

REC2: Logic

Problem 1 Overflow 2's Complement

For each sub-part below, please use a 8-bit two's complement representation:

- i Give the decimal (base-10) for 2's complement value $(10000000)_2$
- ii Give the decimal (base-10) for 2's complement value $(11110011)_2$
- iii Give the decimal (base-10) for 2's complement value $(01111111)_2$
- iv When adding the following values (from immediately above) $(10000000)_2 + (11110011)_2$ in 8-bit two's complement, does an overflow occur? Why or why not?
- v When adding the following values (from immediately above) $(01111111)_2 + (11110011)_2$ in 8-bit two's complement, does an overflow occur? Why or why not?

Problem 2 Mod Arithmetic

Find all values of x which meet all of the following criteria:

- $x > 3$
- $x < 30$
- $x \bmod 5 = 2$

Problem 3 Truth Table

Complete the truth table below for the statement

$$\neg[(x \wedge y) \vee z]$$

Use the intermediate columns to build the statement incrementally.

x	y	z	$x \wedge y$	$(x \wedge y) \vee z$	$\neg[(x \wedge y) \vee z]$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Problem 4 English to Logic

Consider the documents needed to apply to prove one's identity in a credit card application. Express each sentence using logical operations $\neg$, $\wedge$, $\vee$ and the propositional variables b , d , and p defined below. If there's any confusion about the precise meaning of the sentence, you're welcome to use the given table, which provides the same information in another format. The use of the word "or" in the sentences below always means inclusive or.

b	Applicant has birth certificate
d	Applicant has driver's license
p	Applicant has passport

- i The applicant must present a birth certificate, a driver's license, or a passport.

b	d	p	Desired Outcome
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

- ii Applicant must present a birth certificate or both a driver's license and a passport.

b	d	p	Desired Outcome
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

- iii The applicant must present at least two of the following forms of identification: birth certificate, driver's license, passport.

b	d	p	Desired Outcome
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	T
T	T	T	T

Problem 5 Boolean formula derivation

Apply the laws of logical equivalence to show that the following compound logic statement is logically equivalent to T . As with all proofs, make your steps small and clear; identify and apply one law at a time.

$$(\neg p \vee \neg q) \vee (p \wedge q)$$

Problem 6 ★ 10 wise men in the village (optional, no credit)

10 wise men live in a village. Each man has a color dot on the forehead either R or B not known to him; knowing his color means immediate death. But everyone knows the other men's colors, i.e. a B person sees 5R and 4B.

The men don't speak/communicate to each other, but each morning they meet in a circle and they can see if anyone died. They are extremely smart (can infer anything) and know when someone dies it's because he must have figured out his color.

For quite a few days this goes unchanged, until one day a stranger passes to the village and remarks an obvious statement to everyone: "The number of B colors is not 10".

Prove that eventually everyone in the village will figure out his color and die.