Deadline: October 16 at 8pm eastern

CS1802 Recitation 5

Fall 2020

October 13 - 16, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

first $\quad$ 2^{nd} $\quad\ldots\quad$ $n^{th}-1$ $\quad$ last n^{th}

$\underline{\quad}$ $\underline{\quad}$ $\underline{\quad}$ – – – $\underline{\quad}$ $\underline{\quad}$

n option $\quad$ $n-1$ $\quad$ $(n-2)$ $\quad\ldots\quad$ 2 $\quad$ 1

Permutation (n) = # of ways to create sequences (in order) wint n elements (no repetition) $= n!$

Counting: Permutation / Combinations / Balls-into-bins

$P(n,k)$ permute k items out of n

$C_{n,k}$ $\binom{n}{k}$ n choose k

n items $n=5$

$\{a,b,c,d,e\}$

choose k items out of n in order $\Rightarrow$ a sequence

$k=3$

b c e _ _

$\neq$

c e b _ _

d b a _ _

b d a _ _

choose a subset of size k without order

$\{b,c,e\} = \{c,e,b\}$

$\{b,d,a\} = \{d,b,a\}$

$\binom{n}{k} = \#$ of subsets of size k

$= ?$

$P(n,k) = \begin{cases} \text{pick a subset of } k \\ \text{permute it in all possib.} \end{cases}$

$$P(n,k) = \binom{n}{k} \cdot k!$$

set permute the set

Think "Generative" ? Generate all permutations of n items

k: first approach

1st ... nth

$\underbrace{\_\ \_\ \_\ \_}_{k}\ \underbrace{\_\ \_\ \_}_{n-k}$

- pick k set ⇒ the first k
- permute them ⇒ order
- permute the other (remain) n-k

⇒ all sequences of n exactly once

1) all sequences are generated
2) each exactly once.

1st ... nth

_ _ _ _ _ _ _

all permutations = n!

ex n=5

(a b c d e)

k=3

- pick set of 3 {a, b, c}
- order (a, b, c)
- order of n-k=2 (d, e)

- pick set {c, e, a}
- order (c, e, a)
- n-k order (d, b)

(c, e, a, d, b)

$\underbrace{\binom{n}{k}}_{\text{choose } k} \cdot \underbrace{k!}_{\text{permute } k} \cdot \underbrace{(n-k)!}_{\text{permute rem } n-k} = n!$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Rules
- order / not order
- repetition / not
- distinguishable / not
- balls into bins.
- count everything?
- don't double count

Question 1.

(a) How many hexadecimal numbers...

- Begin with one of the digits 3 through B, and
- End with one of the digits 5 through F, and
- Are 5 digits long?

order on
rept on

$\boxed{3-B}$ $\boxed{✓}$ ✓ $\boxed{✓}$ $\boxed{5-F}$

9 pos 16 16 16 11 pos

prod. rule $9 \times 16^3 \times 11$

(b) How many hexadecimal numbers...

- Begin with one of the digits 4 through D, and
- End with one of the digits 2 through E, and
- Are 6 digits long?

10 16 16 16 16 13

$10 \cdot 13 \cdot 16^4$

values = 13/suit
#cards = 52

♠ : 2, 3, ... 10, J, Q, K, A
♡ : 2, 3 - 10, J, Q, K, A
♢ :
♣ : . - - -

Question 2.

In a game of poker, we play with a standard deck of 52 cards. There are 4 suits (Clubs, Diamonds, Hearts, Spades), and each suit has 13 values (2, 3, ..., 10, Jack, Queen, King, Ace), but each card is distinct. Let's say we're playing 5-card draw...

(a) How many different poker hands are there? 5-card => set

$k=5$ out of $\underset{n}{52}$ $\binom{52}{5} = C_{52,5}$

(b) How many different hands are there where the player has a flush (all cards the same suit)?

think "Generative"

- choose suit => ♢ 4 $\times$
- choose 5 in that suit (set) $\binom{13}{5}$

(c) How many different hands are there where the player has a full house (the 5 cards are broken into two parts: a pair where both cards have the same value, and a triple where all three cards have the same value)?

$\underline{v_1}$ $\underline{v_1}$ $\underline{v_1}$ $\underline{v_2}$ $\underline{v_2}$ $v_1, v_2 \in$ values

- choose v_1 => "8" 13 $\times$ 8 8 8 K K
- choose v_2 => "K" 12 $\times$
- choose which 3 "8" = v_1 $\binom{4}{3}$ $\times$
- choose which 2 "v_2" $\binom{4}{2}$

Question 3.

Dio wants to start a baseball team which consists of 9 players.

(a) He received 12 applicants. How many teams could he make?

out of order => set
repetition off

choose 9 (subset out of 12)

$\binom{12}{9}$

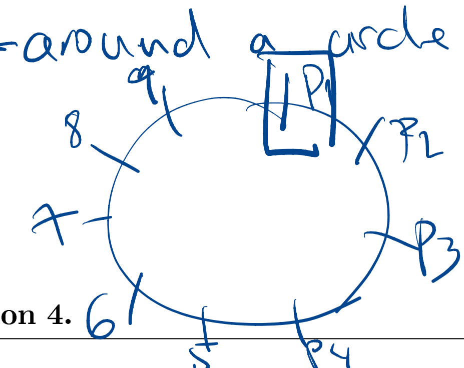
$\binom{n}{k} = \binom{n}{n-k}$

(b) After some hard calls, he has his team of 9. Now he needs to determine the batting order for their first game. How many possible orders are there?

→ sequence of 9 out 9 → 9!

(c) It is time for practice. The players on the team always do warmup stretches in a circle. How many ways are there to order this circle?

- around a circle table order



$\frac{9!}{9}$

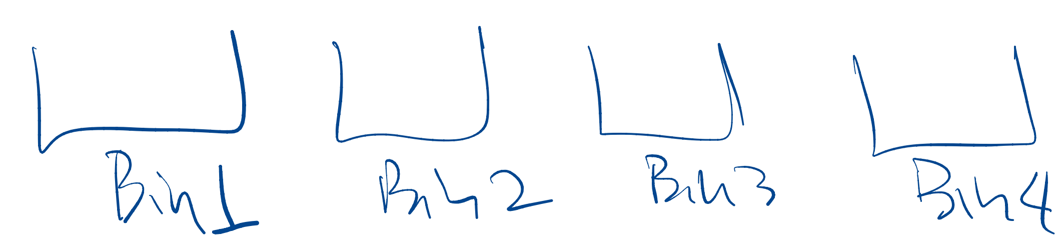
Question 4.

You go to a Halloween party with 3 friends, but you arrive late and there's hardly any candy left. All you can find is 11 pieces of candy corn.

me + 3 = 4 people
- distinguish / "ID"
balls / items
undistinguishable

(a) How many ways are there to distribute the 11 pieces among the four of you?

balls = 11
bins = 4



$\binom{11+4-1}{11}$

$\binom{k+n-1}{n-1} = \binom{k+n-1}{k} \neq \binom{k+n-1}{n}$?

4-4-2-1 $\quad \underbrace{0\ 0\ 0\ 0}_{\text{bin1}} \mid \underbrace{0\ 0\ 0\ 0}_{\text{bin2}} \mid \underbrace{0\ 0}_{\text{bin3}} \mid \underbrace{0}_{\text{bin4}}$

11 candies
4 bins $\Leftrightarrow$ 3 separators

4-0-5-2 $\quad 0\ 0\ 0\ 0 \mid \mid 0\ 0\ 0\ 0\ 0 \mid 0\ 0$

empty bin 2

balls-and-bins

$\Rightarrow$ # ways to write 11 balls (no id) with 3 separators

$\Rightarrow$ choose 3 spots for separators out of 11+3

$\binom{11+3}{3} = \binom{11+4-1}{11}$

(b) How many ways are there to distribute the 11 pieces among the four of you, such that you personally get at least 5 pieces?

(c) Assume that each person can eat a maximum of 8 pieces before they get tired of candy corn. Given this limitation, how many ways are there to distribute the 11 pieces among the four of you?

Question 5.

Suppose a password system has the following restrictions:

- Must be 6-8 characters long
- At least one digit required
- Every character must be an uppercase letter or a digit

How many possible passwords are there?

Question 6.

In the game of Mastermind, one player sets up a secret code made up of four colored pegs and their opponent tries to guess the code. There are six possible colors for each peg: Red, Orange, Yellow, Green, Blue, Purple. Order matters. Repeated colors are allowed.

(a) How many possible codes are there?

(b) How many possible codes are there that contain exactly two yellow pegs?

(c) How many possible codes are there that contain exactly two pegs of the same color?

(d) How many possible codes are there that contain at least two pegs of the same color?

(e) You're the second player, the one trying to guess the code. You guess ROYG. Player One gives you feedback in the form of one white peg, which indicates you have exactly one correct color but it's in the incorrect location. What can you conclude about repeated colors in the code you're trying to guess?

Questions to take home (optional)

★★ Question 4 with distinguishable candies

Same three questions as in Problem 4 (a) (b) (c), except the 11 candies are not the same, they are now identifiable: $k_1, k_2, k_3, ..., k_{11}$

★ Bytes without pair "11"

How many bytes (i.e. sequence of 8 bits) dont have repeated “1”?

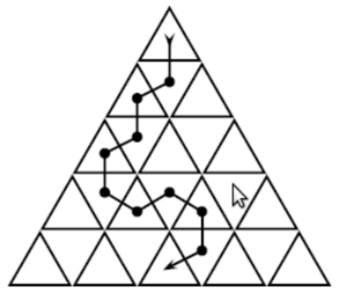
★★ More on pair "11"

How many sequences of 16 bits dont have repeated “1”?

★ Derangements

A *derangement* of 1 2 3 $\ldots n$ is a permutation that leaves none of these numbers in place. By inspection, the derangements of 123 are 312 and 231. Find the number of derangements of 1 2 3 4 5 using Inclusion-Exclusion

★ Ways to go down in a triangle Consider an equilateral triangle of side length n, which is divided into unit triangles, as shown. A valid path runs from the triangle in the top row to the middle triangle in the bottom row, such that adjacent triangles in our path share a common edge and the path never travels up (from a lower row to a higher row) or revisits a triangle.



An example of one such path is illustrated below for n = 5. Compute the the number of paths.

Hint: Construct a bijective mapping between valid paths and ordered lists of positive integers $(a_1, a_2, ..., a_n)$ with $a_i \leq i$. Then count the ordered lists.