HW2: Logic

HW instructions

Problem 1 Arithmetic Operators

Evaluate each of the following statements as True or False. Show intermediate steps for full credit. Try by simplifying only one operator $(\wedge, \vee, >, =, \ldots)$ at a time if the whole expression is difficult to understand.

i $(7 \geq 7) \wedge (8 < 5)$

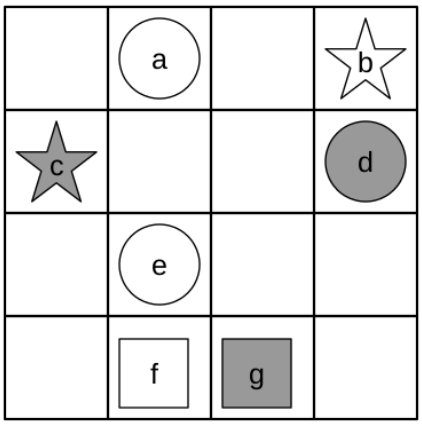
ii $(7 \geq 7) \vee (8 < 5)$

iii $(6 = 3 + 3) \wedge (10 < 1)$

iv $\neg(9 = 8)$

v $\neg((6 = 3 + 3) \vee (10 < 1))$

Problem 2 Tarski



Using the following predicates:

`square(x)` is true if x is a square (otherwise it is false)

`star(x)` is true if x is a star (otherwise it is false)
`circ(x)` is true if x is a circle (otherwise it is false)
`shade(x)` is true if x is shaded (otherwise it is false)
`next_to`(x, y) is true if x and y are adjacent horizontally, vertically or diagonally. No object is `next_to` itself.

For each of the statements below:

- write a simple, equivilent statement in english as the one given
- determine whether the statement is true or false

i $\texttt{star}(c) \wedge \neg\texttt{shade}(c)$

ii $\exists\, x\, \texttt{circ}(x) \wedge \texttt{shade}(x)$

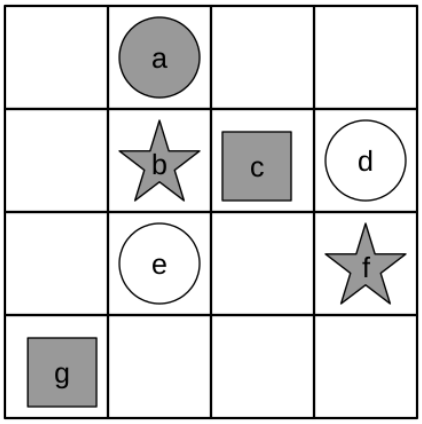
iii $\forall\, x\, \texttt{square}(x) \rightarrow \neg\texttt{shade}(x)$

iv $\forall x \forall y(\texttt{star}(x) \wedge \neg\texttt{shade}(x) \wedge \texttt{next\_to}(x, y)) \rightarrow (\texttt{shade}(y) \wedge \texttt{circ}(y))$

v $\exists\, x\, \forall\, y\, \texttt{next\_to}(x, y)$

vi $\forall\, y\, \exists\, x\, \texttt{shade}(x) \wedge \texttt{next\_to}(x, y)$

Problem 3 Converse



Use the following predicates:

- `circle`(x) is True when x is a circle

- $\texttt{rect}(x)$ is True when x is a rectangle
- $\texttt{star}(x)$ is True when x is a star
- $\texttt{shade}(x)$ is True when x is shaded in
- $\texttt{next\_to}(x, y)$ is True when the squares containing x and y touch (i.e. they're immediate neighbors left-right, top-bottom, or they share a corner).

Given the statement:

$$\forall x \neg \texttt{shade}(x) \rightarrow \texttt{circle}(x)$$

i Tell if the statement is true or false.

ii Write the **contrapositive** of this statement using logical symbols (not english). Tell if this new statement is true or false.

iii Write the **converse** of this statement using logical symbols (not english). Tell if this new statement is true or false.

iv Write the **inverse** of this statement using logical symbols (not english). Tell if this new statement is true or false.

v Using the grid above, give an example of a True conditional statement whose converse is False. (If this isn't possible, explain why not in one sentence)

vi Using the grid above, give an example of a True conditional statement whose contrapositive is False. (If this isn't possible, explain why not in one sentence)

Problem 4 English to Logic (hard)

Consider the eclectic animal collection at the local zoo. Express each sentence using logical operations $\neg$, $\wedge$, $\vee$ and the propositional variables h, w, and d defined below. The use of the word "or" in the sentences below always means inclusive or.

h	They have hampsters
w	They have whales
d	They have dinosaurs

These are more challenging than the previous examples, you may find it helpful to go case by case through a truth table to ensure your expression is consistent with the sentence.

i The zoo doesn't have any hampsters or whales, but it does have dinosaurs.

ii The zoo has at least two of the three groups of animals.

iii The zoo has exactly[1] two of the three groups of animals.

iv The zoo has, at most, one of these groups of animals.

Problem 5 Truth table, Vend

Consider the construction of a vending machine's logic. Assume that every soda costs a quarter, and that the machine accepts only quarters.

- E = True indicates the machine is empty, it has no more sodas
- S = True indicates the user has made a selection
- P = True indicates the user has paid a quarter

The machine may send two control commands to its machinery:

- V = True indicates machine will give the user a soda (i.e. "vend")
- R = True indicates machine will return the user's quarter

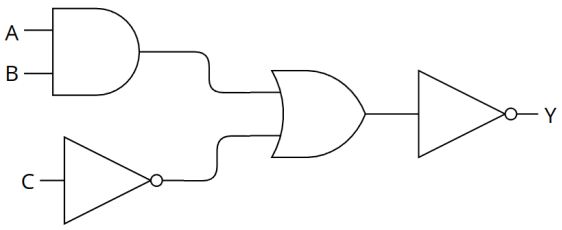
The machine should return a user's quarter only when they have paid and the machine is empty. The machine should give a soda only when a user has paid, made a selection and the machine is not empty.

i Write a truth table for V and R in terms of E, S, P

ii Write an expression for V in terms of E, S, P using boolean operators $\vee, \wedge, \neg$

iii Write an expression for R in terms of E, S, P using boolean operators $\vee, \wedge, \neg$

Problem 6 Circuit



[1] no more or less

i Using the circuit diagram above, express Y in terms of A, B, C and the logical operators $\wedge, \vee, \neg$.

ii Using the logic identities (i.e. "Laws of Logic"), simplify your statement of Y above. Label each step with the name of the logic identity you've used. (A simplified statement uses as few logical operators as possible)

iii Draw the logic circuit corresponding to your simplified statement above. (Notice: this is equivalent to the circuit above and yet it uses fewer circuit elements, requiring less resources to produce!)