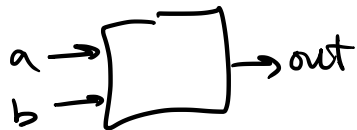
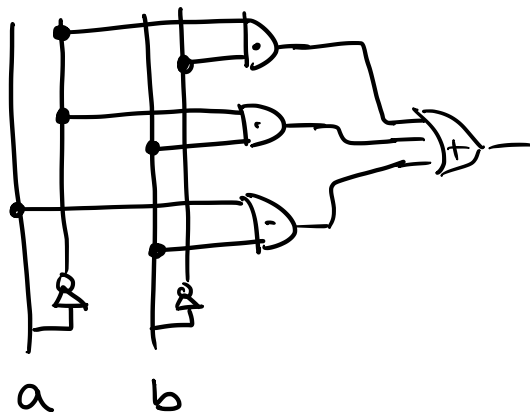


DNF Construction Example

a	b	out
0	0	1
0	1	1
1	0	0
1	1	1



$$(x-2)(x+3) = x^2 + x - 6$$



$$\equiv (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

?

$$\equiv \neg a \vee b$$

a	b	$\neg a \vee b$
0	0	1
0	1	1
1	0	0
1	1	1



Laws of (Boolean) Logic/Algebra

"normal algebra"

comm.
laws

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

dist.
laws

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

ident.

$$a \cdot 1 = a$$

$$a + 0 = a$$

De Morgan

Boolean algebra

$$a \vee b \equiv b \vee a$$

$$a \wedge b \equiv b \wedge a$$

$$p \wedge (q \vee r) \equiv p \wedge q \vee p \wedge r$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

Example (Logical Equivalence)

$$(\neg a \wedge \neg b) \vee [(\neg a \wedge b) \vee (a \wedge b)]$$

$$\equiv (\neg a \wedge \neg b) \vee [b \wedge (\neg a \vee a)]$$

$$\equiv (\neg a \wedge \neg b) \vee [b \wedge \top]$$

$$\equiv (\neg a \wedge \neg b) \vee b$$

$$\equiv (\neg a \vee b) \wedge (\neg b \vee b)$$

$$\equiv (\neg a \vee b) \wedge \top$$

$$\equiv \neg a \vee b \quad \checkmark$$

$$\begin{aligned} b &\rightarrow p \\ \neg a &\rightarrow q \\ a &\rightarrow r \end{aligned}$$

$$a \vee b \equiv b \vee a$$

$$a \wedge b \equiv b \wedge a$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge \top \equiv p$$

$$p \vee \text{F} \equiv p$$

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

a	b	out	out'
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

DNF construction: focus on 1's of output

$$\text{out} \equiv (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

$\vdots$ $\leftarrow$ lots of Boolean algebra

$$\equiv \neg a \vee b$$

Apply DNF construction to out', get

$$\text{out}' = a \wedge \neg b$$

$$\text{out} \equiv \neg \text{out}' \equiv \neg (a \wedge \neg b)$$

$$\equiv \neg a \vee \neg \neg b$$

$$\equiv \neg a \vee b$$

By construction, $\text{out} \equiv \neg \text{out}'$

Example 2

a	b	out	out'
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

DNF:

$$\text{out} \equiv (\neg a \wedge b) \vee (a \wedge \neg b)$$

$$\text{out}' \equiv (\neg a \wedge \neg b) \vee (a \wedge b)$$

but...

$$\text{out} \equiv \neg \text{out}'$$

$$\equiv \neg \left[(\neg a \wedge \neg b) \vee (a \wedge b) \right]$$

$$\equiv \neg (\neg a \wedge \neg b) \wedge \neg (a \wedge b)$$

$$\equiv (\neg \neg a \vee \neg \neg b) \wedge (\neg a \vee \neg b)$$

$$\equiv (a \vee b) \wedge (\neg a \vee \neg b)$$

↖
CNF Form

conjunctive normal form

Formal Logic : if-then $\rightarrow$ implications
if-and-only-if

① Implies : $A \Rightarrow B$ "A implies B"

A : "hot outside"

B : "not many people outside"

implication: $A \Rightarrow B$ "if hot outside, then not many people outside"

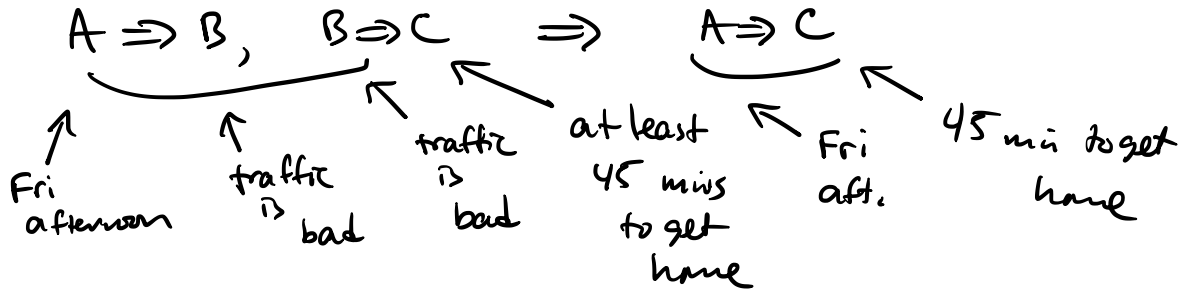
Converse : $B \Rightarrow A$ "if not many people outside, then hot outside"

* Converse not necessarily true

Contrapositive: $\neg B \Rightarrow \neg A$ "if many people outside, then not hot outside"

* Implication and its contrapositive are logically equivalent

Implications chain together



First-order Logic

↙ if and only if

① Consider : "any object is a square iff it is a rectangle with equal length sides"

$$P \Leftrightarrow Q$$

$P =$ "any object is a square"

↗ what does that mean?

* need variables and
predicates

$$\text{Square}(x) \Leftrightarrow \text{Rectangle}(x) \wedge \text{Equal sides}(x)$$

↗ predicate ↗ variable

②

Quantifiers

$\exists$ = "there exists"

$\forall$ = "for all"

$$\exists x : 2x = 4$$

TRUE

$$\forall x \exists y : x + y = 0$$

additive inverses
always
exist

$$\forall x : \text{Even}(2x)$$

FALSE

$$\exists y \forall x : x + y = 0$$

TRUE

$$\exists y \forall x : x \cdot y = 0$$

$y = 0$