REC 3: Logic, Sets

Problem 1

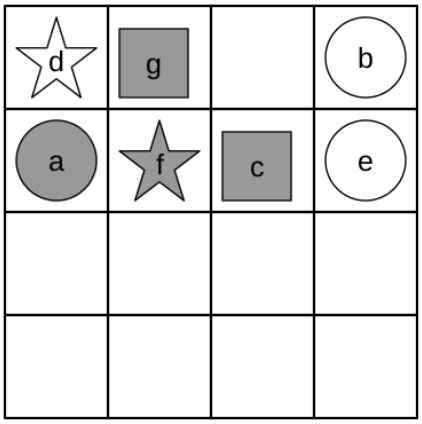
i Using the statements below, write the conditional $P \to Q$ as well as its contrapositive, converse & inverse in english.

- P = it is raining
- Q = my dog is wearing a rain jacket

ii Which of the four statements generated above are logically equivalent to others? (and what does it mean to be logically equivalent?)

iii Assume that, when it rains, I always put a rain jacket on my dog. Is the statement $P \leftrightarrow Q$ true? Explain why or why not.

Problem 2



Use the following predicates:

- `circle`(x) is True when x is a circle
- `rect`(x) is True when x is a rectangle
- `star`(x) is True when x is a star
- `shade`(x) is True when x is shaded in
- `next_to`(x, y) is True when the squares containing x and y touch (i.e. they're immediate neighbors left-right, top-bottom, or they share a corner diagonally).

to determine if each of the statements below is true or false. For statements which are true, justify your answer by describing all relevant elements (maybe all of them). For statements which are false, justify your answer by giving a single counter-example.

i `star`(d)

ii $\exists\, x$ `star`$(x) \wedge$ `shade`(x)

iii $\forall\, x$ `star`$(x) \wedge$ `shade`(x)

iv $\forall x$`circle`$(x) \rightarrow \neg$`shade`(x)

Problem 3

When my son was young, he and the dog had a bad habit of sharing chew toys![1]

Let us define the following predicates on chew toys x

- D(x) = my dog is chewing on toy x right now
- S(x) = my son is chewing on toy x right now

Translate each of the following logic statements into plain english, easily understood by a non-technical reader.

i $\exists x D(x)$

ii $\forall x S(x)$ (he's an impressive kid :))

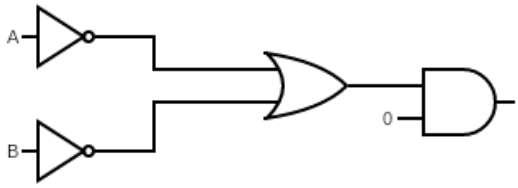
iii $\forall x S(x) \rightarrow D(x)$ (definitely gross, maybe cute too?)

iv $\forall x \exists y S(y) \rightarrow D(x)$

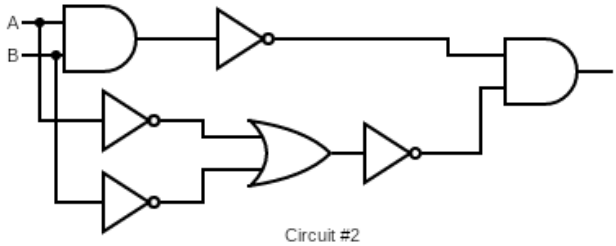
v $\exists y \forall x S(y) \rightarrow D(x)$

[1] Maybe it builds immunity ... he's made it a few years after the fact so it couldn't have been that bad, right?

Problem 4



Circuit #1



Circuit #2

i Write out logical expressions representing each of the two circuits. Show that they are equivalent using the laws of logical equivalence.

ii There are many other circuits that would be equivalent to these two. Draw one that uses three AND gates, one NOT gate, and no other gates. Write its logical expression.

Problem 5 Statements, Implication
Consider the statements P, Q, R, and S below; because, ok, it could be going better for my Red Sox this year, but at least they're playing!

- P = Jackie Bradley Jr (JBJ) plays center field.
- Q = The Red Sox win.
- R = Chris Sale gets the win.
- S = The Yankees lose.

Translate each of the sentences below into logic statements using conjunction ($\wedge$), disjunction ($\vee$), negation ($\neg$), and/or implication ($\implies$). English can be squishy but logic can't be, so if a sentence seems ambiguous, choose an interpretation that makes the most sense to you and make it crystal-clear in your logic statement.

i If JBJ plays center field, then the Red Sox win.

ii The Yankees lose, but Chris Sale doesn't get the win.

iii The Yankees lose if the Red Sox don't win or JBJ doesn't play center field.

iv Chris Sale gets the win whenever the Red Sox win.

Problem 6 ★ Logic Puzzle
(Work in a group of 2-3 to debate the possibilities)

A teacher writes six words on a board: `cat dog has max dim tag`
She picks one of the words, and gives each of the three letters of that word to three students, Albert, Bernard and Cheryl. Then she asks, "Albert, do you know the word?" Albert replies yes. She asks, "Bernard, do you know the word?" He replies yes. Then she asks Cheryl the same question. She thinks and then replies yes. What is the word?
Explain Cheryl's reasoning