College of Computer & Information Science
Northeastern University
CSG250: Wireless Networks

Spring 2007
Handout 1
12 January 2007

Sample Solution to Problem Set 1

(Problem numbers indicated below refer to the problems in the second edition of the course text. The problem numbers in parentheses refer to the problems in the first edition of the course text.)

1. (8 points) Applying low-pass and bandpass filters to a digital signal

A square periodic signal is represented as the following sum of sinusoids:

$$s(t) = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} cos((2k+1)\pi t).$$

(Note that this is just a rewriting of the formula we discussed in class.)

(a) Suppose that the signal is applied to an ideal low-pass filter with bandwidth 15 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 10 Hz; for 5 Hz. What happens as the bandwidth decreases.

(b) Suppose that the signal is applied to a bandpass filter that passes the frequencies from 4 to 8 Hz. Plot the output from the filter and compare to the original signal.

For your plots, use an appropriate plotting tool. One such tool is gnuplot, available in Unix. (You may even be able to generate these plots using Excel).

Answer:

(a) The non-zero frequency components of the signal $s(t)$ correspond to the frequencies 1/2 Hz, 3/2 Hz, 5/2 Hz, ... If the signal is passed through an ideal low-pass filter with bandwidth 15Hz, we obtain the output signal as:

$$\frac{2}{\pi} \sum_{k=0}^{14} \frac{(-1)^k}{2k+1} cos((2k+1)\pi t).$$

Signals can be obtained for the other cases similarly. On plotting, we obtain Figures 1, 2, and 3, respectively.

(b) See Figure 4.

2. (5 points) Shannon capacity theorem

Assume the following bandwidth & available channel capacity for the transmision media *twisted pair*, *coaxial cable*, and *optical fiber* of a certain grade, respectively: 4 Mbps & 3 MHz, 500 Mbps

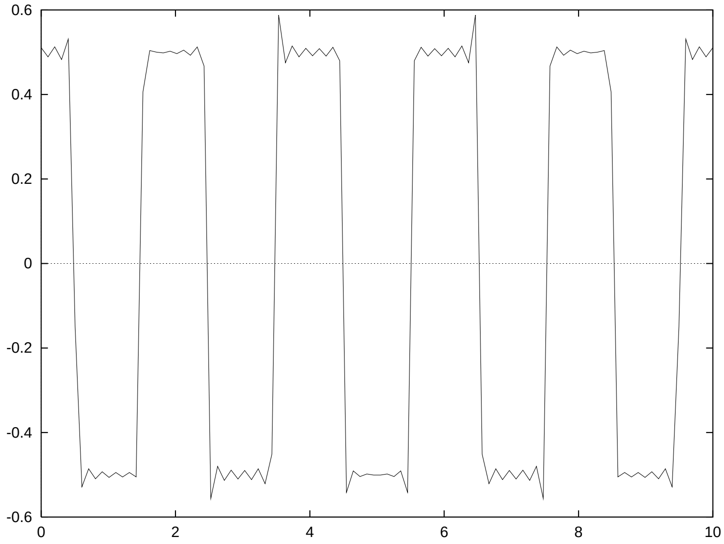

Figure 1: Passing through a low-pass filter of 15Hz bandwidth.

& 350 MHz, and 2 Gbps & 2 GHz. Assuming these numbers capture the fundamental limitation imposed by Shannon's capacity theorem, estimate the signal-to-noise ratio underlying the three media.

Answer: By Shannon's theorem, we have

$$C = W \log(1 + \frac{S}{N}).$$

where C is the available channel capacity, W is the bandwidth, and $\frac{S}{N}$ is the signal-to-noise ratio. So we have $\frac{S}{N} = 2^{\frac{C}{W}} - 1$.

For twisted pair, $\frac{S}{N} = 1.52$; for coaxial cable, $\frac{S}{N} = 1.69$; and for optical fiber, $\frac{S}{N} = 1$.

3. (6 points) Antenna gains and power attenuation

A wireless receiver with an effective diameter of 250 cm is receiving signals at 10 GHz from a transmitted that transmits at a power of 30 mW and a gain of 30dB.

(a) What is the gain of the receiver antenna?

Answer: The antenna gain is given by

$$\frac{4\pi A_e f^2}{c^2} = \frac{4\pi \cdot \pi 1.25^2 \cdot 10^{20}}{9 \cdot 10^{16}} = 68469.$$

(b) What is the received power if the receiver is 5 km away from the transmitter?

Answer: The gain of the transmitting antenna is $10^{30/3} = 1000$. The received power is given by

$$\frac{G_t G_r P_t c^2}{(4\pi f d)^2} = \frac{68469 \cdot 1000 \cdot 30 \cdot 9 \cdot 10^{16}}{16\pi^2 10^{20} \cdot 25 \cdot 10^6} mW = 46216.5 \times 10^{-6} mW = .046 mW.$$

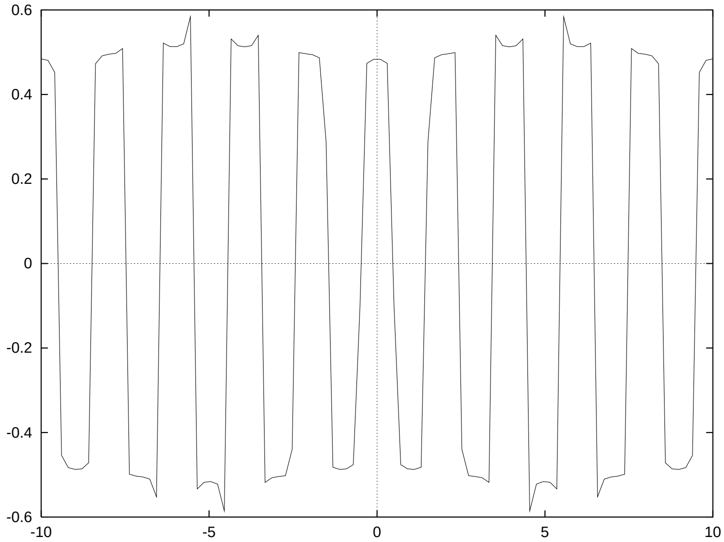

Figure 2: Passing through a low-pass filter of 10Hz bandwidth.

4. (5 points) Transmitting voice by radio waves

Problem 5.4 (Chapter 5, Problem 4).

Answer:

- The length of the antenna is $(3 \times 10^8/300)/2 = 5 \times 10^5$ m.
- The carrier frequency would be $3 \times 10^8/2 = 150$ MHz.

5. (6 points) Parabolic antennas

A microwave transmitter with an output of 0.5 W at 2 GHz is used in a transmission system, where both the transmitting and receiving antennas are parabolas, each 1 m in diameter. Suppose the two antennas are directionally aligned and are 10 kms apart.

(a) What is the effective radiated power of the transmitted signal, in W and dB?

Answer: The gain of a parabolic antenna is given by $0.56 \cdot 4\pi\pi r^2/\lambda^2$, where r is the radius of the antenna and λ is the wavelength. We have $r = 0.5$m and $\lambda = 3 \times 10^8/(2 \times 10^9) = 0.15$m. Substituting the numbers, we obtain the gain of each antenna to be 245.39.

Effective radiated power of the transmitted signal is $0.5 \cdot 245 = 122.5\text{W} = 20.88$ dB.

(b) What is the available signal power out of the receiving antenna?

Answer:

$$P_r = \frac{245.39^2 \cdot 0.15^2 \cdot 0.5}{16\pi^2 \cdot 10^2 \cdot 10^6} = 4.2 \times 10^{-8} W$$

6. (10 points) Height and line of sight

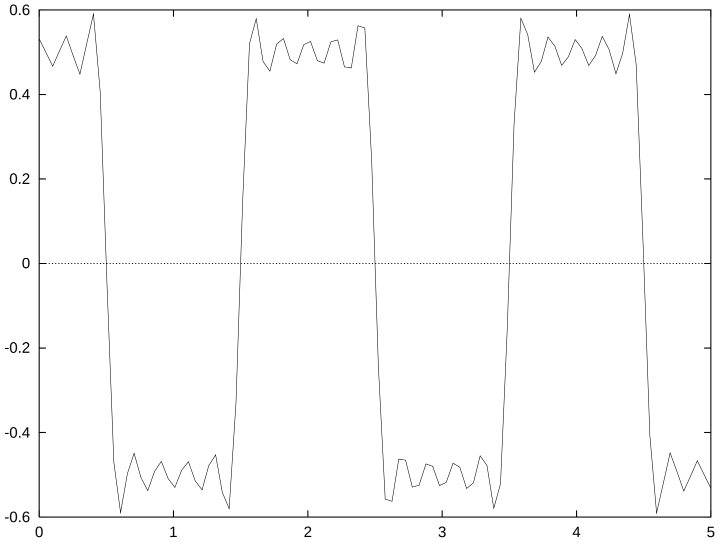

Figure 3: Passing through a low-pass filter of 5Hz bandwidth.

Problems 5.12 and 5.13 (Chapter 5, Problems 12 and 13).

Answer: Let d be the distance in kms, and h be the height of the antenna in ms. Then, since the line of sight from the top of the antenna forms a tangent to the earth's surface, it follows that

$$(6370 + h/1000)^2 = 6370^2 + d^2,$$

which implies that

$$2 \cdot 6370 \cdot h/1000 + h^2/10^6 = d^2.$$

Since $h^2/10^6$ is so much smaller than $2 * 6370 * h/1000$, we obtain as an approximation:

$$d^2 = 12.74h \Rightarrow d = 3.57\sqrt{h}.$$

Therefore, if $d = 80$ km, then the height of the antenna must be at least $((80/3.57)^2)/K = 502.1/K$ m, where K is the adjustment factor. If we use $K = 4/3$, we get 376.575 m.

7. (5 points) QAM demodulation

Problem 6.1 (Chapter 6, Problem 1).

Answer: The encoded signal is $d_1(t)\cos(2\pi f_c t) + d_2(t)\sin(2\pi f_c t)$. When this signal is modulated on carrier signal $\cos(2\pi f_c t)$, we obtain

$$d_1(t)\cos^2(2\pi f_c t) + d_2(t)\cos(2\pi f_c t)\sin(2\pi f_c t) = \frac{d_1(t)}{2}(1 + \cos(4\pi f_c t)) + \frac{d_2(t)}{2} sin(4\pi f_c t).$$

When the above signal is passed through a low-pass filter, the high frequency components (of frequency $2f_c$) are filtered out, leaving the signal $d_1(t)/2$.

A similar calculation explains the recovery of $d_2(t)/2$.

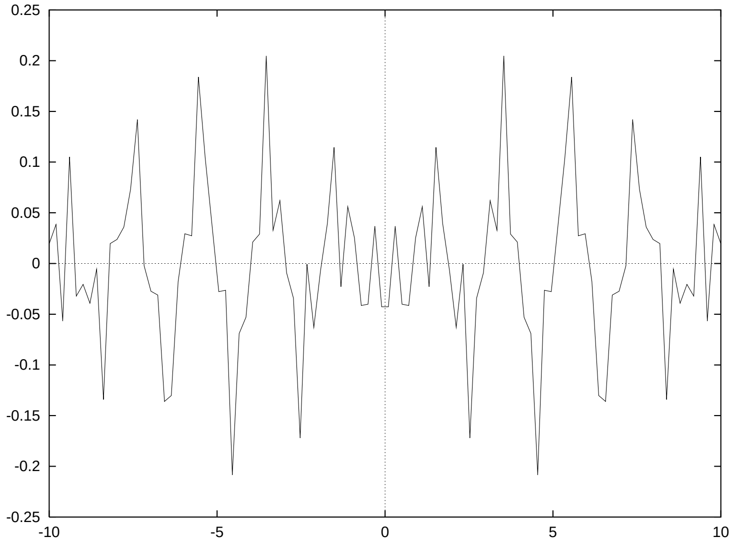

Figure 4: Passing through a 4-8Hz bandpass filter.

8. (5 points) Pulse Code Modulation

Problem 6.12 (Chapter 6, Problem 11).

Answer: The notions of normalization and resolution in this question were quite confusing. We assume that the normalization interval is $[0, 1]$. If you adopted different notions for your solutions, that is fine.

- **(a)** Since we have 2^8 levels, the normalized step size is $1/2^8$.
- **(b)** The actual step size in volts is $10/2^8$.
- **(c)** The actual maximum quantized level in volts is $10(1 - 1/2^8)$.
- **(d)** The normalized resolution is 8-bits. Expressed in terms of quantization error, we get a normalized quantization error of $1/2^8$.
- **(e)** The actual resolution, in terms of quantization error, is $10/2^8$.
- **(f)** The percentage resolution varies with the data value. One upper bound on the percentage resolution is $(1 - 1/2^8) \cdot 100\% = 99.6\%$.