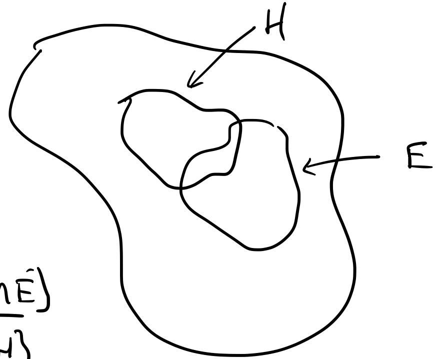
Conditional Probability

prob of E given H

$$\Pr\{H \cap E\} = \Pr\{H\} \cdot \underline{\Pr\{E \mid H\}}$$



$$\Rightarrow \Pr\{E \mid H\} = \frac{\Pr\{H \wedge E\}}{\Pr\{H\}}$$

$$= \Pr\{E\} \cdot \Pr\{H \mid E\}$$

$$\Rightarrow \Pr\{H \mid E\} = \frac{\Pr\{H \wedge E\}}{\Pr\{E\}}$$

Independence

$$\Pr\{E \mid H\} = \Pr\{E\}$$

$$\Pr\{H \mid E\} = \Pr\{H\}$$

$$\Rightarrow \Pr\{H \cap E\} = \Pr\{H\} \cdot \Pr\{E \mid H\} = \Pr\{H\} \cdot \Pr\{E\}$$

Bayes Law:

$$\underline{\Pr\{E\}} \cdot \underline{\Pr\{H \mid E\}} = \Pr\{H \wedge E\} = \underline{\Pr\{H\}} \cdot \underline{\Pr\{E \mid H\}}$$

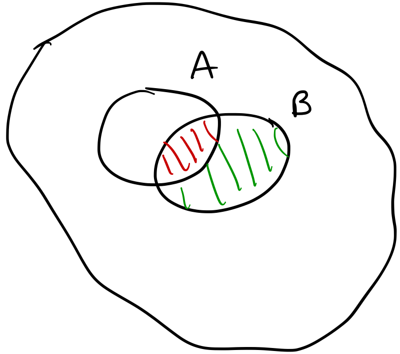
$$\Pr\{H \mid E\} = \frac{\Pr\{E \mid H\} \cdot \Pr\{H\}}{\Pr\{E\}}$$

H = hypothesis: do have Zika?

E = evidence: did blood test come back pos?

Bayes Law

$$\Pr(B|A)\cdot\Pr(A) = \Pr(A\cap B) = \Pr(A|B)\cdot\Pr(B)$$



$$\Pr(A|B) = \frac{\Pr(B|A)\cdot\Pr(A)}{\Pr(B)}$$

$$= \frac{\Pr(B|A)\cdot\Pr(A)}{\Pr\{B|A)\cdot\Pr(A\}+\Pr\{B|\bar{A}\}\cdot\Pr\{\bar{A}\}}$$

what is $\Pr(B)$?

$$p(B) = \quad + $$

$$= p(B|A)\cdot p(A) + p(B|\bar{A})\cdot p(\bar{A})$$

$$\Pr\{H|E\} = \frac{\Pr\{E|H\}\cdot\Pr\{H\}}{\Pr\{E|H\}\cdot\Pr(H\}+\Pr\{E|\bar{H}\}\cdot\Pr\{\bar{H}\}}$$

Example: Zika in FL 2016

- Prevalence of Zika in S.FL. $p(\text{Zika}) = 10^{-5}$ (1 in 100,000)
- accuracy of blood test is 99 %

  i.e. $p(\text{pos. test} \mid \text{Zika}) = 0.99$ ← test subjects who had Zika

  $p(\text{pos. test} \mid \text{no Zika}) = 0.01$ ← control group who don't have Zika

- You test <u>positive</u> : what it is the chance that you have Zika?

$\Longrightarrow$ **<u>Not</u>** $p(\text{pos. test} \mid \text{Zika}) = 0.99$

$\Longrightarrow$ You want $p(\text{Zika} \mid \text{pos. test})$!

$$p(\text{zika} \mid \text{pos test}) = \frac{p(\text{pos. test} \mid \text{zika}) \cdot p(\text{zika})}{p(\text{pos test})}$$

$$= \frac{p(\text{pos. test} \mid \text{zika}) \cdot p(\text{zika})}{p(\text{pos test} \mid \text{zika}) \cdot p(\text{zika}) + p(\text{pos test} \mid \text{not zika}) \cdot p(\text{not zika})}$$

$$= \frac{0.99 \times 10^{-5}}{0.99 \times 10^{-5} + 0.01 \times (1 - 10^{-5})}$$

$$= \frac{0.0000099}{0.0000099 + 0.0099999}$$

$$\approx 0.00099$$

$$\approx 0.1\%$$ i.e. only 1 in 1,000 !

Seems wildly counter intuitive, but...

10,000,000 people in FL. 10^7

w/ zika : $10^7 \cdot 10^{-5} = 10^2 = 100$

w/o zika : 9,999,900

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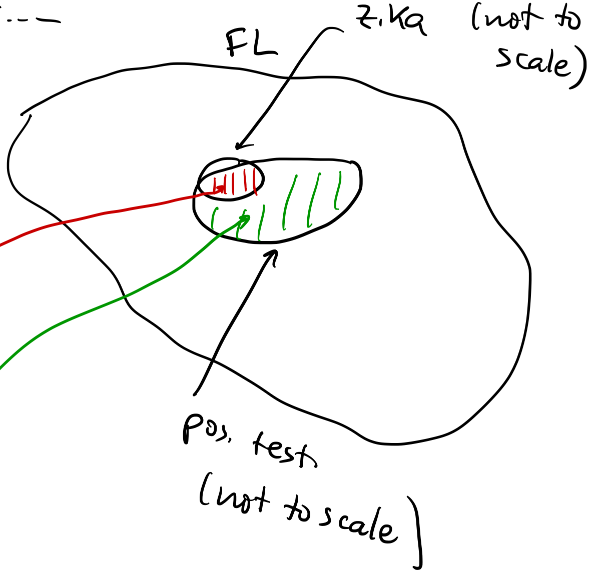
test pos w/ zika : $100 \cdot 0.99 = 99$

test pos. w/o zika : $9{,}999{,}900 \times 0.01$

$= 99{,}999$



$\therefore$ If you test pos.,

you're about 1,000 times more likely to be among 99,999 who <u>don't</u> have zika but test pos. than among the 99 who do have zika and test pos.

Monty Hall Problem

- Let A be door that player picks
- Let B be door that Monty Hall opens
  - ↳ always a goat (not prize)

$$P(\text{prize in } A \mid \text{MH opens } B) = \frac{P(\text{MH opens } B \mid \text{prize } A) \cdot P(\text{prize } A)}{P(\text{MH opens } B)} = \frac{1/2 \cdot 1/3}{1/2} = 1/3$$

$$P(\text{prize in } B \mid \text{MH open } B) = \frac{P(\text{MH opens } B \mid \text{prize } B) \cdot P(\text{prize } B)}{P(\text{MH opens } B)} = \frac{0 \cdot 1/3}{1/2} = 0$$

$$P(\text{prize in } C \mid \text{MH open } B) = \frac{P(\text{MH open } B \mid \text{prize } C) \cdot P(\text{prize } C)}{P(\text{MH opens } B)} = \frac{1 \cdot 1/3}{1/2} = 2/3$$

$$\begin{aligned} P(\text{MH opens } B) &= 1/2 \cdot 1/3 + 0 \cdot 1/3 + 1 \cdot 1/3 \\ &= 1/6 + 1/3 \\ &= 1/2 \end{aligned}$$

Example: Zika

- prevalence of Zika in S. Florida, $p(\text{zika}) = 10^{-5}$, i.e. 1 in 10,000

Correction: 1 in 100,000

- accuracy of blood test is 99%

i.e. $p(\text{pos. test} \mid \text{zika}) = 0.99$ ← test subjects

$p(\text{pos. test} \mid \text{no zika}) = 0.01$ ← control group

- You test <u>positive</u>: what is chance that you have zika?

$\Rightarrow$ <u>Not</u> $p(\text{pos test} \mid \text{zika}) = 0.99$

$\Rightarrow$ you <u>want</u> $p(\text{zika} \mid \text{pos. test})$!

$$p(\text{zika} \mid \text{pos test}) = \frac{p(\text{pos test} \mid \text{zika}) \cdot p(\text{zika})}{P(\text{pos test})}$$

$$= \frac{p(\text{pos test} \mid \text{zika}) \cdot p(\text{zika})}{p(\text{pos test} \mid \text{zika}) \cdot p(\text{zika}) + p(\text{pos test} \mid \text{not zika}) \cdot p(\text{not zika})}$$

$$= \frac{0.99 \times 10^{-5}}{0.99 \times 10^{-5} + 0.01 \times (1 - 10^{-5})}$$

$$= \frac{0.0000099}{0.0000099 + 0.0099999}$$

$$\approx 0.00099$$

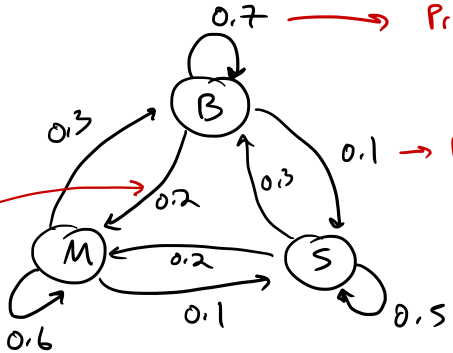
$$\approx 0.1\%$$ 1 in 1,000

Markov chains

B: Bertucci's

M: Margaritas

S: Sato



$\Pr\{B|B\}$

$\Pr\{S|B\}$

$\Pr\{M|B\}$

3 reds $\Rightarrow$ must add to 1