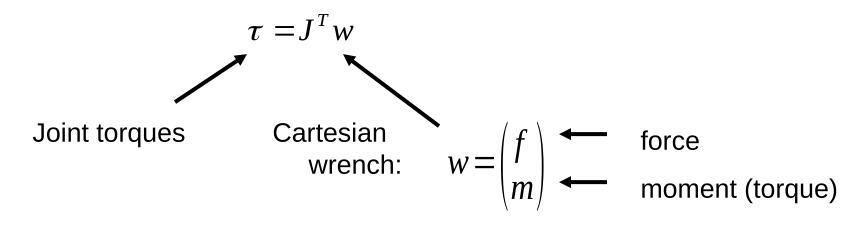
# Cartesian Control (Wrenches)

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### Using the Jacobian for Statics

Up until now, we've used the Jacobian in the twist equation,  $\xi = J\dot{q}$ 

Interestingly, you can also use the Jacobian in a statics equation:



## Using the Jacobian for Statics

It turns out that both wrenches and twists can be understood in terms of a representation of displacement known as a *screw.* 

• Therefore, you can calculate work by integrating the dot product:

$$W = \int (v \cdot f + \omega \cdot m) = \int \begin{bmatrix} v \\ \omega \end{bmatrix}^T \begin{bmatrix} f \\ m \end{bmatrix} \leftarrow \text{Work in Cartesian}$$
  
space  
$$W = \int \tau^T \dot{q} \leftarrow \text{Work in joint space}$$

Conservation of energy:

$$\tau^{T} \dot{q} = \int \begin{bmatrix} v \\ \omega \end{bmatrix}^{T} \begin{bmatrix} f \\ m \end{bmatrix}$$

#### Using the Jacobian for Statics

Incremental work (virtual work)

Wrench-twist duality:

$$\tau = J^T w$$
 vs  $\xi = J \dot{q}$ 

 $\tau = J^T w$ 

#### New perspective on J^T control

Input: x\* Output: q\* 1. repeat until dx is small: 2. init q to random joint configuration 3. repeat K times: 4. x = FK(q)5.  $dx = x^*-x$ 6.  $dq = stepsize * J^T dx$ 7. q = q + dq8. return  $q^* = q$ 

Original statement of J^T transpose control

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Statement of J^T transpose control as forces and torques