Problem 5 - 12 points MIDTERM 2019Fall $A=\{1,2,3,4,5,6,7,8,9,10\}$

The following questions all consider the set of natural numbers between 1 and 10 inclusive.

i. (2 points) Using set builder notation, write this set. $A=\{x \in \mathbb{N} \mid 1 \le x \le 10\}$

ii. (4 points) How many subsets of size three contain only even numbers

iii. (3 points) How many subsets of size three contain only even numbers, but not 2 and 4 both?

iv. (3 points) How many sequences of length 15 with elements from the set are never decreasing? For example, 1,1, 2, 2, 2, 3, 5, 5, 5, 5, 5, 6, 7, 8, 10 is one such sequence. (Hint: Notice that a sequence is determined only by count and value, the example is two 1s, three 2s, one 3, five 5s, one 6, one 7, one 8, and one 10.)

II subsets of size 3 out of k elements set (not entire A) $\Rightarrow \binom{k}{3}$

B = even elements of A set, $B=\{2,4,6,8,10\}$ $|B|=k=5$

Answer is $\binom{5}{3}$

III all subsets in part II, except the ones that fail the restriction

Solution1 $\binom{5}{3}$ − #subsets that contain both 2 and 4.

$\{2,4,x\}$ $x \in \{6,8,10\}$

3 such subsets

$\binom{5}{3}-3$

Solution 2 (III): $B = \{2,4\} \cup \{6,8,10\}$ ($B1$, $B2$)

want subsets of size 3 excluding ones with both 2 and 4.

disjoint:
- one element from $B1$ (2 or 4) + 2 elem from $B2$ $\binom{2}{1}\binom{3}{2}$
- no elem from $B1$, choose all 3 from $B2$ $\binom{3}{3} = 1$ subset

Total (sum Rule) $2 \cdot 3 + 1 = 7$

Sanity check: Solution 1 $\binom{5}{3} \rightarrow = \frac{5!}{3!2!} - 3 = \frac{4 \cdot 5}{2} - 3 = 7$ ✓

$A = \{1,2,3,4,5,6,7,8,9,10\}$

iv. (3 points) How many sequences of length 15 with elements from the set are never decreasing? For example, 1,1, 2, 2, 2, 3, 5, 5, 5, 5, 5, 6, 7, 8, 10 is one such sequence. (Hint: Notice that a sequence is determined only by count and value, the example is two 1s, three 2s, one 3, five 5s, one 6, one 7, one 8, and one 10.)

15 elements in sequence, fixed order (increasing)

chunks

"1"	"2"	"3"	"4"	"5"	"6"	"7"	"8"	"9"	"10"
2 times	3	1	0	5	1	1	1	0	1

total count = 15

Bin 1, Bin 2, Bin 3, no 4 Bin 4 ... no 9, Bin 10

15 items balls → 10 Bins valid elements from A

seq values

$$\binom{15 + 10 - 1}{10 - 1}$$

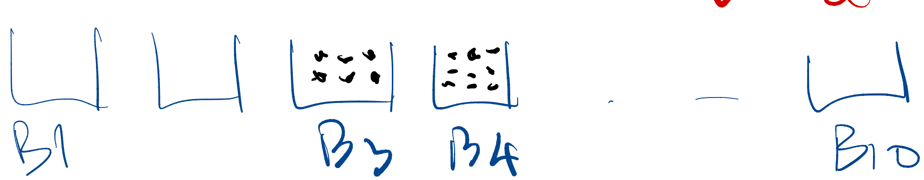
(balls, bins)

✓ • every valid sequence can be obtained by balls-into-bins ✓

✓ • balls into bins procedure only produces valid sequences

→ any arrangement balls → bins that is NOT valid sequence?

✓ • double counting? 2 balls → bins arrang same sequence.



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