byte: 8-bits

nibble: 4-bits

128 64 32 16 8 4 2 1

1111 1111 = $2^8 - 1 = 255$

$87|65_{10}$

Other Representations: Octal & Hexadecimal

Octal: base 8 — binary 3 bits at a time

Hexadecimal: base 16 — 4 bits at a time

53	–	1
26	–	0
13	–	1
6	–	0
3	–	1
1	–	1

LSB

MSB

Octal:

- Base 8 representation
- Binary 3-bits at a time

e.g.

53_{10}

... 64 8 1

$6\ 5_8$

$$\begin{array}{r} 53 \\ -\ 48 \\ \hline 5 \end{array} \quad 48 = 6\times 8$$

32 16 8 4 2 1

$53_{10} = 110\,|\,101$

6 5

Hexadecimal :

- Base 16 rep.
- Binary 4-bits at a time

53_{10}

256 16 1

3 5 $_{16}$

$$\begin{array}{r} 53 \\ -48 \\ \hline 5 \end{array} = 16\times 3$$

... 32 16 | 8 4 2 1

0011 | 0101

3 5 $_{16}$

47_{10}

256 16 1

2 F $_{16}$

$$\begin{array}{r} 47 \\ 32 \\ \hline 15 \end{array}$$

0, 1, 2, ——, 9, A, B, C, D, E, F

10 11 - - —— 15

Binary mathematical operations

Addition

$$
\begin{array}{rcccccccl}
 & ^{1} & & ^{1} & & & & & \\
 & & 1 & 0 & 1 & 1 & 0 & \rightarrow & 22_{10} \\
+ & & 1 & 0 & 1 & 0 & 1 & \rightarrow & 21_{10} \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 1 & \rightarrow & 43_{10} \checkmark
\end{array}
$$

32 16 8 4 2 1

Subtraction

$$
\begin{array}{rccccccl}
 & 16 & 8 & 4 & 2 & 1 & & \\
 & & 0 & 1 & {}^{1}0 & & & \\
 & 1 & \cancel{1} & \cancel{0} & \cancel{1} & {}^{1}0 & & 26_{10} \\
- & 0 & 0 & 1 & 1 & 1 & & 7_{10} \\
\hline
 & 1 & 0 & 0 & 1 & 1 & & 19_{10} \checkmark
\end{array}
$$

Representing negative #s : signed magnitude, one's complement,
two's complement

Signed magnitude :

$+5$

-5

sign magnitude

	sign	4	2	1
+5 :	0	1	0	1
-5 :	1	1	0	1
	sign	mag.		

4-bit signed mag. rep.

[-7 , +7]

+0 : 0 000

-0 : 1 000

One's complement

4-bit 1's comp.

largest pos #

0111 → +7

most neg #

1000 → −7

[−7, +7]

① Positive #s → represented in straight binary

e.g. $5_{10} \rightarrow \boxed{0101}$

② Negative #s → write the magnitude in straight binary and flip all the bits

+0 0000

−0 1111

e.g. $-5_{10} \rightarrow$ mag: 0101

flip: $\boxed{1010}$

leading bit is sign

0 → +

1 → −

$0\}\underbrace{110}_{6} \rightarrow +6$

+

$1\}011 \rightarrow -4$

− → $\overset{4\,2\,1}{100} = 4$

Observation:

what is "flip all the bits"

$0101 \xrightarrow{\text{flip}} 1010$

subtract from 1111

$$\begin{array}{rcccc} & 1 & 1 & 1 & 1 \\ - & 0 & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array} \quad \begin{array}{l} = 2^4 - 1 = 15 \\ = \quad -5 \\ \hline \leftarrow \quad 10 \end{array}$$

neg #'s in 1's comp $\quad -x : (2^4-1)-x$ in straight bin

2's complement: pos #'s → rep. in straight bin $\quad +5 \rightarrow 0101$

neg #'s → · rep. mag. in bin $\quad -5 \rightarrow \underbrace{0101}_{\text{mag.}}$

· flip all bits $\quad \rightarrow 1010$

· add 1 $\quad \rightarrow \boxed{1011}$

$+x \rightarrow x$ in straight binary

$-x \rightarrow 2^4 - x$ in straight binary

Examples

2's complement : subtraction $\underline{\underline{\text{IS}}}$ addition

① $4_{10} - 3_{10} = 1_{10}$

$$\begin{array}{rr} 4 & 0100 \\ + \ -3 & + \ 1101 \\ \hline +1 & 1 \mid 0001 \end{array}$$

✓

mag 3	0011
flip	1100
+1	1101

→ 2's comp rep. of −3

$3_{10} - 4_{10} = -1_{10}$

$$\begin{array}{rr} 3 & 0011 \\ + \ -4 & + \ 1100 \\ \hline & 1111 \end{array}$$

✓

mag 4	0100
flip	1011
+1	1100

→ 2's comp for −4

−1 :

mag	0001
flip	1110
+1	1111

✓

Range of 4-bit 2's comp.

biggest pos # : 0111 → +7

Smallest neg # : −8
(most)

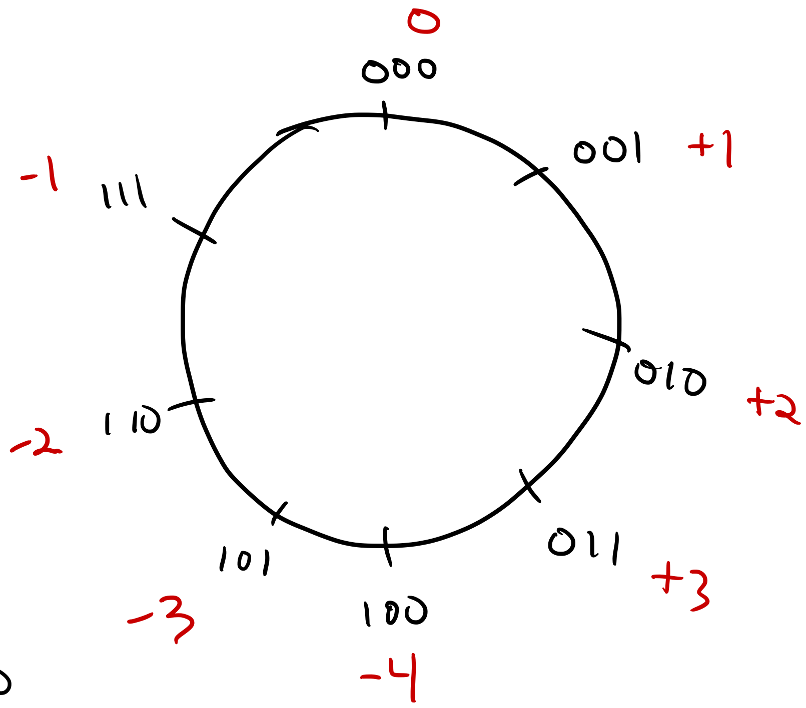
1000

neg: 1000
flip: 0111
+1 1000

[−8, +7]

Visual 2's comp. 3-bit 2's comp

#	
+0	000
+1	001
+2	010
+3	011
-4	100
-3	101
-2	110
-1	111



-2

2	010
flip	101
+1	110

Logic Gates

Digression:

motivating example: addition

$$\begin{array}{rrrr} a: & 1 & 0 & 1 \\ b: + & 1 & 1 & 0 \\ \hline & & & 1 \end{array}$$

carry bit 0

a_0

b_0

sumbit s_0

$$\begin{array}{rrrr} & 1 & 1 & 1 \\ + & 1 & 0 & 1 \\ \hline & & & 0 \end{array}$$

carry bit 1

a_0

b_0

s_0 sum bit

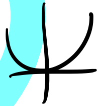
Sum bit

a_0	b_0	s_0
0	0	0
0	1	1
1	0	1
1	1	0

a_0	b_0	c
0	0	0
0	1	0
1	0	0
1	1	1

$f(x) = x^2$

x	$f(x)$
-2	4
-1	1
0	0



Recap 2's comp.

4-bit 2's comp.

	sign 4 2 1	math
① pos #'s → rep. in straight binary e.g. +6	0 1 1 0	x
② neg #'s → ⓐ rep mag. in binary e.g −6	0 1 1 0	x
ⓑ flip all bits	1 0 0 1	$(2^4-1)-x$
ⓒ add 1	1 0 1 0	2^4-x

$16-6=10$

$$
\begin{array}{rl}
1111 & \leftarrow 2^4-1 \\
-\ 0110 & \leftarrow 6 \\
\hline
1001 & (2^4-1)-6
\end{array}
$$

E.g.

3: 0011

4: 0100

-3: 0011
1100
$\boxed{1101}$

-4: 0100
1011
$\boxed{1100}$

① $4-3 \Rightarrow 1$

$$
\begin{array}{rr|l}
 & & 1\,1\,1 \\
4: & & 0100 \rightarrow 4 \\
+\ -3: & & 1101 \rightarrow 2^4-3 \\
\hline
 & 1 & \boxed{0001} \quad 2^4+1
\end{array}
$$

② $3-4 \Rightarrow -1$

$$
\begin{array}{r|l}
3: & 0011 \rightarrow 3 \\
+\ -4: & 1100 \rightarrow 2^4-4 \\
\hline
 & 1111 \quad 2^4-1
\end{array}
$$

Converting back from 2's comp:

one way

$$\begin{array}{rl} 1\,\cancel{1}\,\cancel{0}\,0 & 2^4 - x \\ -\;0\,0\,0\,1 & -1 \\ \hline 1\,0\,1\,1 & (2^4 - x) - 1 \\ 0\,1\,0\,0 \quad \text{flip} & (2^4 - 1) - [2^4 - x - 1] \\ \rightarrow 4 & x \end{array}$$

$\Rightarrow -4$

	1100	$2^4 - x$
flip	0011	$(2^4 - 1) - [2^4 - x]$
add 1	0100 ✓	$(2^4 - 1) - [2^4 - x] + 1$
		$= x$

to convert a <u>neg</u> 2's comp. # to its magnitude:

1. flip all bits
2. add 1
3. result is neg. mag.

Overflow Rules

-6

$$\begin{array}{r} 0110 \\ 1001 \\ \boxed{1010} \end{array}$$

$$\begin{array}{rr} 4: & \overset{1}{0}100 \\ +4: & 0100 \\ \hline & 1000 \end{array}$$

$$\begin{array}{rr} -6 & \overset{1}{1}\overset{1}{0}10 \\ +-6 & 11010 \\ \hline & 1\,|\,\boxed{0100} \end{array}$$

1. pos + pos must be pos

   ⇒ If you have leading zeroes for both #s, but answer has a leading one

   ⇒ overflow

2. neg + neg must be neg

   ⇒ if you have leading ones for both #s, but answer has leading zero

   ⇒ underflow