Exam 1 Practice B 25X

Problem 1

Consider the addition of $(1111)_2$ and $(0101)_2$ in a 4-bit two's complement number system:

  i Compute the addition in binary (your output must also be 4-bits).

  ii Write the decimal equivalent of the operation above. For example, you might write 5 + 3 = 8.

  iii Is there an overflow in this operation? Explain why or why not in one sentence.

Problem 2

The headlights of a car turn on for either of the following two reasons:

- the manual switch is on

- the automatic switch is on and the car does not detect any light (i.e. it's dark outside)

1. Construct a truth table for H in terms of A, L, M.

      - A = 1 when automatic headlight switch is on
      - L = 1 when light detected by light sensor
      - M = 1 when manual headlight switch is on
      - H = 1 when headlights on

A	L	M	H

2. Write a logic expression[1] for H in terms of A, L, M.

Problem 3
Simplify each of the following expressions by applying (and labelling) one law at time from logic_set_identities.pdf. Do not use the set difference operator in your simplifications. Note that the set U in the second item is the universal set, which includes all elements.

i $A \cap A$

ii $(A^C \cap B^C)^C \cap U$

iii $(A \cup A) \cap (B \cup A^C)$

Problem 4
Using induction, show that $n! < n^n$ for all $n = 2, 3, 4, \ldots$.

Problem 5 ★. Wilson characterization of primes
A positive integer p has the property $(p-1)! = -1(\mod p)$. Prove that p must be prime.

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[1]For example, a logical expression for X in terms of R and E is $X = R \wedge E$