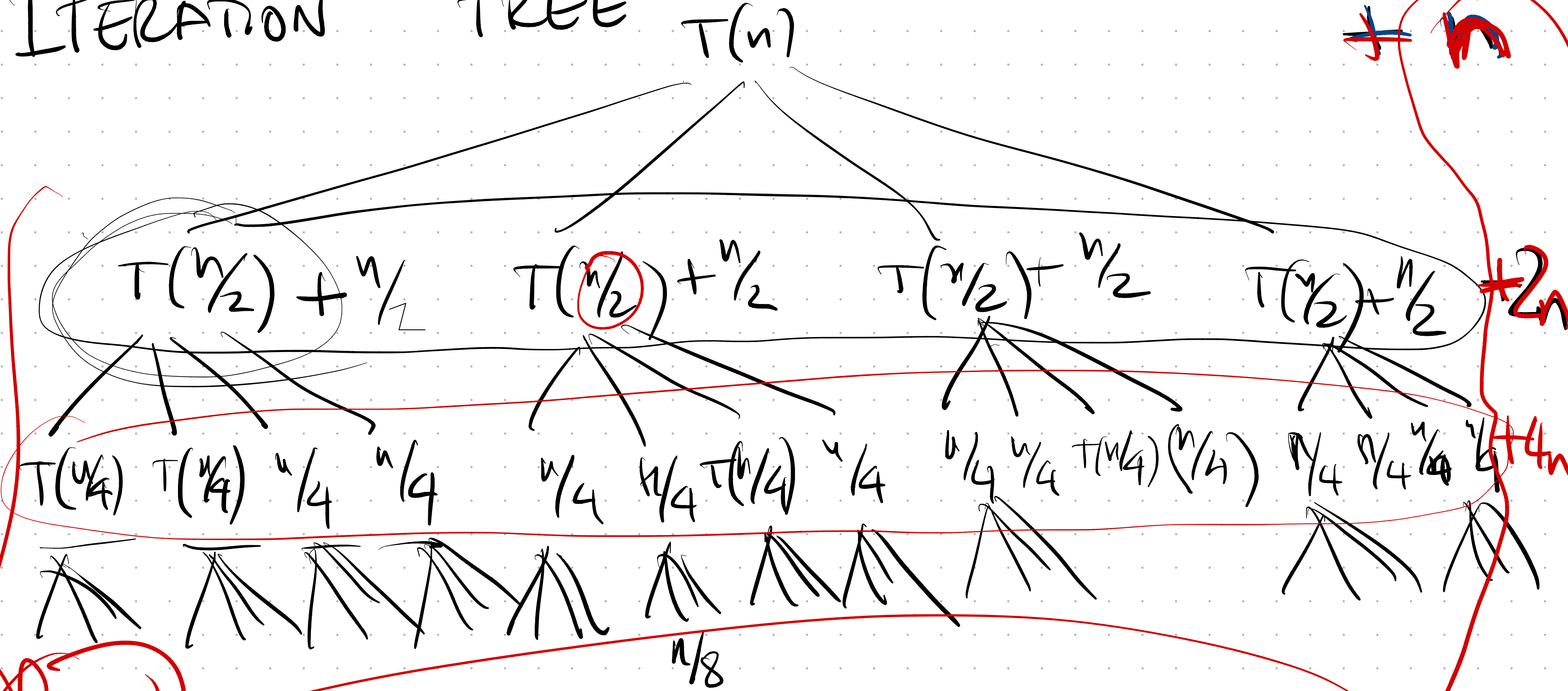
$$= n\cdot\left(2^{\log(n)}-1\right) + 4^{\log(n)}\,T(1)$$

$$= n\cdot(n-1) + 2^{2\log n}\,T(1)$$

$$\Theta(n^2) + \Theta(n^2)$$

$$\Theta(n^2)$$

ITERATION TREE



$\log_2(n)$ layers

rec component

non rec

Total calls?

$4^{\log(n)}$ leaves in tree

$n + 2n + 4n + \dots + ?n$

$\dots \Theta(n^2)$

practice : ITERATIONS / TREE $\longrightarrow$ clean pattern, & done

$\downarrow$

Guess? intuition (not clean)

$\downarrow$

induction proof

$$\boxed{T(n) = n + 4T(n/2)}$$

iterations

not clean pattern

$$\boxed{\text{Guess} \quad \Theta(n^2)}$$

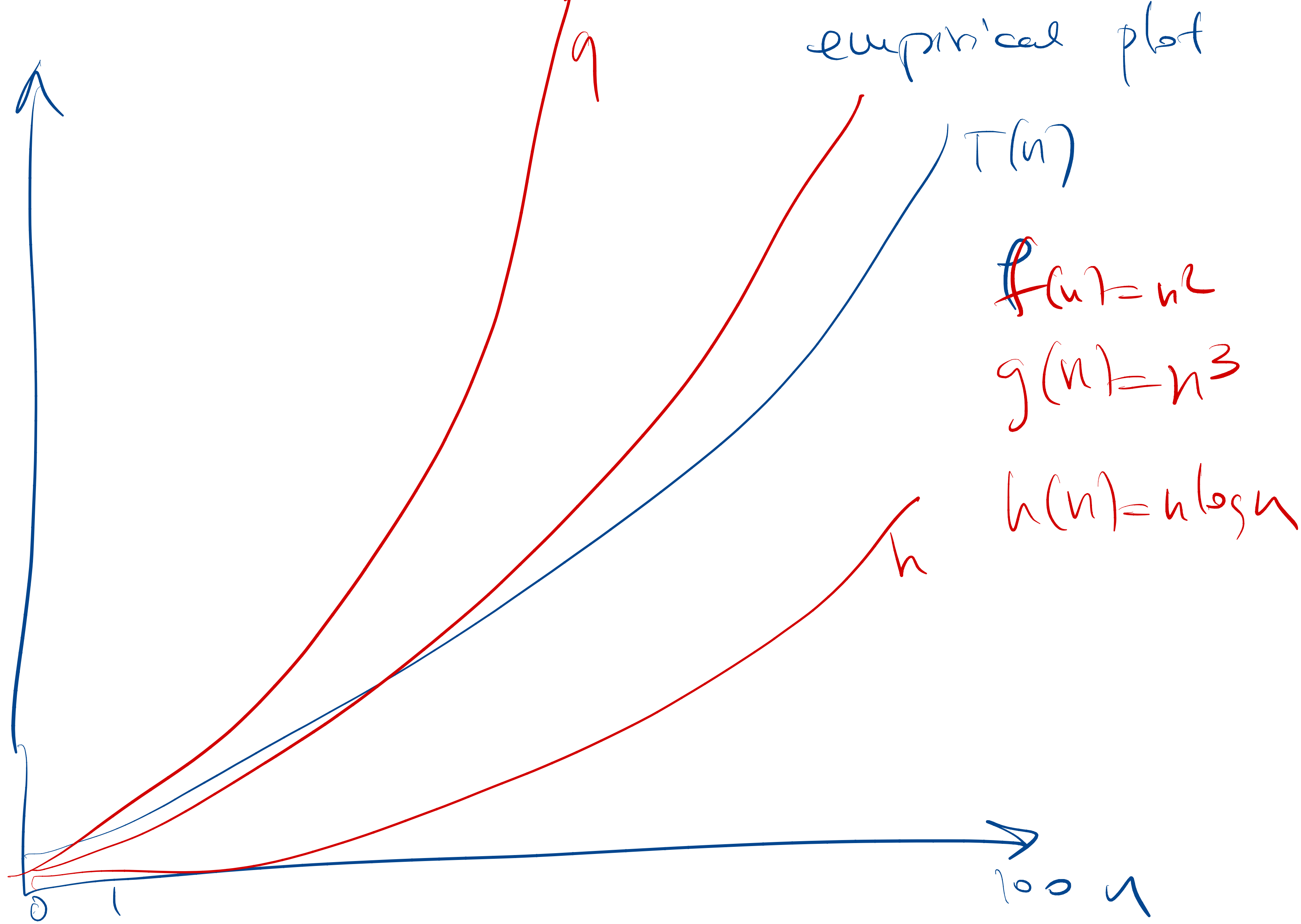
$$c_1 n^2 \leq T(n) \leq c_2 n^2$$

ind step

$$c_1\left(\frac{n}{2}\right)^2 \leq T\left(\frac{n}{2}\right) \Longrightarrow c_1 n \leq T(n)$$

$$T\left(\frac{n}{2}\right) \leq c_2\left(\frac{n}{2}\right)^2 \Longrightarrow$$

$$T(n) \leq c_2 n^2$$



empirical plot
g
T(n)
f(n) = n^2
g(n) = n^3
h(n) = n log n
h
0
1
100
n

$T(n) = 4T(n/2) + n$

Susstitution
- guess
- proof

guess $\Theta(n^2) \iff c_1 n^2 \le T(n) \le c_2 n^2$

ind proof lower bound

$\boxed{c_1 (n/2)^2 \le T(n/2) \implies c_1 n^2 \le T(n)}$ $\left(c_1 n^2 \le T(n) \implies c_1 (2n)^2 \le T(2n) \right)$

proof

$$T(n) = 4T(n/2) + n \ge 4\left[c_1 (n/2)^2\right] + n$$

$$= 4 c_1 n^2/4 + n$$

$$= c_1 n^2 + n \ge c_1 n^2$$

say ex $c_1 = 1$

ind proof Upper Bound

$$T\left(\frac{n}{2}\right) \le c_2\left(n/2\right)^2 \Longrightarrow T(n) \le c_2 n^2$$

Proof

$$T(n) = 4T(n/2) + n \le 4\left[c_2(n/2)^2\right] + n$$

$$4c_2\, n^2/4 + n = c_2 n^2 + n$$

want $\le c_2 n^2$

not true

it only means

its not strong enough

$T(n) \le cn^2$

Ind Proof Upper Bound (Stronger Claim)

$$T(n/2) \le c(n/2)^2 - d\,n/2 \implies T(n) \le cn^2 - dn$$

Proof $T(n) = 4T(n/2) + n \le 4\left[c(n/2)^2 - d\,n/2\right] + n$

$$= 4cn^2/4 - 4d\,n/2 + n$$

$$= cn^2 - n(2d-1)$$

want $\overset{?}{\le} cn^2 - dn$

$c = 1$

$\iff d\cancel{n} \overset{?}{\le} (2d-1)\cancel{n}$

$\iff d \overset{?}{\le} 2d - 1$ say choose $d = 2$

$$T(n) = n^2 + T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right)$$

$$= n^2 + \left[\left(\frac{n}{2}\right)^2 + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right)\right] + \left[\left(\frac{n}{4}\right)^2 + T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)\right]$$

$$= n^2 + \frac{5}{16}n^2 + T\left(\frac{n}{4}\right) + 2T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)$$

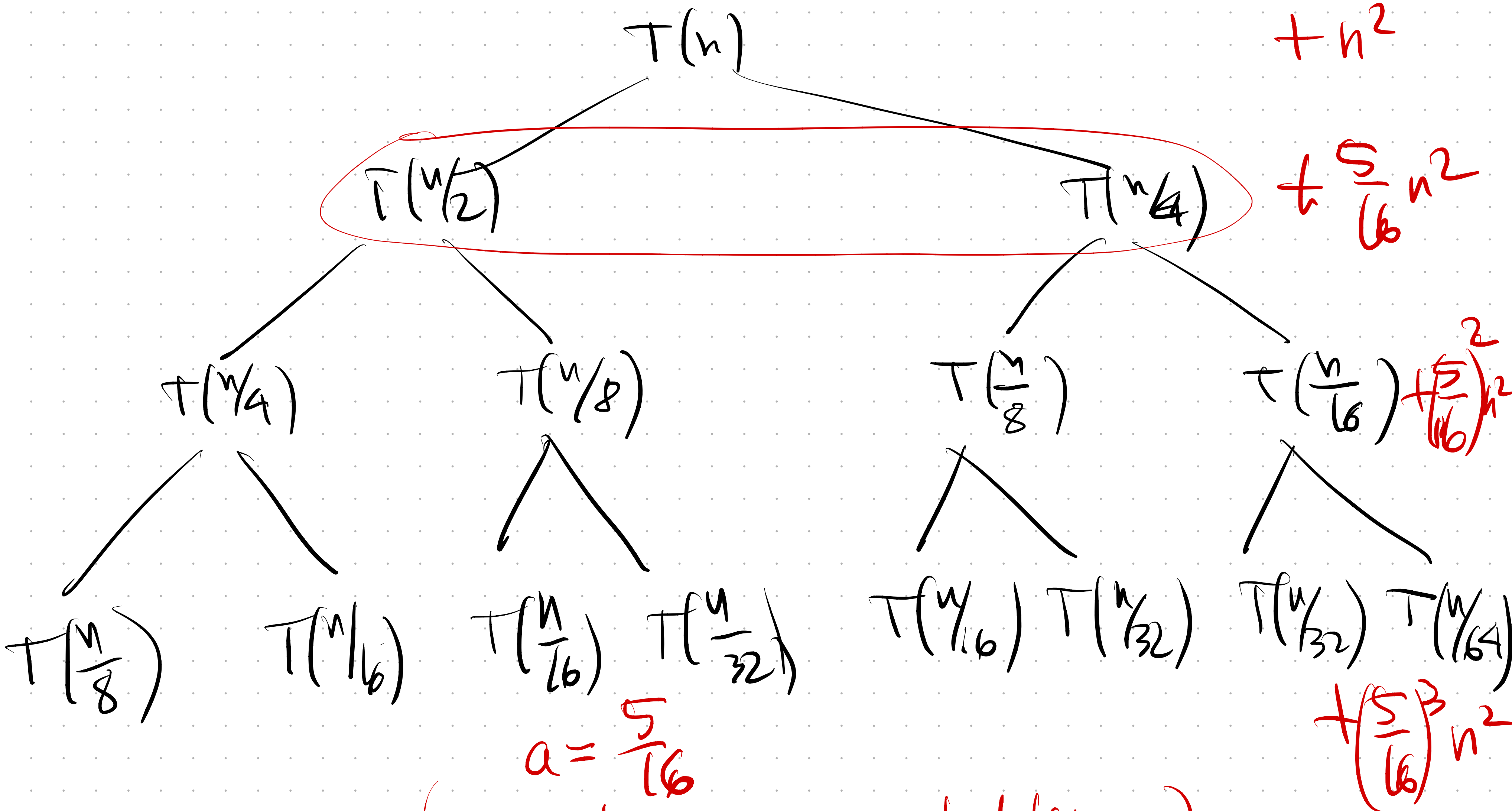
$$= n^2 + \frac{5}{16}n^2 + \left[\left(\frac{n}{4}\right)^2 + T\left(\frac{n}{8}\right) + T\left(\frac{n}{16}\right)\right] + 2\left[\left(\frac{n}{8}\right)^2 + T\left(\frac{n}{16}\right) + T\left(\frac{n}{32}\right)\right] +$$

$$+ \left[\left(\frac{n}{16}\right)^2 + T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right)\right]$$

$$= n^2 + \frac{5}{16}n^2 + \frac{25}{256}n^2 + T\left(\frac{n}{8}\right) + \underline{\underline{3}}T\left(\frac{n}{16}\right) + 3T\left(\frac{n}{32}\right) + T\left(\frac{n}{64}\right)$$

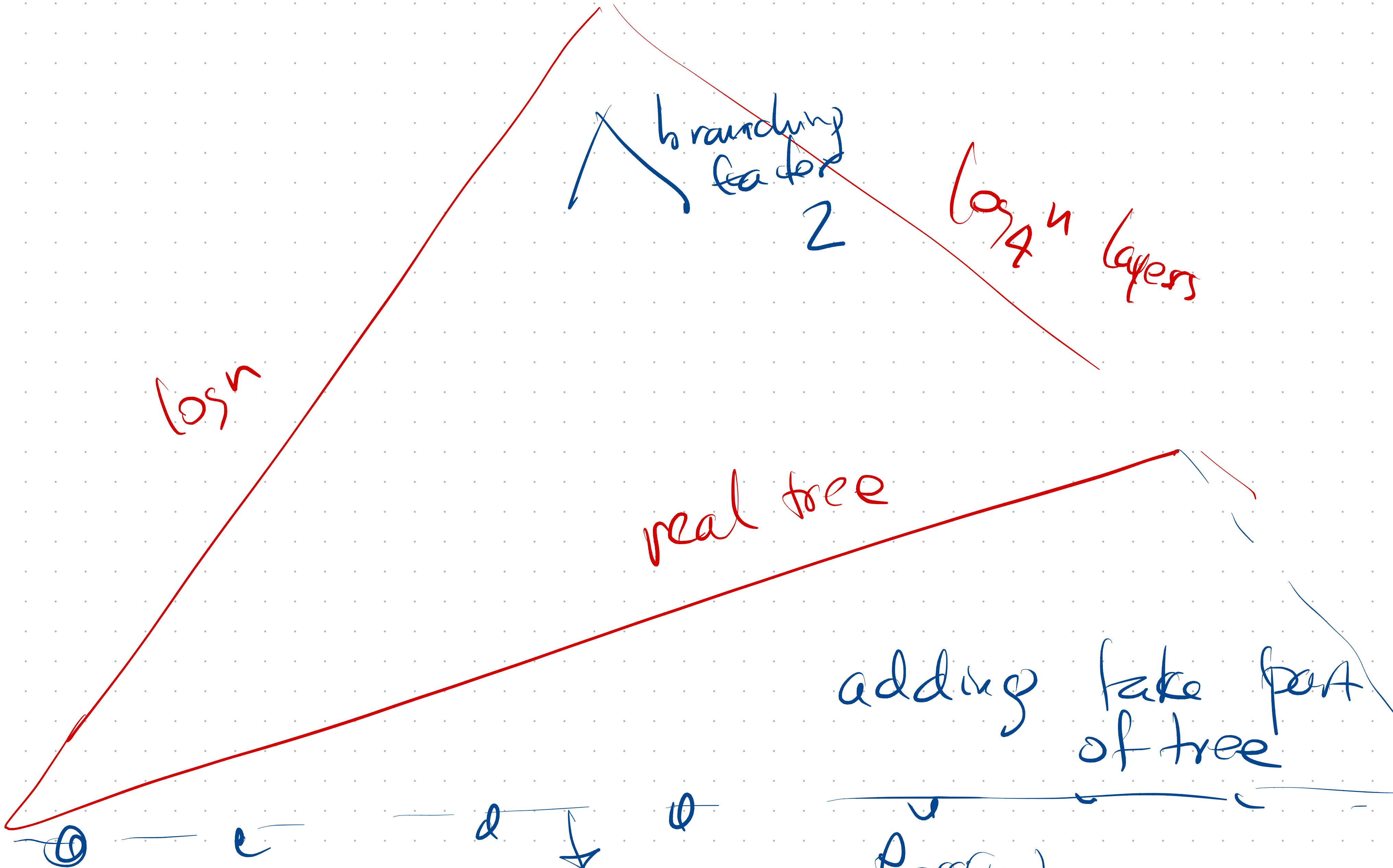
$$= \qquad + \left(\frac{5}{16}\right)^3 n^2$$

~Pascal Δ ??



$$a = \frac{5}{16}$$

non rec $n^2\left(\underbrace{a^0 + a^1 + \dots + a^{\text{last layer}}}_{\text{constant} < \frac{1}{1-a}}\right) = \Theta(n^2)$



total

$2^{\log_2(n)} = n$ $\Theta(n)$

total $\Theta(n^2)$ smaller than non-rec load $\Theta(n^2)$

This PB short cut T( ) rec part is too small to matter

$$\Rightarrow \Theta\left[n^2\left(1+5/16+\left(\frac{5}{16}\right)^2+\dots+\left(\frac{5}{16}\right)^{\text{lastle}}\right)\right]$$

base $= \frac{5}{16} < 1 \Rightarrow$ FINITE $\Theta(1)$

$$\Rightarrow \Theta(n^2)$$

Substitute / Guess $T(n) = T(n/2) + T(n/4) + n^2 = \Theta(n^2)$

proof ind Upper Bound

$$\left.\begin{aligned} T(n/2) &\le c(n/2)^2 \\ T(n/4) &\le c(n/4)^2 \end{aligned}\right\} \Longrightarrow T(n) \le cn^2$$

proof :

$T(n) = T(n/2) + T(n/4) + n^2 \leq c(n/2)^2 + c(n/4)^2 + n^2$

$$\boxed{= \cancel{n^2}\left(c/4 + c/16 + 1\right) \overset{?}{\leq} \cancel{n^2} \cdot c}$$

want

$\Longleftrightarrow \quad \frac{c}{4} + \frac{c}{16} + 1 \leq c$

$T(n) = T(n/2) + T(n/4) + n \qquad n/2 \quad n/4$

$\Theta(n) \qquad \frac{c}{2} + \frac{c}{4} \leq c$

$T(n) = T(n/2) + T(n/4) + T(n/4) + n$

$c/2 + c/4 + c/4 \leq c$

Simple **Master Theorem** $T(n) = aT(n/b) + \Theta(n^c)$

a branches, $\frac{n}{b}$ subproblems each, n^c non rec load

$i=0$ $T(n)$ $+ n^c \left(\frac{a}{b^c}\right)^0$

a

$i=1$ $T(n/b)+\left(\frac{n}{b}\right)^c$ $T(n/b)+\left(\frac{n}{b}\right)^c$ $T(n/b)+\left(\frac{n}{b}\right)^c$

$a \cdot (n/b)^c$

a

$i=2$ $T\left(\frac{n}{b^2}\right)$ $T(n/b^2)$ $T(n/b^2)$ $T(n/b^2)+\left(\frac{n}{b^2}\right)^c$

a^2 branches

$a^2\left(\frac{n}{b^2}\right)^c$

a

layers $\log_b(n)$

$i=3$ $T\left(\frac{n}{b^3}\right)$

$a^3 \cdot \left(\frac{n}{b^3}\right)^c$

$\left(\frac{a}{b^c}\right)^3 \cdot n^c$

Total $\displaystyle n^c \sum_{i=0}^{\log_b n - 1} \left(\frac{a}{b^c}\right)^i + a^{\log_b n}$

$\left(n^{\log_b a}\right)$

$\frac{a}{b^c}$ = base = r

$\underbrace{n^c \sum_{i=0}^{\log_b n - 1} \left(\frac{a}{b^c}\right)^i}_{\text{non-rec geom series}}$

rec #calls

Case 1 – exp

base $\frac{a}{b^c} > 1$

$c < \log_b a$

Case 2 – linear

base $\frac{a}{b^c} = 1$

$c = \log_b a$

Case 3 – const

base $\frac{a}{b^c} < 1$

$c > \log_b a$

case 1 $c < \log_b a \iff \frac{a}{b^c} > 1$

$x^0 + x^1 + \dots + x^r = \frac{x^{r+1} - 1}{x - 1}$

$$T(n) = n^c \sum_{i=0}^{\log_b n - 1} \left(\frac{a}{b^c}\right)^i + \Theta\left(n^{\log_b a}\right)$$

$x = \frac{a}{b^c}$

$r = \log_b n - 1$

$$= n^c \frac{\left(\frac{a}{b^c}\right)^{\log_b n} \cancel{-1}}{\cancel{\left(\frac{a}{b^c}\right) - 1}} + \Theta\left(n^{\log_b a}\right)$$

$$= \Theta\left(n^c \frac{a^{\log_b n}}{\left(b^c\right)^{\log_b n}}\right) + \Theta\left(n^{\log_b a}\right)$$

$$= \Theta\left(\cancel{n^c} \frac{n^{\log_b a}}{n^{\log_b (b^c)}}\right)$$

$\cancel{n^c}$

???

$$\boxed{a^{\log_b x} = x^{\log_b a}}$$

$$= \Theta\left(n^{\log_b a}\right)$$ ✓

<u>Case 2</u> $\quad c = \log_b a \qquad \frac{a}{b^c} = 1$

$$T(n) = n^c \sum_{i=0}^{\log_b n} 1^i + \Theta\left(n^{\log_b a}\right) =$$

$$= n^c \cdot \log_b n + \Theta\left(n^{\underbrace{\log_b a}_{=c}}\right)$$

$$\Theta\left(n^c \cdot \log_b n\right) + \Theta\left(n^c\right)$$

$$\Theta\left(n^c \log_b n\right) = \Theta\left(n^{\log_b a} \cdot \log n\right)$$

case 3 $\boxed{c > \log_b a}$ $\qquad \frac{a}{b^c} < 1$

$$T(n) = n^c \underbrace{\boxed{\sum_{i=0}^{\log_b n - 1} \left(\frac{a}{b^c}\right)^i}}_{\text{bound constant}} + \Theta\left(n^{\log_b a}\right)$$

$$\Theta(n^c) + \Theta\left(n^{\log_b a}\right)$$

$$\Theta(n^c) \quad ✓$$

Book - Master Th $T(n) = aT(n/b) + f(n)$

$f = n^2/\log n$

$T(n) = 4T(n/2) + \Theta(n^3)$ $a=4$ $b=2$ $c=3$

case $\frac{a}{b^c} < 1$ case 3

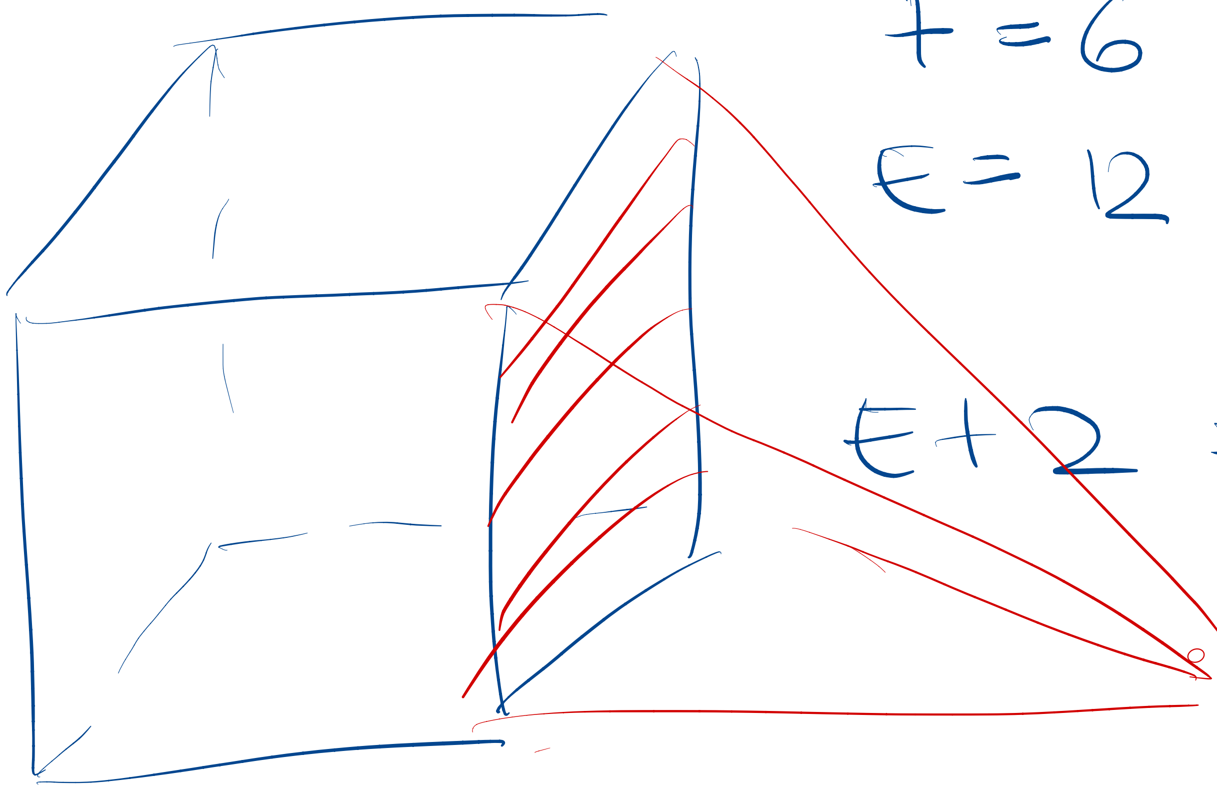
$\Theta(n^c)$

binary search

$T(n) = T(n/2) + 1 \Rightarrow$ $a=1$ $b=2$ $c=0$

$\frac{a}{b^c} = 1$ case 2

$\Theta(n^{\log_b a} \cdot \log n) = \Theta(\log n)$

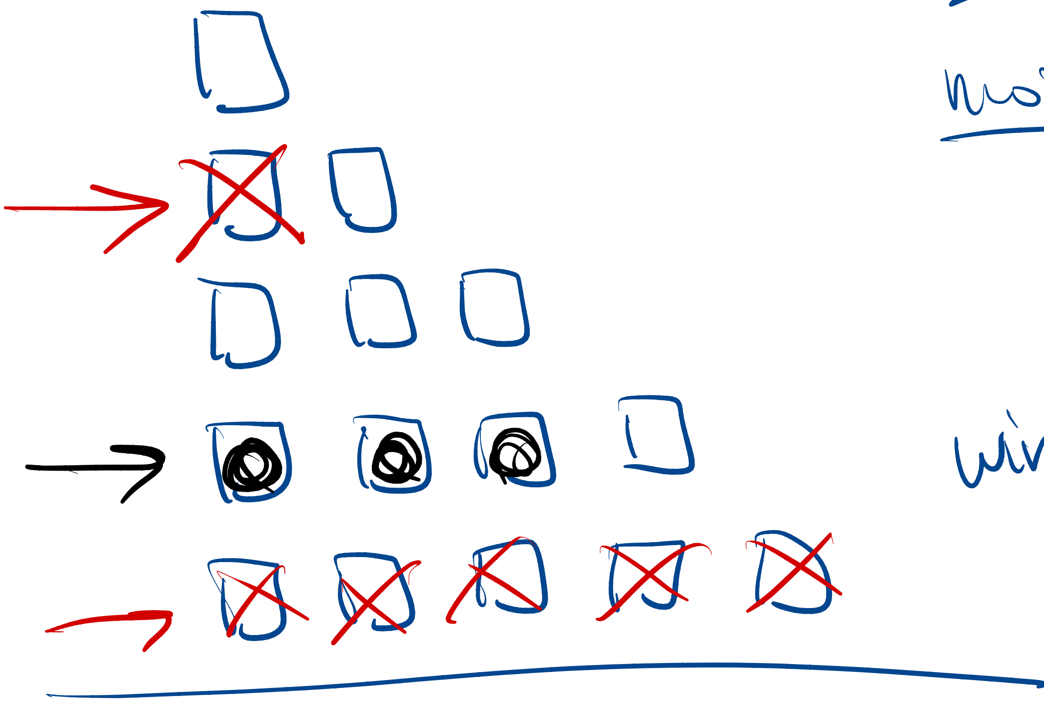


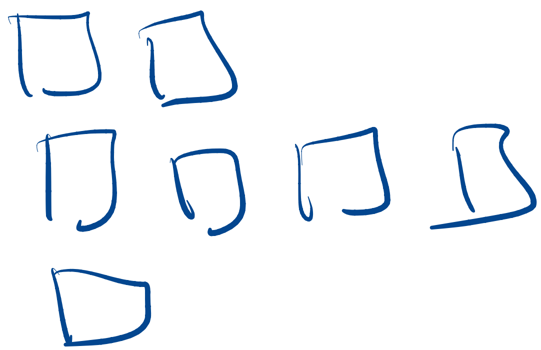
$F = 6 \quad V = 8$

$E = 12$

$E + 2 = F + V$

Square Game (Diversion, not lecture)





2 players, alternate turns

move: pick a row
remove any squares ≥ 1 from that row

wins: who picks the last sq.

$$n^{\log_b a} = a^{\log_b n} \quad ??$$

Induction Method for solving rec eq

iterations
tree
chang var

→ Guess ? → prove by induction

→ clear pattern → calculations → result

$T(n) = 2T(n/2) + n$ guess $\Theta(n \log n)$

$c_1 n \log n \leq T(n) \leq c_2 n \log n \quad \forall n \geq n_0$

LB UB

LB ind step $n/2 \to n$:

$$T(n/2) \geq c_1 \frac{n}{2} \log \frac{n}{2} \Longrightarrow T(n) \geq c_1 n \log n$$

ind hyp ind concl

proof

ind hyp

$$T(n) = \underline{2T(n/2)} + n \geq 2 c_1 \frac{n}{2} \log\left(\frac{n}{2}\right) + n \overset{\text{want}}{\geq} c_1 n \log n$$

$$c_1 n (\log n - \log 2) + n \overset{\text{want}}{\geq} c_1 n \log n$$

$$\cancel{c_1 n \log n} - c_1 n + n \overset{\text{want}}{\geq} \cancel{c_1 n \log n}$$

$$n \overset{\text{want}}{\geq} c_1 n$$

$$1 \overset{\text{want}}{\geq} c_1 \quad \checkmark$$

ex. $c_1 = 0.5$

UB ind step $n/2 \to n$ $\quad T(n/2) \le c_2 n/2 \log(n/2) \implies$

$T(n) \le c_2 n \log n$

<u>proof</u>

$$T(n) = 2T\left(\tfrac{n}{2}\right) + n \le 2 \underbrace{c_2 n/2 \log(n/2)}^{\text{ind hyp}} + n \overset{\text{want}}{\le} c_2 n \log n$$

$$c_2 \cdot n \cdot (\log n - 1) + n \overset{\text{want}}{\le} c_2 n \log n$$

$$\cancel{c_2 n \log n} - c_2 n + n \overset{\text{want}}{\le} \cancel{c_2 n \log n}$$

$$n(1 - c_2) \overset{\text{want}}{\le} 0 \quad \checkmark \quad c_2 > 1$$

$$T(n) = 4T(n/2) + n \qquad \Theta(n^2)$$

$$c_1 n^2 \le T(n) \le c_2 n^2$$

LB ind. step: $n/2 \to n$ $\quad \underbrace{c_1\left(\frac{n}{2}\right)^2 \le T(n/2)}_{\text{ind hyp}} \Longrightarrow \underbrace{c_1 n^2 \le T(n)}_{\text{cond}}$

proof $T(n) = 4T(n/2) + n \ge 4c_1\left(\frac{n}{2}\right)^2 + n \overset{\text{want}}{\ge} c_1 n^2$

$$\cancel{4}c_1 \frac{n^2}{\cancel{4}} + n \overset{\text{want}}{\ge} c_1 n^2$$

$$c_1 n^2 + n \overset{\text{want}}{\ge} c_1 n^2 \quad \checkmark$$

$$n \ge 0$$

UB ind step $T(n/2) \le c_2 (n/2)^2 \Longrightarrow T(n) \le c_2 n^2$

proof → broken

$$T(n) = 4T(n/2) + n \le 4 c_2 \frac{n^2}{4} + n \overset{\text{want}}{\le} c_2 n^2$$

$$c_2 n^2 + n \overset{\text{want}}{\le} c_2 n^2$$

Impossible

proof 2

$$T(n) = 4T(n/2) + n \le 4 c_2 \left(\frac{n}{2}\right)^2 + n$$

$$= c_2 n^2 + n = O(n^2) \le \text{constant } n^2$$

diff constant

proof 3 Stronger induction claim $T(n) \le c_2 n^2 - dn$

ind step

$$T\left(\frac{n}{2}\right) \le c_2\left(\frac{n}{2}\right)^2 - d\frac{n}{2} \Longrightarrow T(n) \le c_2 n^2 - dn \quad \forall n \ge n_0$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n \underset{\text{ind hyp}}{\le} 4\left(c_2\left(\frac{n}{2}\right)^2 - d\frac{n}{2}\right) + n \overset{\text{want}}{\le} c_2 n^2 - dn$$

$$4c_2\frac{n^2}{4} - 4d\frac{n}{2} + n \overset{\text{want}}{\le} c_2 n^2 - dn$$

$$\cancel{c_2 n^2} - 2dn + n \overset{\text{want}}{\le} \cancel{c_2 n^2} - dn$$

$$n \overset{\text{want}}{\le} dn \quad \checkmark$$

$d \ge 1 \quad d = 2$ ex $\quad \cancel{c_2 \le 5}$

$$c_2 > \frac{T(n_0+1)}{(n_0+1)^2}$$

$$T(n) \le c_2 n^2 - dn \le c_2 n^2$$

$T(n) = n^2 + T(n/2) + T(n/4)$ $\quad\quad \Theta(n^2)$

$$\underbrace{c_1 \cdot n^2 \leq}_{\text{obvious}} \boxed{T(n) \leq c_2 n^2}$$

UB ind step $\left.\begin{array}{l} T(n/2) \leq c_2 \left(\frac{n}{2}\right)^2 \\ \text{and} \\ T(n/4) \leq c_2 \left(\frac{n}{4}\right)^2 \end{array}\right\} \Longrightarrow T(n) \leq c_2 n^2$

proof: $T(n) = n^2 + \underline{T(n/2)} + T(n/4) \leq n^2 + c_2\left(\frac{n}{2}\right)^2 + c_2\left(\frac{n}{4}\right)^2$

want

$\leq c_2 n^2$

$\cancel{n^2} + c_2 \frac{\cancel{n^2}}{4} + c_2 \frac{\cancel{n^2}}{16} \overset{\text{want}}{\leq} c_2 \cancel{n^2}$ $\quad \Big| \cdot 16$

$16 + 4c_2 + c_2 \leq 16 c_2$ $\quad \Big\| \; 16 + 5c_2 \leq 16 c_2$ ✓

$$T(n) = \frac{n^2}{\log n} + 4T\left(\frac{n}{2}\right) \qquad k=1$$

iterations

$$= \frac{n^2}{\log n} + 4\left[\frac{(n/2)^2}{\log(n/2)} + 4T\left(\frac{n}{4}\right)\right]$$

$$= \frac{n^2}{\log n} + \frac{n^2}{\log(n/2)} + 4^2 T\left(\frac{n}{4}\right) \qquad k=2.$$

$$= \frac{n^2}{\log n} + \frac{n^2}{\log(\frac{n}{2})} + \cancel{4^2}\left[\frac{\cancel{(n/4)^2}}{\log(\frac{n}{4})} + 4T\left(\frac{n}{8}\right)\right]$$

$$= \frac{n^2}{\log n} + \frac{n^2}{\log \frac{n}{2}} + \frac{n^2}{\log(\frac{n}{4})} + 4^3 T\left(\frac{n}{2^3}\right) \qquad k=3$$

General k

$$\frac{n^2}{\log n} + \frac{n^2}{\log \frac{n}{2}} + \frac{n^2}{\log \frac{n}{4}} \cdots + \frac{n^2}{\log\left(\frac{n}{2^{k-1}}\right)} + 4^k T\left(\frac{n}{2^k}\right)$$

last k want $T\left(\frac{n}{2^k}\right) \simeq T(1) \iff k \simeq \log n$

$$\sum_{j=0}^{k-1} \frac{n^2}{\log\left(\frac{n}{2^j}\right)} + 4^k T\left(\frac{n}{2^k}\right)$$

$k = \log n$

$$n^2 \sum_{j=0}^{\log n - 1} \frac{1}{\log(n) - \log(2^j)} + 4^{\log n} \underline{\underline{T(1)}}$$

$$n^2 \boxed{\sum_{j=0}^{\log n - 1} \frac{1}{\log(n) - j}} + \underline{n^2 T(1)} \; \Theta(n^2)$$

$$\sum_{j=0}^{\log n - 1} \frac{1}{\log(n) - j} = \sum_{l=1}^{\log n} \frac{1}{l}$$

$$\frac{1}{\log n} + \frac{1}{\log n - 1} + \ldots + \frac{1}{2} + \frac{1}{\log n - (\log n - 1)} = \frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{\log n}$$

$$\boxed{H_n = \boxed{\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{n}} \simeq \ln(n) + \text{const}} = \Theta(\log(\log n))$$

$$\Theta(\log n)$$

$$n^2 \cdot \Theta(\log(\log(n))) + \Theta(n^2)$$

$$\Theta(n^2 \log \log n)$$

---

compounded logs $t=4$

$$f(n) = \log(\log(\log(\log(n)))) \simeq 10$$

$$\log(\log(\log(n))) \simeq 2^{10}$$

$$\log(\log(n)) = 2^{2^{10}}$$

$$n = 2^{2^{2^{2^{10}}}}$$

$$\log_a x = \log_b x \cdot \log_a b$$

const

$$\Theta(\log_a n) = \Theta(\log_b(n)) ✓$$

$$\Theta\left(2^{\log_a n}\right) \neq \Theta\left(2^{\log_b(n)}\right)$$

$$T(n) = \frac{n^2}{\log n} + 4T\left(\frac{n}{2}\right)$$

$\neq n^2$

MT? no

class

$$f(n) = \frac{n^2}{\log n}$$

$$f = \frac{n^2}{\log n}$$

$a=4 \quad b=2 \quad \log_b a = 2$

f ? vs $n^{\log_2 a} = n^2$

$$\lim_{n\to\infty} \frac{f}{n^2} = \frac{1}{\log(n)} = 0$$

MT book $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$\boxed{f(n) \ll n^{\log_b a}}$

$f(n) = O\left(n^{\log_b a - \varepsilon}\right)$ $\Big\} \Rightarrow \Theta\left(n^{\log_b a}\right)$

$f(n) = \Theta\left(n^{\log_b a}\right) \Rightarrow \Theta\left(n^{\log_b a} \cdot \log n\right)$

$f(n) \gg n^{\log_b a}$

$f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right) \Rightarrow \Theta(f(n))$

$af(n/b) < \text{const} \cdot f(n)$

$f(n) \overset{\text{want}}{\leq} c \cdot n^{2-\varepsilon}$

$\frac{n^2}{\log n} \overset{\text{want}}{\leq} c\, n^{2-\varepsilon}$

$n^{\varepsilon} \overset{\text{want}}{\leq} c \cdot \log(n)$

not true

$$\boxed{T(n) = \frac{n}{\log n} + 4T\left(\frac{n}{2}\right)}$$

$f(n) = \frac{n}{\log n}$ vs n^2

$\frac{n}{\log n} < n < n^{2-\varepsilon}$ ✓

$$T(n) = \frac{n^3}{\log n} + 4T\left(\frac{n}{2}\right)$$

$f(n)\ \frac{n^3}{\log n}$ vs n^2

wait

$\frac{n^3}{\log n} > c\ n^{2+\varepsilon}$ yes

$\frac{n^3}{\log n} > n^{2.5} > n^{2+\varepsilon}$ ✓

$a f\left(\frac{n}{b}\right) < \text{const}\ f(n)$ controll growth