$|A \cup B \cup C| = \boxed{|A| + |B| + |C|}$

everything counted once

• So+ R_1, R_2, R_3 counted once (correct)

• R_4, R_5, R_6 counted twice

R_4 counts in A and in B

$- \boxed{|A \cap B| - |B \cap C| - |A \cap C|}$

• R_4 R_5 R_6 are counted once ✓

• R_7 is counted $+3-3 = 0$ times

$\boxed{+|A \cap B \cap C|}$

R_7: $+3-3+1$ ✓

Deadline: October 9 at 8pm eastern

CS1802 Recitation 4

Fall 2020

October 6 - 9, 2020

Instructions: The problems in this recitation are based on the course material covered in the CS1800 lecture videos and are meant to prepare you for upcoming homework assignments. You earn full credit for a recitation by using your time well and demonstrating effort on the assignment. Submit your solution on Gradescope by uploading images of hand-written work, or uploading a PDF.

Logistics for Fall 2020: The recitation assignments are designed to be completed within the official 65-minute time. However, we know that schedules are harder to work with this semester, and so the deadline for recitations will officially be on Fridays at 8pm eastern. We recommend submitting your work in real-time at the end of your section, but it's OK if your preference is to submit later as long as you meet that last deadline.

- In-person: If you're able to join in-person, please come to the classroom where instructors will be there to help. Work on the assignment, ask us questions, and submit whatever you have when time is up.
- Synchronous, remote: If you're not able to join in-person but you can remotely join at the designated time, please join the recitation remotely. Work on the assignment, post any questions in the meeting chat, and submit whatever you have when the time is up.
- Asynchronous, remote: If you're both remote and not able to join in real-time, we suggest you register for the asynchronous online section. Dedicate 65 minutes to work on the assignment, and submit your solution by the Friday deadline.

Question 1.

$B = \{1, 2, 3, \ldots, 19\}$

Q: $0 \in \mathbb{Z}^+$? depends on who you ask. more often $0 \notin \mathbb{Z}^+$

Let $A = \{1, 5, 14, 15, 27, 28\}$, $B = \{x | (x \in \mathbb{Z}^+) \wedge (x < 20)\}$, $C = \{x | (x \in A) \wedge (x \text{ is even})\}$ and our universe $U = \{1, 2, 3, ..., 30\}$. Use the inclusion/exclusion

$C = \{\text{even in } A\} = \{14, 28\}$

$A \cap B = \{1, 5, 14, 15\}$ $\quad |A \cap B| = 4$

manual + tricks

$\bar{C} = \{1, 2, 3, 4 \ldots 13, 15, 16, \ldots 27, 29, 30\}$

missing 14, 28

$A \cup \bar{C}$ A ~~puts~~ adds back 14, 28

$= U$

formulas (where appropriate) from this week's module to determine the ~~cardinality~~ size of the following sets:

(a) $|A \cup B| = |A| + |B| - |A \cap B| = 6 + 19 - 1 = 24$

$|\text{Union}| \Rightarrow$ Inclusion-Exclusion (no enumeration)

(b) $|A \cap B| = 4$ enumeration

$|\text{intersection}|$

(c) $(A \cap C) \cup (B \cap C) = \boxed{(A \cup B) \cap C}$

$|(A \cup B) \cap C| = |C| = 2$

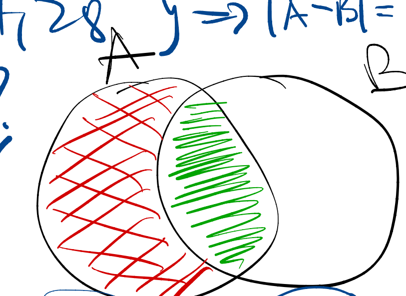
trick $C \subseteq A \Rightarrow C \subseteq A \cup B$

$\Rightarrow (A \cup B) \cap C = C$

(d) $A - B = \{\text{in } A, \text{not in } B\} = \{27, 28\} \Rightarrow |A - B| = 2$

$|A - B| = |A| - |A \cap B|$?

$|A - B| + |A \cap B| = |A|$ ✓



Sum rule

A partitioned into $(A \setminus B) \cup (A \cap B) = A$

$(A \setminus B) \cap (A \cap B) = \emptyset$

$|A| = |A \setminus B| + |A \cap B|$

(e) $A \cup \overline{C}$

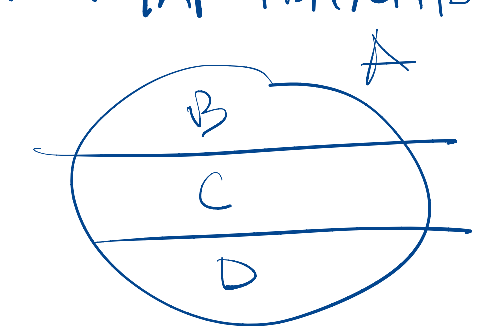
$-|A \cap \overline{C}| + |A| + |\overline{C}|$?

Sum rule (partitions)

$A = B \cup C \cup D$

$B \cap C = \emptyset$, $B \cap D = \emptyset$, $C \cap D = \emptyset$

Then $|A| = |B| + |C| + |D|$



Question 2.

A character in the game Dungeons and Dragons is assigned a moral alignment based on exactly one element of lawfulness $\{lawful, neutral, chaotic\}$, and exactly one element of goodness $\{good, neutral, evil\}$. For example, Captain Holt from Brooklyn 99 is clearly Lawful Good.

How many moral alignments are there?

PS, If you take this moral alignment quiz during recitation it totally counts as work:
https://play.howstuffworks.com/quiz/whats-your-moral-alignment

$|M| \times |N| = |M \times N|$

$3 \times 3 = 9$ ✓

$N \times M \neq M \times N$

Question 3.

In the Strange household, when we're just hanging out watching TV, we have basically three categories of TV shows: my favorites, my husband Tom's favorites, and then some compromise shows that we'll both watch.

Let's define our TV-watching sets:

- $L = \{Ozark, CobraKai, Resident\}$
- $T = \{Archer, FamilyGuy\}$
- $S = \{Brooklyn99, Archer\}$

(a) Assuming we watch one Laney show and one Tom show, how many ways are there to spend an evening at Strangehouse? Order definitely matters, because I want to watch my stuff first.

$6 = |L \times T|$

(b) Suppose I don't actually *need* to get my own way, and we can just watch something from L and something from T, in either order. Is the answer the same as above? Explain why or why not.

$6 + 6 \quad |L \times T| + |T \times L|$

enumerate exercise

(c) Write the following set in list notation: $(S \times S) - (S \times T)$ $= S \times (S - T)$

$\{(arc, b99), (b99, b99)\}$

(d) Write the following set in list notation: $(L \times T) \cap (S \times T)$

$\{\ \} = \emptyset$

Pigeon Principles k items $\rightarrow$ m boxes

$\Rightarrow$ one box $\geq \lceil \frac{k}{m} \rceil$ items.

Virgil PP

$$\frac{x_1 + x_2 + x_3 \dots + x_m}{m} = \mu = \text{arit. average}$$

$\exists x_i \geq \mu$ x_i = # items in box i

$\exists x_j \leq \mu$ $\mu = \frac{k}{m}$

Question 4.

The seven CS 1802 instructors need to make 11 recitations and 11 videos. What is the minimum number of new materials (recitations or videos) that the hardest-working instructor is guaranteed to do?

Total 22 assig

P.P. k items $k = 22$ items $\rightarrow$ boxes

m boxes $m = 7$ boxes contain multiple items

one box instruct $\geq \lceil \frac{k}{m} \rceil = 4$

Question 5.

There are 919 students registered in CS1800/1802 (true story).

(a) There must be at least one birthday (month/day, excluding year) that is shared by at least how many students?

(b) Suppose that you're given additional information: A *maximum* of 3 people share any one birthday. At least how many dates must have *exactly* 3 people?

Question 6.

In a standard deck of 52 cards, there are 4 suits (clubs, diamonds, hearts, spades), and each suit has 13 values (2, 3, ..., 10, Jack, Queen, King, Ace). The clubs and spades are called black cards, and the hearts and diamonds are red.

(a) How many cards do you need to draw to guarantee at least ten of the same color?

(b) How many cards do you need to draw to guarantee at least two of the same suit?

(c) If I draw 27 cards, how many different values are guaranteed to be represented?

(d) How many cards do you need to draw to guarantee that you have at least three cards each from at least two different suits (e.g., three hearts and three clubs)?

(e) How many cards do you need to draw to guarantee that you have at least three hearts?

(f) Let's get rid of Aces and Face Cards (J, Q, K). Now the only values we have in the remaining deck are 2 - 10. How many cards do you need to draw to guarantee that two of them sum to 18?

Questions to take home (optional)

Set size inequality

Prove that $|A| + |B| + |A \cap B \cap C| \geq |A \cap B| + |A \cap C| + |B \cap C|$

★ Pigeonhole cascade

In a class of 20 students, each student has at least 14 friends (friends are reciprocal). Show that there are 4 students that form a *clique*, that is all 4 are pairwise friends.

★★ Modulo 3,5 cases

Given any 7 integers, show that there are 2 of them with either sum or difference or product = multiple of 15

★★ Pigeonhole in plane

There is a plane where every point is colored either Red, Green, or Blue. Prove that there exists a rectangle in the plane that has all four corners the same color.