Matrix Multiplication using Strassen's Eq.

$$\underbrace{\begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}}_{C_{n\times n}} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}}_{A_{n\times n}} \times \underbrace{\begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}}_{B_{n\times n}}$$

$C_1, C_2, C_3, C_4, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$ – $n/2 \times n/2$

$$\left.\begin{aligned}
C_1 &= [A_1 A_2] \cdot \begin{bmatrix} B_1 \\ B_3 \end{bmatrix} = A_1 B_1 + A_2 B_3 \\
C_2 &= [A_1 A_2] \cdot \begin{bmatrix} B_2 \\ B_4 \end{bmatrix} = A_1 B_2 + A_2 B_4 \\
C_3 &= [A_3 A_4] \cdot \begin{bmatrix} B_1 \\ B_3 \end{bmatrix} = A_3 B_1 + A_4 B_3 \\
C_4 &= [A_3 A_4] \cdot \begin{bmatrix} B_2 \\ B_4 \end{bmatrix} = A_3 B_2 + A_4 B_4
\end{aligned}\right\} 8 \text{ multiplic.}$$

trick: only use 7 multiplication (Strassen's)

7 products (matrices) → compute C_1, C_2, C_3, C_4 only using addition/subtraction

$P_1 = A_1 (B_2 - B_4)$

$P_2 = (A_1 + A_2) B_4$

$P_3 = (A_3 + A_4) B_1$

$P_4 = A_4 (B_3 - B_1)$

$P_5 = (A_1 + A_4)(B_1 + B_4)$

$P_6 = (A_2 - A_4)(B_3 + B_4)$

$P_7 = (A_1 - A_3)(B_1 + B_2)$

$C_1 = P_5 + P_4 - P_2 + P_6$

$C_2 = P_1 + P_2$

$C_3 = P_3 + P_4$

$C_4 = P_5 + P_1 - P_3 - P_7$

done! $C = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$

7 multiplications of size $n/2$, n^2 combination ⇒

$T(n) = 7 \cdot T(n/2) + \Theta(n^2)$