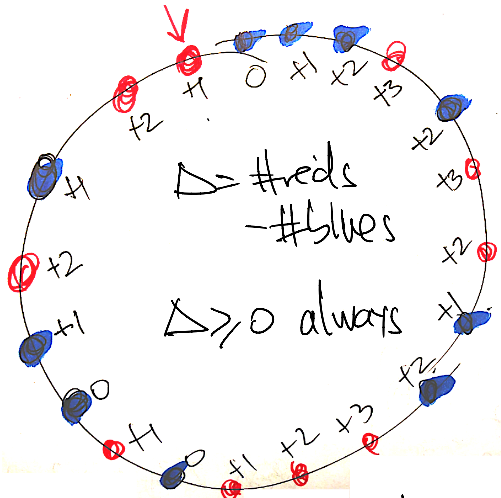
PB 10 ★★

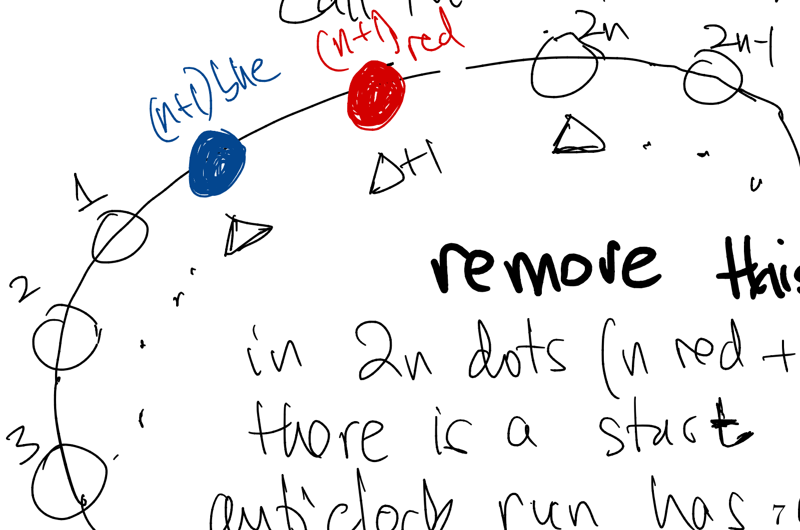
$2n$ dots are placed around the outside of the circle; n of them are colored red and the remaining n are colored blue. Going around the circle anti-clockwise, you keep a count of how many red and blue dots you have passed. If at all times the number of red dots you have passed is at least the number of blue dots, you consider it a successful trip around the circle. Prove that no matter how the dots are placed on the circle, it is possible to have a successful trip around the circle if you start at the right point.



Solution: We want at all times #red dot ≥ #blue dots, and we can choose the start dot (red) to go around anti-clockwise.

ind step if its always possible for (n red, n blue) ⟹ its possible for ($n+1$ red, $n+1$ blue)

proof: given ($n+1$ red, $n+1$ blue) dots we find a pair (red blue) anticlock ordered. and call them the $n+1$ pair. This is possible by starting at a red and going anticlockwise until we find a blue. **We now remove this pair (red, blue)** resulting in $2n$ dots (n red + n blue). By induction hypoth there is a start such that a succesfull anticlock run has at all times Δ = #reds − #blues ≥ 0



That start will work for ($n+1$ reds + $n+1$ blues) ∵ $\Delta \geq 0$ at $2n$, $\Delta + 1 \geq 0$ at $n+1$ red, $\Delta \geq 0$ at $n+1$ blue, the rest same Δ.