Problem 1 [24 pts: (6 each)]: Base conversions

i Convert the number 947 from decimal to binary using Euclid's Division Algorithm.

ii Convert the binary number $(11010011)_2$ to decimal. Show an intermediate step which shows each powers of two to be added.

iii Convert the decimal number 174 to base 16 (hexadecimal) using Euclid's Division Algorithm.

iv Solve for the decimal value x:
$(2F3)_{16} = (x)_{10}$

Problem 2 [23 pts: (14, 9)] 2's Complement & Overflow

i Give the 8-bit two's complement representations of the following decimal values: 73, 97, -65, -94. Give some justification with responses for any negative values.

ii Compute the following operations as a computer would, directly from the (8-bit) 2's complement representations above. Indicate whether each operation results in an overflow or not.

Note: Use the two's complement representations from part ii above.

(a) $-65 + 73$
(b) $-65 - 94$
(c) $97 + 73$

Problem 3 [16 pts: (8 each)]: Expressiveness of a number representation

1. What range of numbers can be represented in 6-bit two's complement?
2. What are the *minimum* number of bits necessary to represent 1234 in two's complement?

Problem 4 [21 pts (7 each)]: Multiplication & Addition
Perform the given arithmetic problems in the given base. Add place values as needed to ensure correctness (i.e. your output can have more digits than either input value). Show every step of your work, the carry operations in particular, to receive full credit. To help us grade the problem more easily, please do not swap the order of values (e.g. if given as a times b, do not compute b times a). You're welcome to use a calculator to check your work, though answers without justification will not receive full credit.

i
$$\begin{array}{rr} & (1\ 1\ 0\ 1\ 0\ 1)_2 \\ \times & (1\ 0\ 1\ 1)_2 \\ \hline \end{array}$$

ii
$$\begin{array}{rr} & (3\ \mathtt{B}\ 4\ \mathtt{A})_{16} \\ + & (\mathtt{C}\ 5\ 2\ 7)_{16} \\ \hline \end{array}$$

iii
$$\begin{array}{rr} & (3\ 6\ 2)_8 \\ \times & (4\ 7)_8 \\ \hline \end{array}$$

Problem 5 [16 pts: (4, 4, 4, 4) + 2 extra]: Modular Arithmetic
For each of the equations below, find **every** x which satisfies the equality. If there are infinitely many x which satisfy an equality, you may find it helpful to list them out in a sequence whose pattern is obvious (e.g. $\{4, 7, 10, 13, 16, \ldots\}$)

i $11 \mod 3 = x$

ii $8 \mod 4 = x$

iii $x \mod 4 = 1$

iv $x \mod 5 = 3$

v This extra credit part has two equalities to compute x for:

$$(145 + 174) \mod 17 = x$$
$$(145 \mod 17) + (174 \mod 17) = x$$

These problems suggest a rule, identify and justify why this rule is True.

Problem 6 [1 extra pts]: Counting Ancestors
How many genetic ancestors does a person have, in total, among the previous n generations? Be sure to simplify your answer as much as possible. Please assume every person has two genetic parents and that a person is their own genetic ancestor too.