Set = collection of items / objects / elements

OUT OF ORDER

$\{1,2,4,3,6,5,10,9,8,7\}$ //

enumeration $U = \{1,2,3,4,5,6,7,8,9,10\}$ $|U| = 10$

set notation / builder / property

$U = \{x \mid x \in \mathbb{Z}, x > 0, x \leq 10\}$

$= \{x \in \mathbb{Z} \mid 0 < x \leq 10\}$

$|A| = 5$

$|B| = 5$

A = "evens in U" $= \{2,4,6,8,10\}$

B = "≤ 5 in U" $= \{1,2,3,4,5\}$

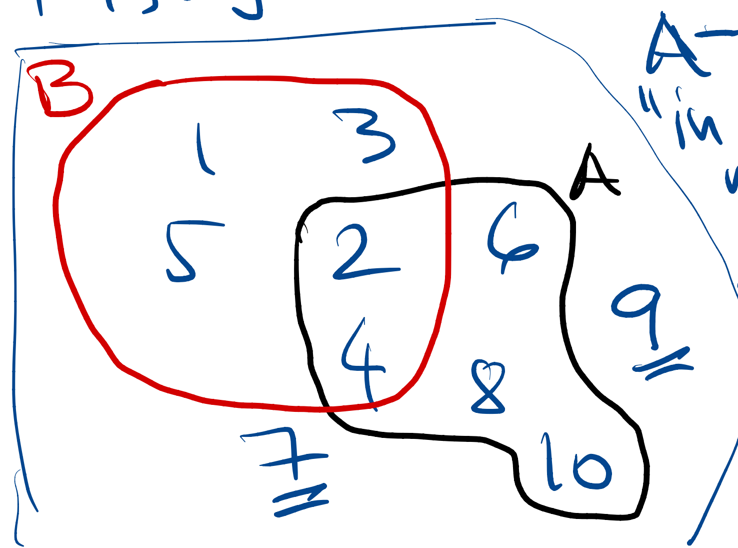
$A \subset U$ subset

$B \subset U$ subset

Venn Diagram

$A \cap B$ = "common elem to A and B" $= \{2,4\}$

$A \cup B$ = "elements in A or B", $\{1,2,3,4,5,6,8,10\}$



$A - B = A \setminus B$ "in A and not B"

$\{6,8,10\}$

$B - A = \{1,3,5\}$

$A - B =$ "elem in A" **and** not in B

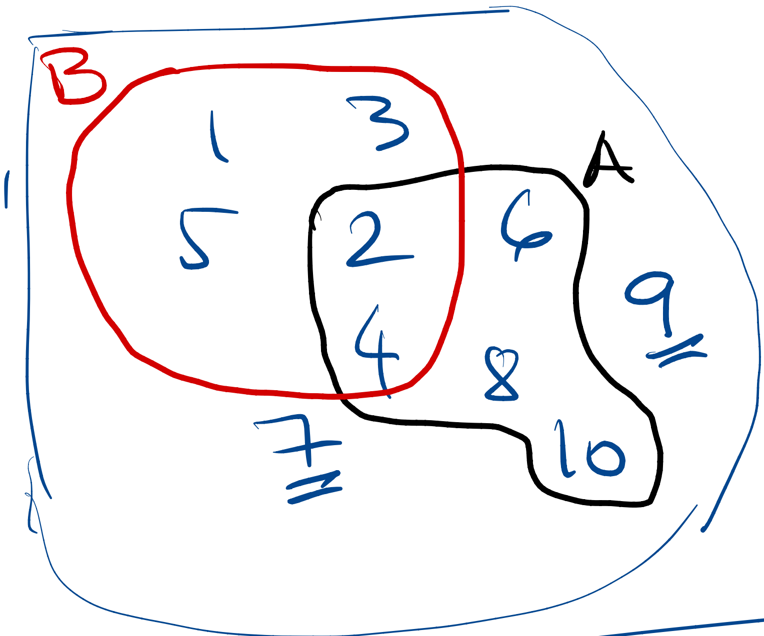
$= A \cap \text{not } B = A \cap \overline{B}$

symmetric difference $\simeq$ XOR

$A \Delta B = (A - B) \cup (B \setminus A)$ } proof?

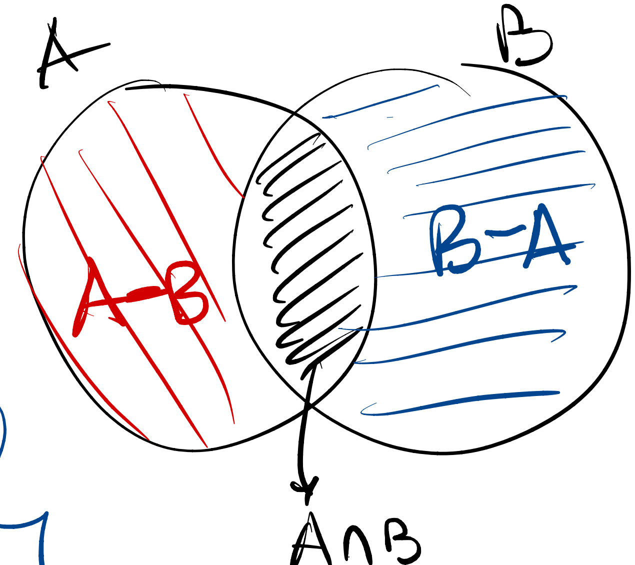
$= (A \cup B) - (A \cap B)$

$= \{1, 3, 5, 6, 8, 10\}$



not B = complement of $B = \overline{B}$

$=$ universe $\setminus B = \{6, 7, 8, 9, 10\}$



power set = set of subsets

$\mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}$
$\{1,2\}\ \{1,3\}, \{1,4\}, \ldots$
$\{1,2,3\}\ \{1,2,4\}\ \{1,2,5\}, \ldots \}$

$|\mathcal{P}(B)| = 2^{|B|} = 2^5$

Bit-vector

$U = \{A, B, \ldots Z\}$

$[\,0/1 \;\_\;\_\; \ldots \;\_\,]$

$\longleftarrow 26 \longrightarrow$

$S_1 = \{A, C, Z\} = [1\,0\,1\,0 \ldots 0\,1]$

$S_2 = \{B, C\} = [0\,1\,1\,0 \ldots 0]$ $\quad \vee$

$S_1 \cup S_2 = S_1 \vee S_2 = [1\,1\,1\,0\,0 \ldots 0\,1]$

($\vee$ ↑) OR bitwise

$S_1 \cap S_2 = S_1 \wedge S_2$

↖ AND bitwise.

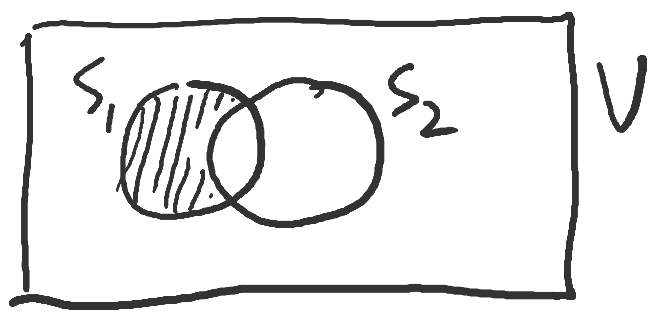
$\overline{S}$ complement $\neg S$ flip bits.

↖ NOT bitwise

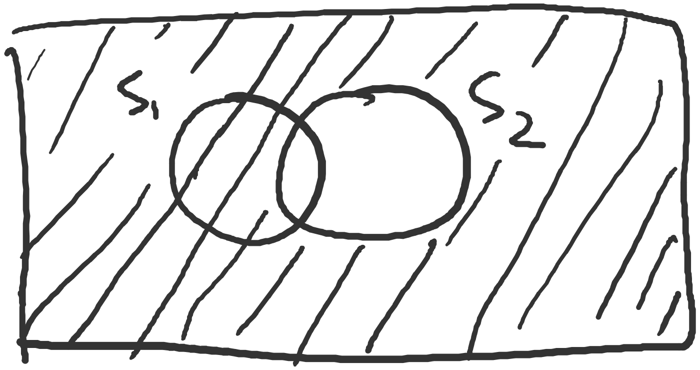
$S_1 - S_2 = S_1 \cap \overline{S_2} = S_1 \wedge (\neg S_2)$

Venn diagram

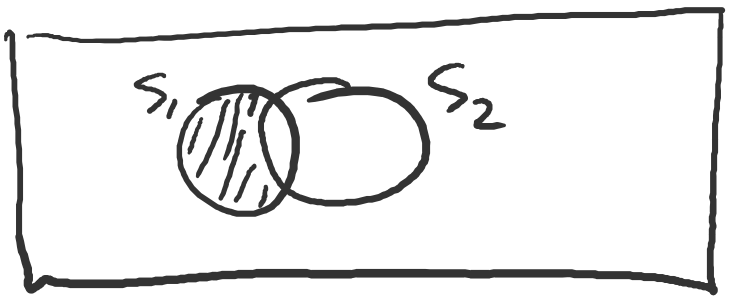
$S_1 - S_2$



$\overline{S_2} = \neg S_2$

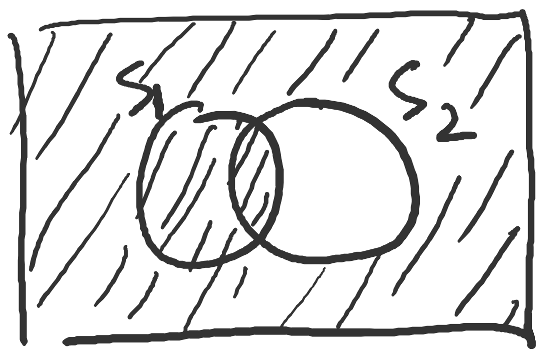


$S_1 \cap \overline{S_2}$



$= S_1 - S_2$

$S_1 \cup \overline{S_2}$



$S_1 \triangle S_2 = S_1 \oplus S_2 = [1\ 1\ 0\ \_\ \_\ .\ 0\ 1]$

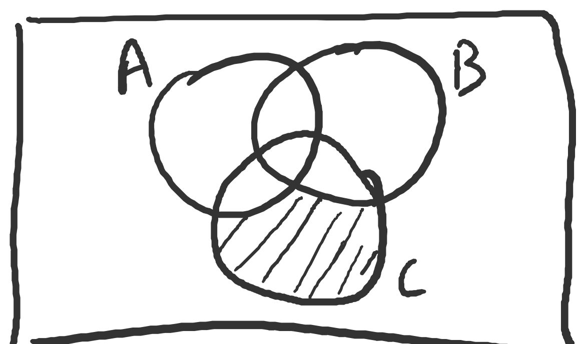
$\oplus$ ← XOR, parity

$= \{A, B, Z\}$

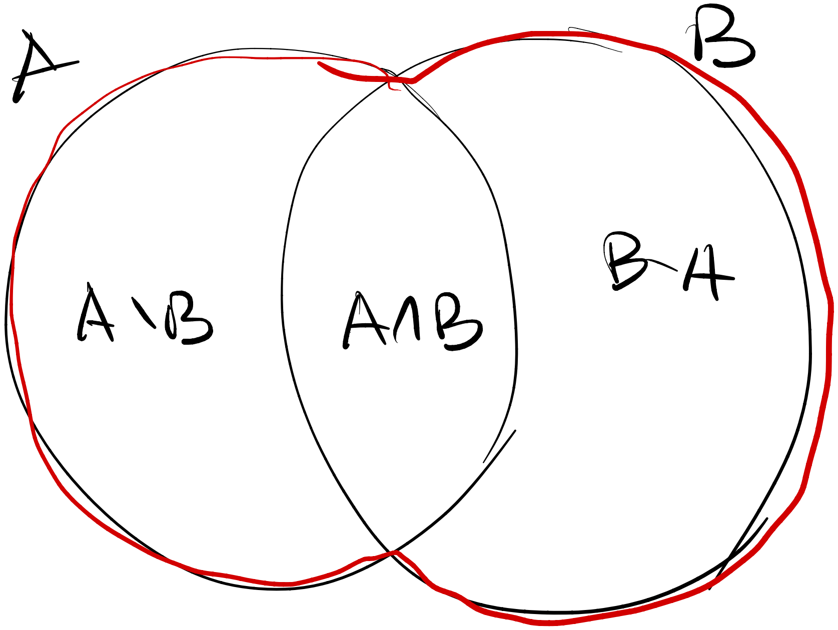
(C-A)-B

not

$\overline{(A \cup B)} \cap C = (\neg(A \vee B)) \wedge C$



Sum Rule: size of union.



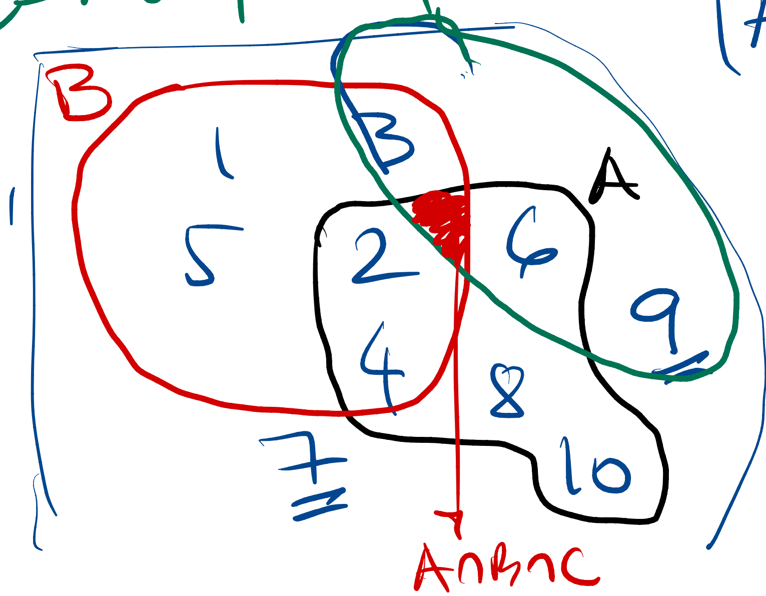
$|A\cup B| = |A\setminus B| + |A\cap B| + |B\setminus A|$

- **partition** into 3 sets
  - → disjoint sets
  - → union = total

$$|A\cup B| = |A| + |B| - |A\cap B|$$

$|A\cap B|$ → was double counted.

C = multiples of 3 $= \{x\in U \mid x = 3k \;\; k\in\mathbb{Z}\}$



$|A\cup B| = |A| + |B| - |A\cap B|$

$= 5 + 5 - 2 = 8$

$|A\cup B\cup C| = |A| + |B| + |C|$

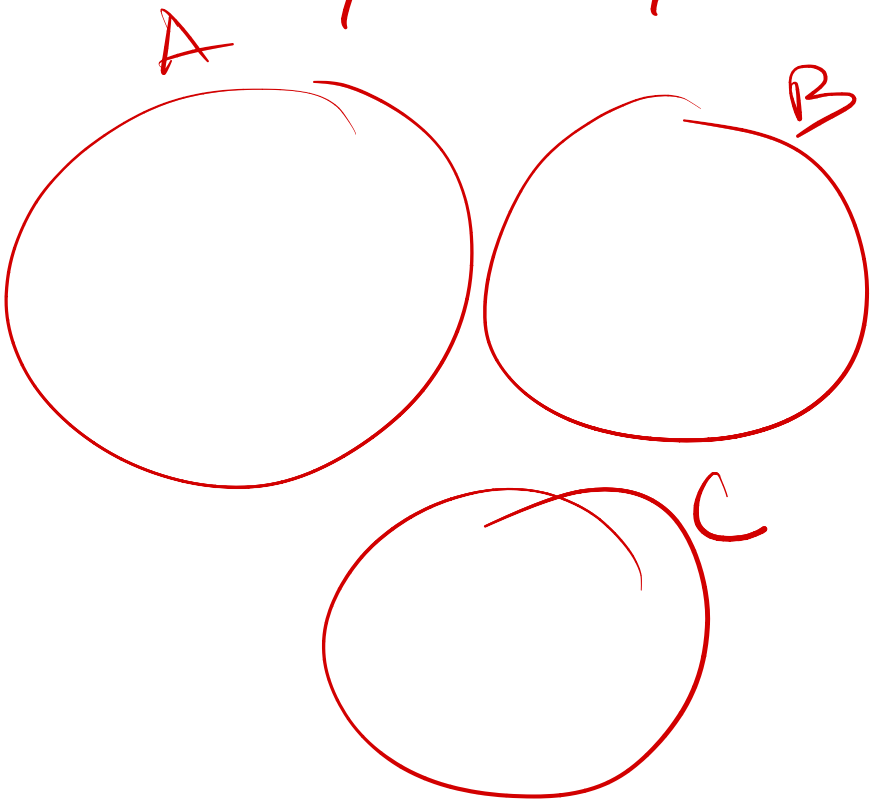
$- |A\cap B| - |B\cap C| - |C\cap A|$

$+ |A\cap B\cap C|$

Principle of Inclusion-Exclusion

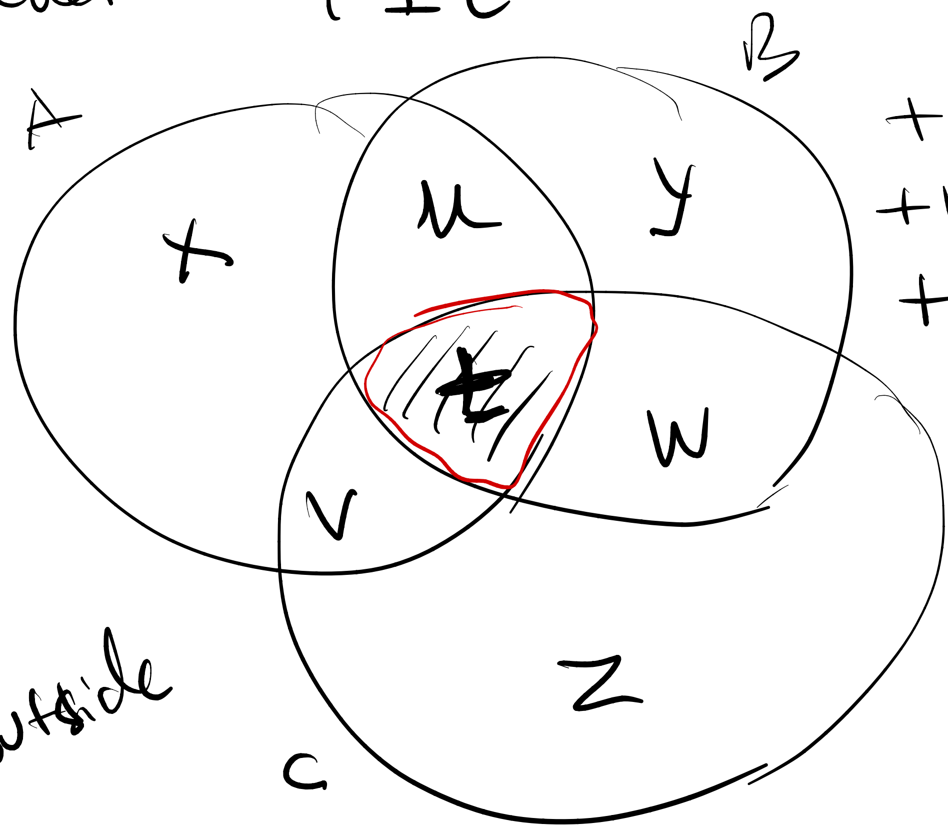
Sum Rule for Partition (Partition Rule) (no intersection)

$A \cap B = \emptyset$, $B \cap C = \emptyset$, $C \cap A = \emptyset$



$|A \cup B \cup C| = |A| + |B| + |C|$

General PIE



m = outside

$|A \cup B \cup C| = x+u+y+v+t+w+z$

$+|A| \rightarrow x+u+t+v$

$+|B| \rightarrow u+y+t+w$

$+|C| \rightarrow z+t+v+w$

$-|A \cap B| \rightarrow -u-t$

$-|B \cap C| \rightarrow -t-w$

$-|C \cap A| \rightarrow -t-v$

$+|A \cap B \cap C| \rightarrow +t$

$x+v+u+y+z+w+t$

Partition: Split everything into <u>disjoint</u> parts

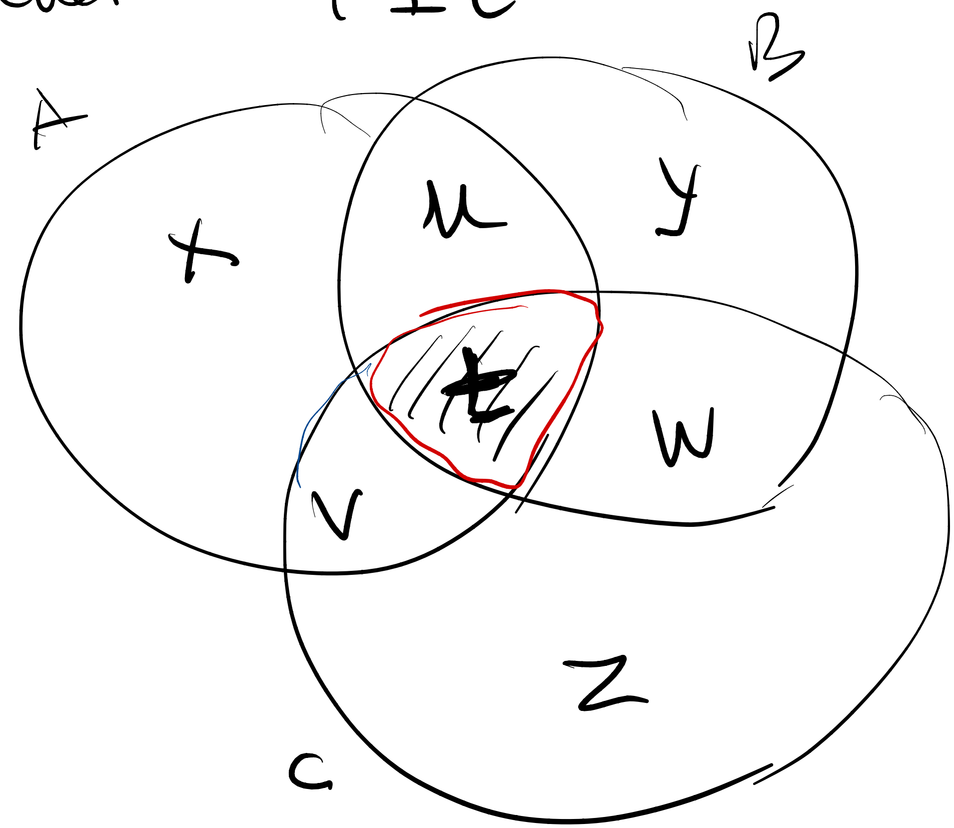
$x = A \setminus B \setminus C$

$u = A \cap B \setminus C$

$m = \overline{A \cup B \cup C}$

⋮

General PIE



PB1

$(A\cap B)\cup C \overset{?}{=} A\cap(B\cup C)$

$u+t+v+w+z = u+v+t$

$u+t+v+w+z$ ≠ $u+v+t$

NO

PB2

$(A\setminus C)-B \overset{?}{=} A-(C-B)$

$x \overset{?}{=} x+u+t$

NO

PB3

$(A\cup B)-C \overset{?}{=} (A\cup B\cup C)-C$

$x+u+y \overset{\text{YES}}{=} x+u+y$

PB4

$|A\cup B\cup C| = |A|+|B|-|A\cap B|-|A\cap C|-|B\cap C|+|A\cap B\cap C|$

$x+u+y = x+u+t+v+y+u+t+w - u-t - v-t - t-w + t$

$u+y+t+w$

YES

$$|(A\cup B)\setminus C| \overset{?}{=} |A|+|B|-|A\cap B|-|A\cap C|-|B\cap C|+|A\cap B\cap C|$$

Set algebra:

$$|(A\cup B)\setminus C| = |A\cup B\cup C \setminus C| = |A\cup B\cup C| - |C| =$$

subset of first term

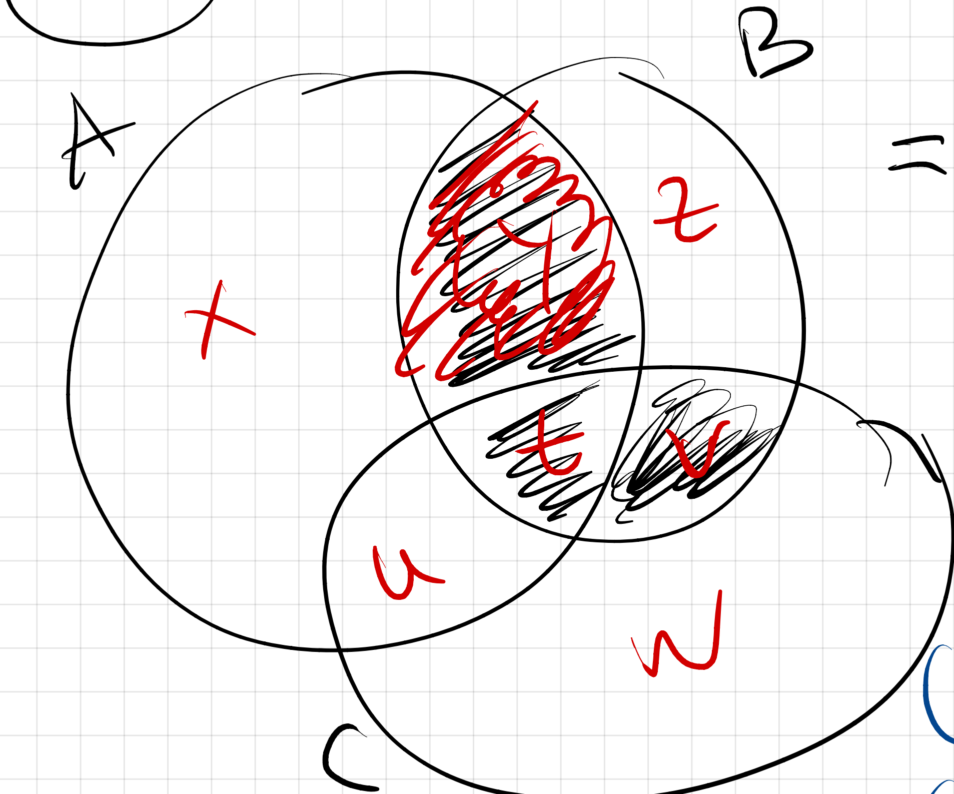
$$|A|+|B|+\cancel{|C|} - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C| - \cancel{|C|}$$

Sum Rule $A, B, C, \ldots$ disjoint ($\cap = \emptyset$) (partition)

$$|A\cup B\cup C \ldots| = |A|+|B|+|C|+\ldots$$

- counting objects in set S:
  - partition $S = S_1\cup S_2 \ldots S_n$ ($S_i\cap S_j=\emptyset$)
  - count each part $|S_i|$
  - sum up $|S_1|+|S_2|+\ldots+|S_n|$

PB3 Sets Rule Algebra



$y \cup t \cup u \cup v$

$(A \cup C) \cap B =$

$= (A \cap B \setminus C) \cup (A \cap B \cap C) \cup (B \cap C \setminus A)$

$y \qquad t \qquad t \cup v \setminus A$

$(A \cap B \cap \bar{C}) \cup (A \cap B \cap C) \cup (B \cap C \cap \bar{A})$

$(A \cap B \cap \bar{C}) \cup ((B \cap C) \cap (A \cup \bar{A}))$ (all)

$(A \cap B \cap \bar{C}) \cup (B \cap C)$

$B \cap ((A \cap \bar{C}) \cup C)$

$B \cap ((A \cup C) \cap (C \cup \bar{C}))$ (all)

$B \cap (A \cup C) = (A \cup C) \cap B$

Counting

$|A \dot{\cup} B| = |A| + |B|$ - sum rule

(A, B disjoint)

$|A \times B| = |A| * |B|$ - product rule

password length 6, digits + upper-case + lowercase + 12 special

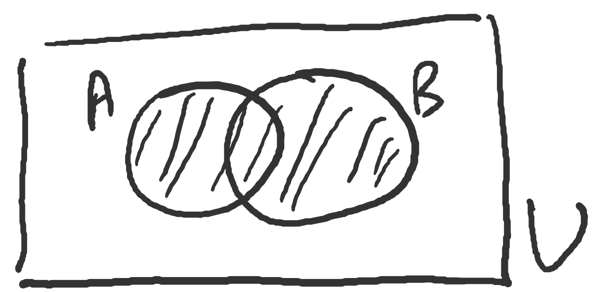
$|\text{passwords}| = \underset{=74}{(10+26+26+12)} \times 74 \ \_ \ \_ \ \_ \ \_ \times 74 = 74^6.$

Same as before : password length at least 4 and at most 6.

$|\text{password}| = 74^4 + 74^5 + 74^6$

Inclusion - Exclusion

2 sets A & B $\quad |A \cup B| = |A| + |B| - |A \cap B|$



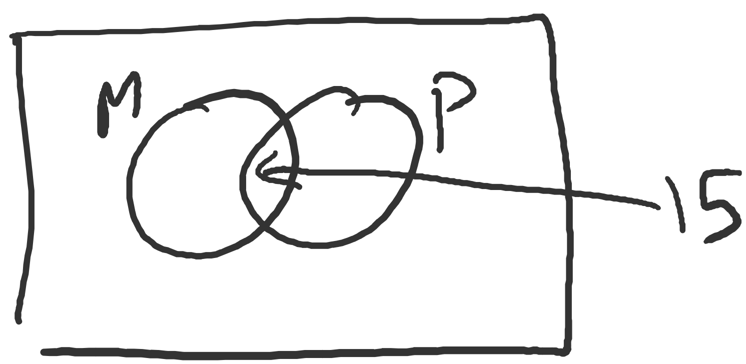
<u>Example</u> 20 courses contain some math

30 " " " programming

15 " " both

How many courses containing math or programming

<u>Answer</u> 35



$20+30-15$

$=35.$

Example initials - 2 upper case RS

How many initials with a C?

Ans C_1 – initials starting with C

C_2 – " ending " "

$$|C_1 \cup C_2| = 26 + 26 - 1 = 51.$$

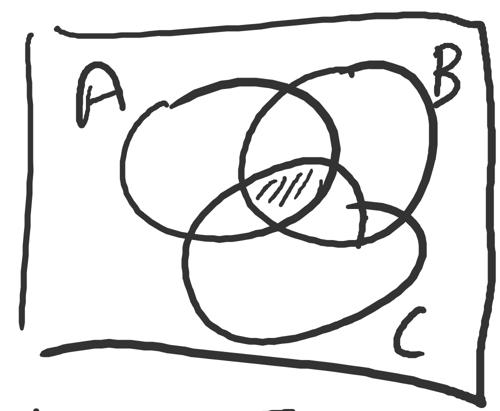
Inclusion - Exclusion (3 sets).

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Example

Ans (62)

video game club - 30
anime society - 20
fencing club - 30



10 VG are fencers, 2 fencers are anime, 7 anime are VG, 1 is in all, How many in Total?

Example initials - 3 upper case.

How many with a C?

C_i - initials with C in i'th place $1 \le i \le 3$

$|C_i| = 26^2$.

$|C_i \cap C_j|$, $1 \le i < j \le 3$ $= 26$

$|C_1 \cap C_2 \cap C_3| = 1$

So $3 * 26^2 - 3 * 26 + 1 = 1951$