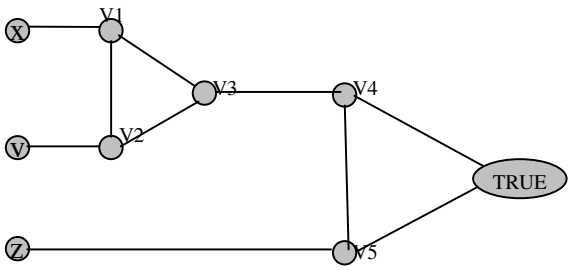
c) According to part b, since graph coloring problem can be self-reducible to corresponding decision problem, 3-color can be self-reducible to corresponding decision problem with number of colors equal to 3. In that case, if 3-color is NP-complete then corresponding decision problem will be NP-complete.

d) If we construct the graph in the way specified on page 1019, one type of literal edges forms a triangle on x_i, $\neg x_i$ and RED. Then in any 3-coloring graph, the variable can be constructed as this kind of triangle and the variable could take true or false when form clause with other variables.

e)

a. 3 colorable => at least one of x, y or z is colored true:
Suppose x, y and z are all false, then v3 has to be false, and v4 has to be RED. In that case, since z is false, v5 has to be RED, v4 and v5 conflicts. So it can't be 3-colorable. (see figure following)



b. At least one if true => 3-colorable:
For other seven cases of assignments to variables x, y and z, the figure above can be 3-colorable.

f)

a. It belongs to NP: Given any graph instance and a 3-coloring certificate to the graph, it can be recognized in polynomial time by deciding whether the adjacent vertices have different colors.
b. It is NP-hard: By using the way described in page 1019 and figure 34.20, we can construct a 3-coloring graph such that the graph is 3-colored if and only if the corresponding 3-CNF is satisfiable as we proved in part e. Since 3-CNF is NP-complete then 3-coloring problem, which is reduced in polynomial time from 3-CNF, is NP-complete.