REC 4: Sets, PP, PIE

Problem 1 [24 pts: (6 each)]:
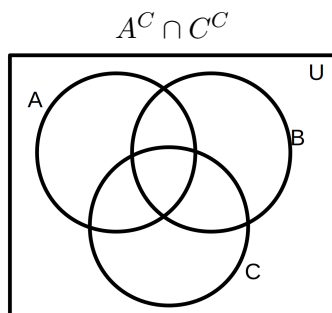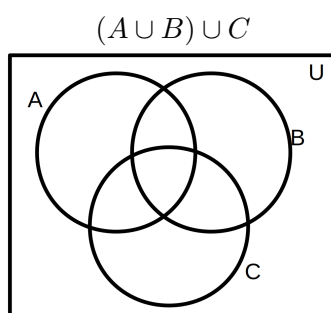
Simplify each of the following expressions by applying one identity (logic_set_identities.pdf) at a time. Label each identity used. Be mindful that you don't mix the boolean or set notations, use whichever the problem is given in.

  i $((A \cap A^C) \cup (A \cap (A \cup B)))^C$

 ii $\neg((\neg P \vee \neg Q) \wedge Q)$

Problem 2

Shade the indicated regions of the following Venn diagrams.

$(A \cup B) \cup C$

$A^C \cap C^C$

Problem 3

i Express the set:

$$S = \{n \in \mathbb{N} | (-11 \leq n) \wedge (n \mod 7 = 4) \wedge (n < 10)\}$$

by explicitly writing each item in a set (e.g. $\{1, 2, 3\}$). Assume that the set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

ii Express the set of all integers whose absolute value is less than 10 using set builder notation.

Problem 4 [24 pts: (6 each)]:

For each of the sets below, write out its powerset by listing the entire set.

  i $\emptyset$

  ii $A = \{1\}$

  iii $B = \{1, 2\}$

  iv $C = \{1, 2, 3\}$

  v How many elements are in the powerset of $D = \{1, 2, 3, \ldots, n\}$ for some $n \in \mathbb{N}$?

Problem 5 [24 pts: (6 each)]:

Assume that the 160 students who take an exam will receive one of 24 grades.

i What does the pigeonhole principle tell us about the exam results of these students?

ii Regardless of the scores assigned, how many other students must get the same grade as you?

Problem 6 [24 pts: (6 each)]:

There are 39 8th graders in a school, each of whom likes at least one genre of books. There are 3 popular genres among the students: Romance, Fantasy, and Mystery:

- 16 like Romance;

- 25 like Fantasy;

- 18 like Mystery;

- 12 like Romance and Fantasy (Romantasy![1]);

- 5 like Romance and Mystery;

- 8 like Fantasy and Mystery.

Answer the following questions:

1. How many students like either Romance, Mystery or both?

2. How many students like all 3 genres of books?

3. How many students like exactly 2 genres?

4. How many students like exactly 1 genre?

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[1]Credit to Prof Hamlin for sharing what is undoubtdely the best-named-intersection in our course :)

Problem 7 [24 pts: (6 each)]:

A package of baseball cards contains 15 cards. In total, there are 550 unique baseball cards. How many packages of cards must be purchased to ensure that they contain two identical cards?