

Quiz 3

Name: _____

(10 points) Consistency of constraints

The following problem occurs in program analysis. For a set of n variables x_1, \dots, x_n , you are given a set of m constraints, of one of two forms: *equality*, of the form $x_i = x_j$; *strict inequality*, of the form $x_i > x_j$. The goal of this problem is to determine whether a given set of constraints can be satisfied.

For example, the following set of constraints can be satisfied.

$$x_1 = x_2; x_1 = x_3; x_2 > x_4; x_3 > x_5$$

One can set $x_1 = x_2 = x_3 = 1$, $x_4 = 0$, and $x_5 = 0$.

The following set of constraints, however, cannot be satisfied.

$$x_1 = x_2; x_1 = x_3; x_2 > x_4; x_4 > x_5; x_5 > x_3$$

Give an algorithm that takes as input m constraints over n variables and determines whether the constraints can be satisfied. Your algorithm only needs to return a “yes/no” answer (“yes” if the constraints can be satisfied; and “no” otherwise).

State the running time of your algorithm. You do not need to prove the correctness of the algorithm. The more efficient your algorithm is, in terms of its worst case running time as a function of n and m , the more credit you will get.