Lower bounds

We prove that SAT cannot be solved by an algorithm that runs in space O(log n) and uses time n^c for a constant $c > 1$.

This algorithm is allowed random-access to input.
(Without this, n^2 time lower bounds hold for palindromes)

The best-known result is

$c = 2 \cos(\pi / 7) = 1.80193...$

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First, two lemmas

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Lemma 1: L ⊆ $U_a \sum_a$ TIME(n)

Lemma 2: NTIME(n) ⊆ TIME(n^c) → $\sum_a$ TIME(n) ⊆ TIME(n^d)

for $d = c^a$

Proof of theorem:

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Proof of theorem:

Pick any A $\in$ TIME(n^2).

We show A $\in$ TIME($n^{1.9}$), contradicting time hierarchy:

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A $\in$ SPACE(????) WHY?

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for $d = c^a$

Proof of theorem:

Pick any $A \in$ TIME(n^2).

We show $A \in$ TIME($n^{1.9}$), contradicting time hierarchy:

$A \in$ SPACE(c log n)	(for some c; assumption + padding)
$\subseteq \Sigma_a$ TIME(n)	(for some a(c); Lemma 1)
$\subseteq$ TIME(???)	WHY?

We'll actually prove:

Theorem: NTIME(n) NOT IN $TIME(n^c) \cap L$, for some $c > 1$.

Lemma 1: $L \subseteq U_a \Sigma_a TIME(n)$

Lemma 2: $NTIME(n) \subseteq TIME(n^c) \rightarrow \Sigma_a TIME(n) \subseteq TIME(n^d)$ for $d = c^a$

Proof of theorem:

Pick any $A \in TIME(n^2)$.

We show $A \in TIME(n^{1.9})$, contradicting time hierarchy:

$A \in SPACE(c \log n)$	(for some c; assumption + padding)
$\subseteq \Sigma_a TIME(n)$	(for some a(c); Lemma 1)
$\subseteq TIME(n^d)$	(for some a(c); assumption+Lemma 2)

For small $c > 1$, have $d \leq 1.9$. ■