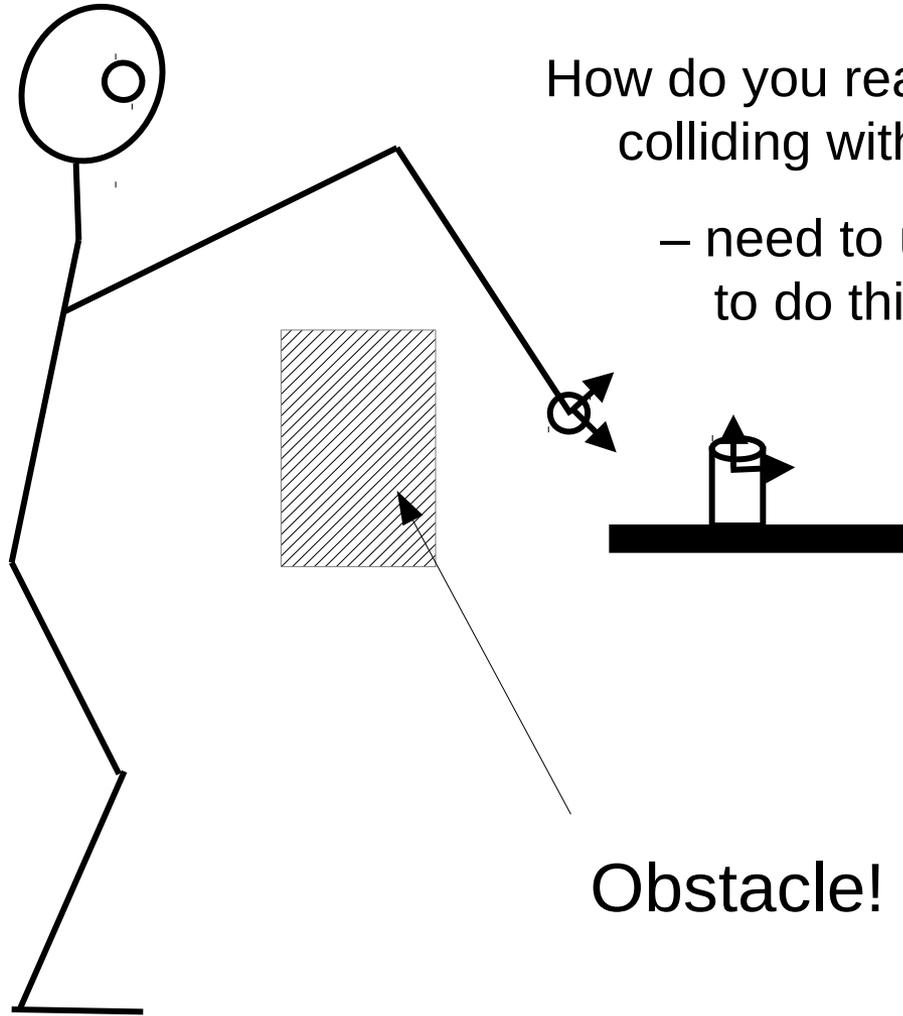


# Configuration space



How do you reach toward something without colliding with obstacles in the environment?

– need to understand configuration space to do this!

Obstacle!

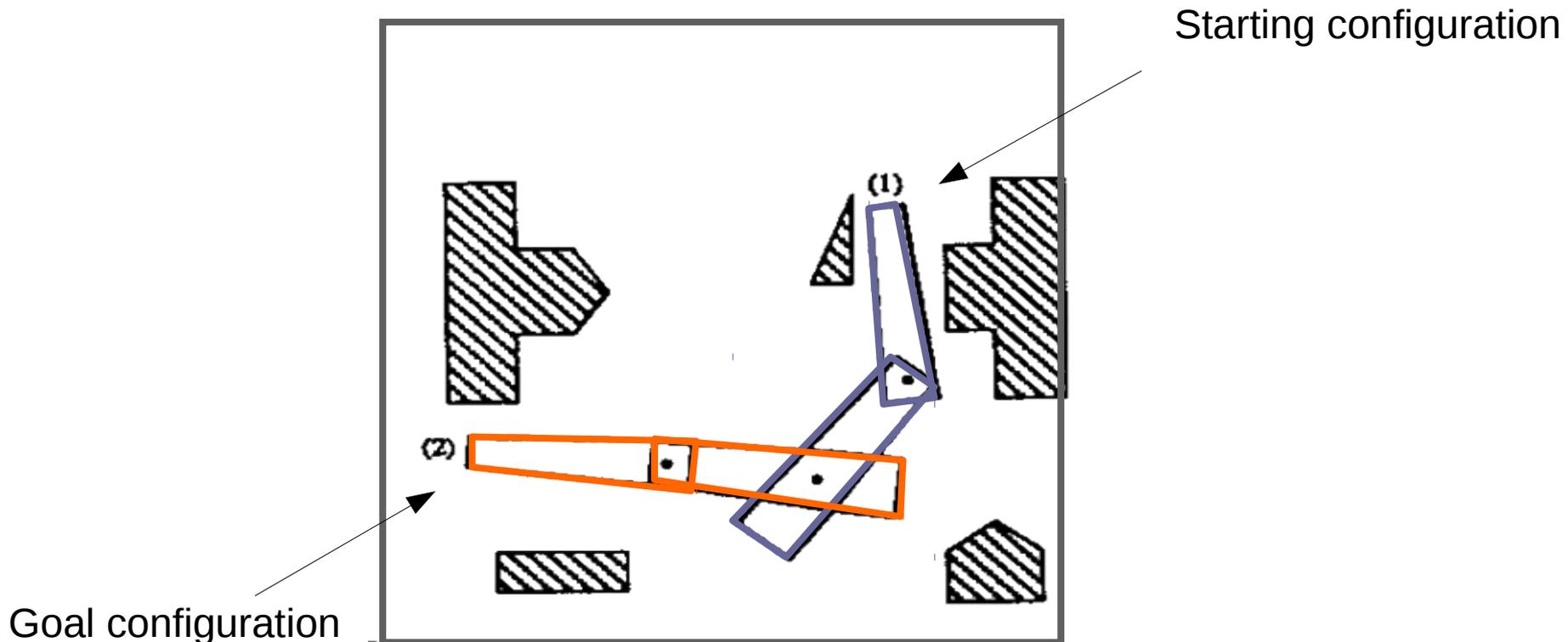
# Problem we want to solve

## Given:

- description of the robot arm (the manipulator)
- description of the obstacle environment

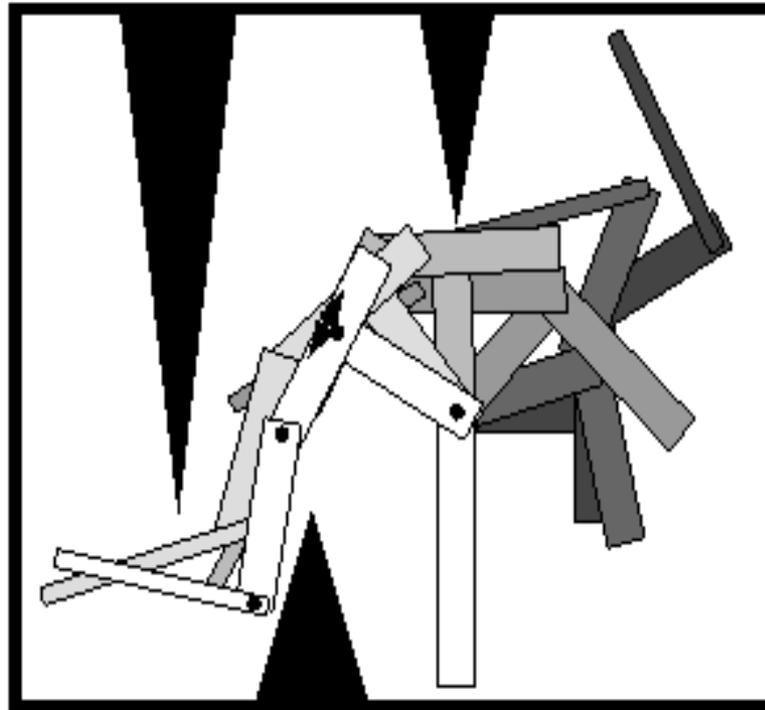
## Find:

- path from start to goal that does result in a collision



# Problem we want to solve

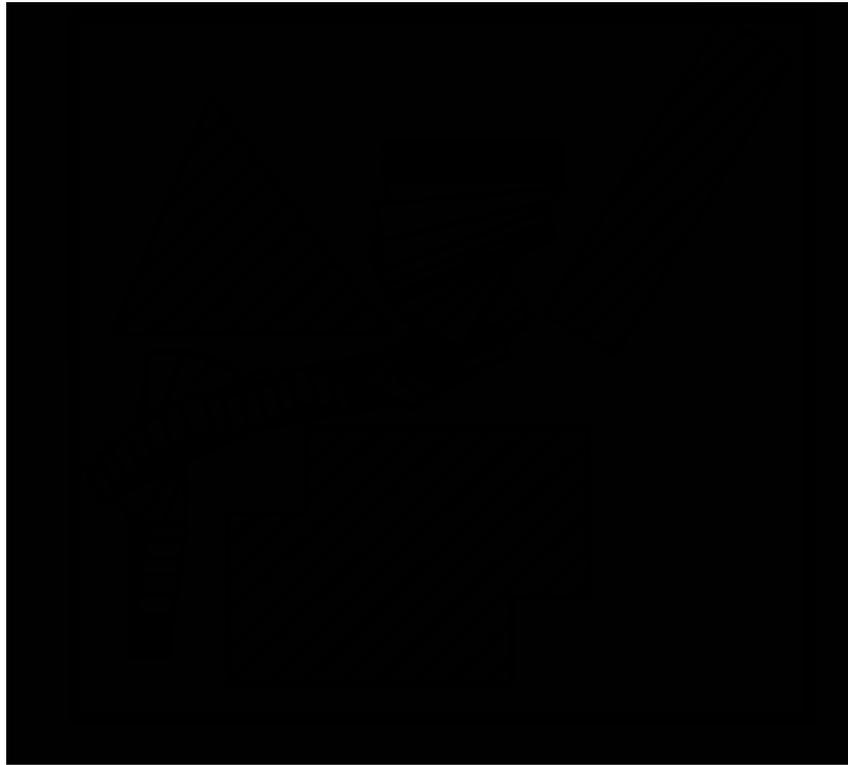
This problem statement is actually very general  
– manipulators



# Problem we want to solve

This problem statement is actually very general

- manipulators
- mobile robots

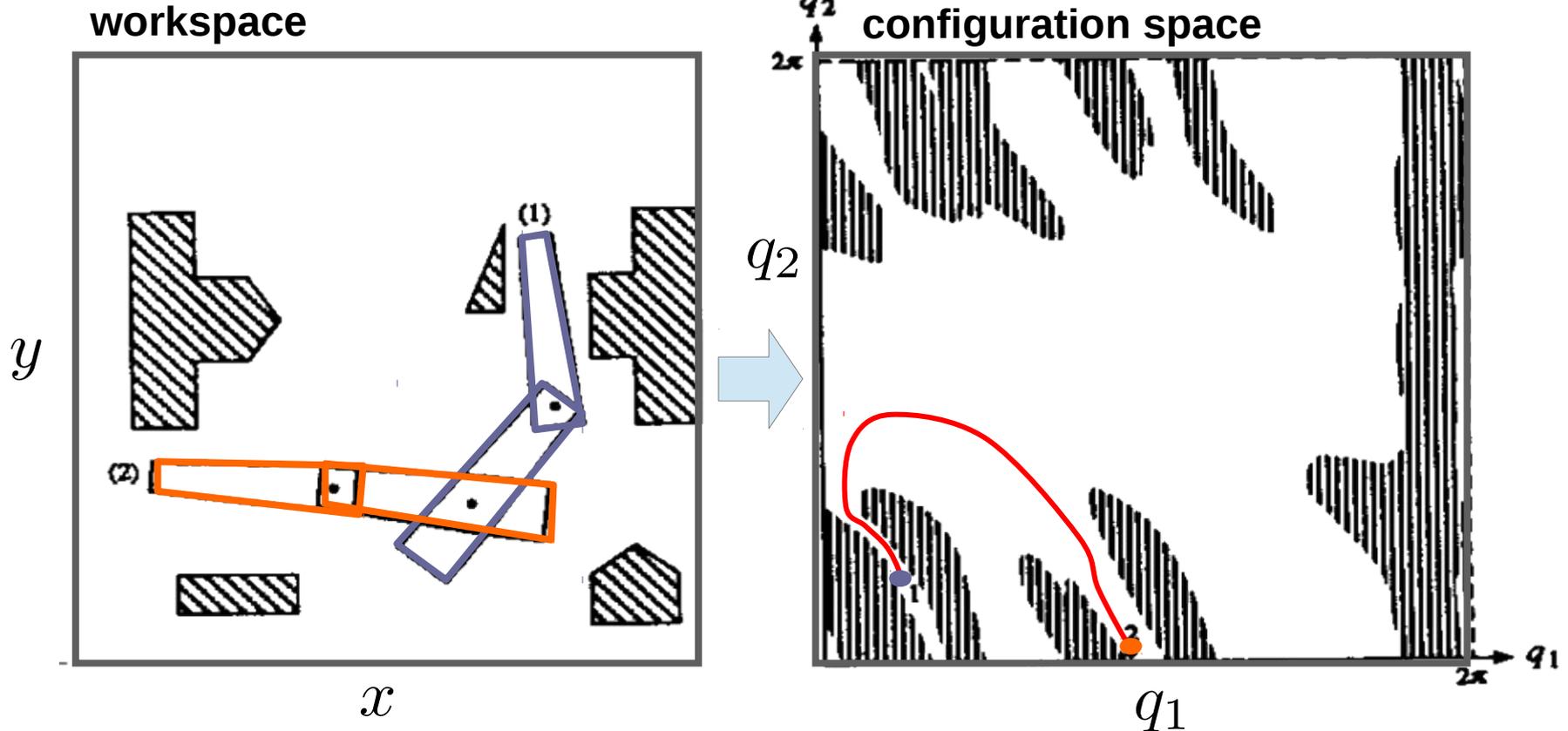


# Approach: plan in “configuration space”

Convert the original planning problem into a planning problem for a single point.

# Approach: plan in “configuration space”

Convert the original planning problem into a planning problem for a single point.

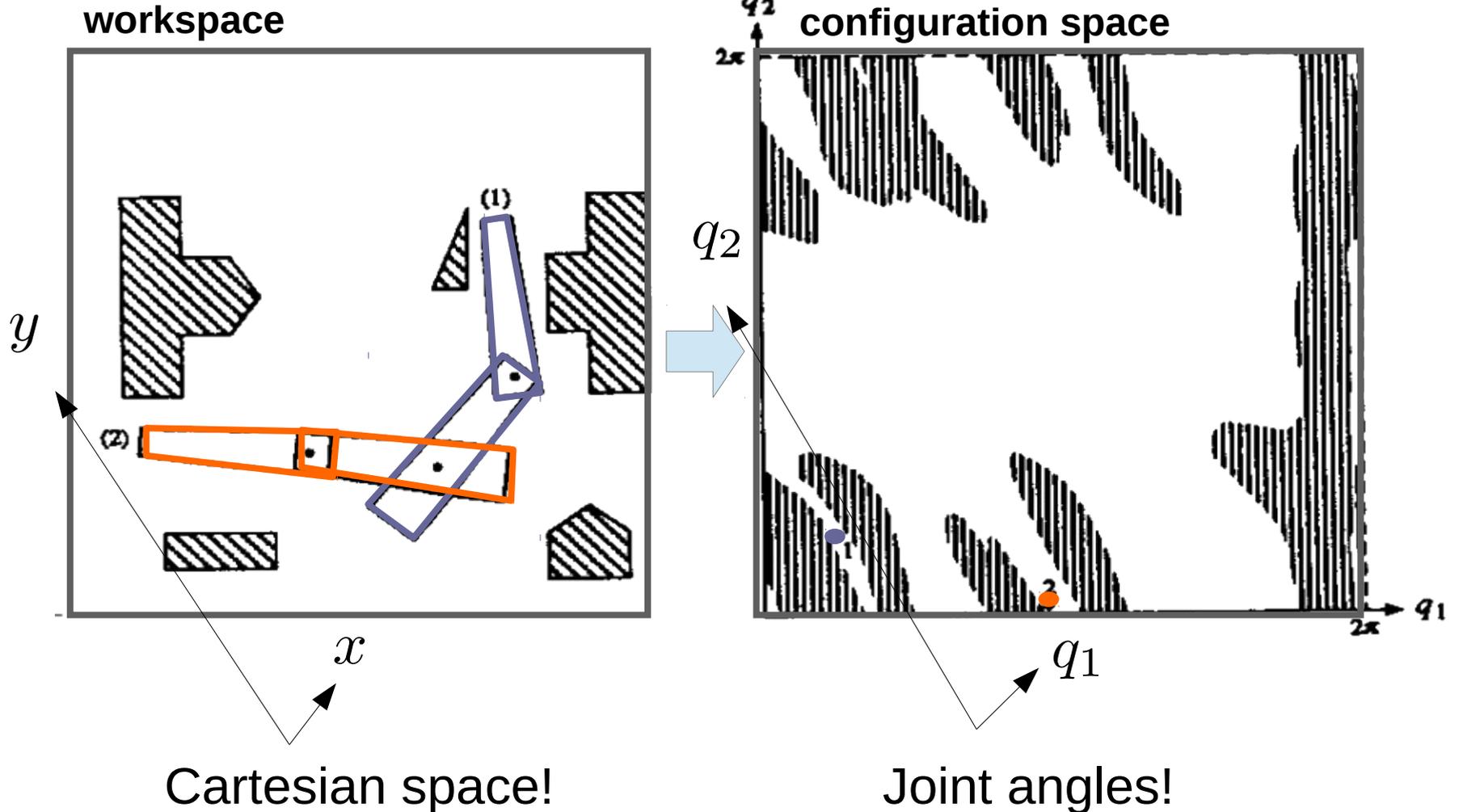


Original problem  
– plan path for robot arm

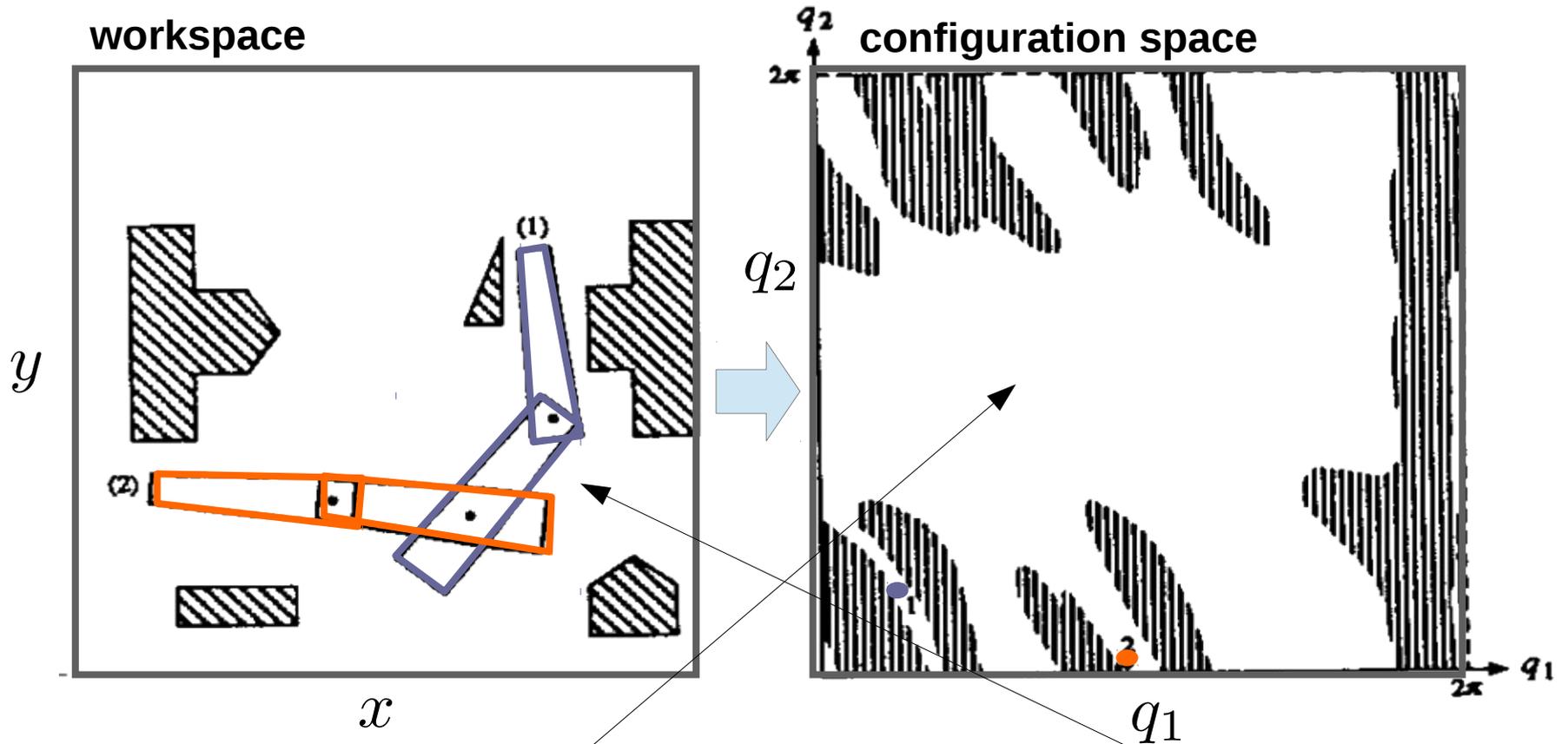
Equivalent problem:  
– plan path for a point

# Approach: plan in “configuration space”

Notice the axes!

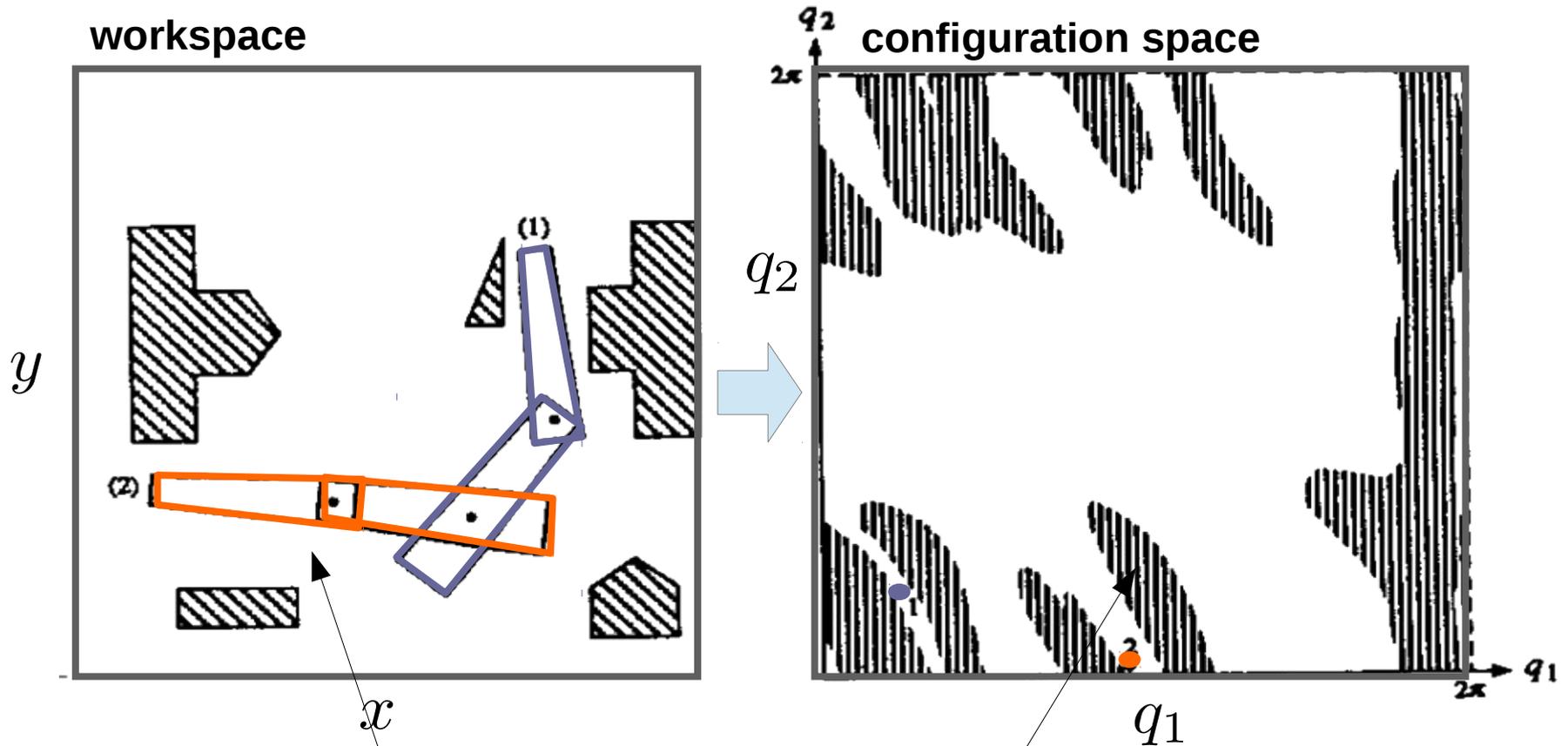


# Approach: plan in “configuration space”



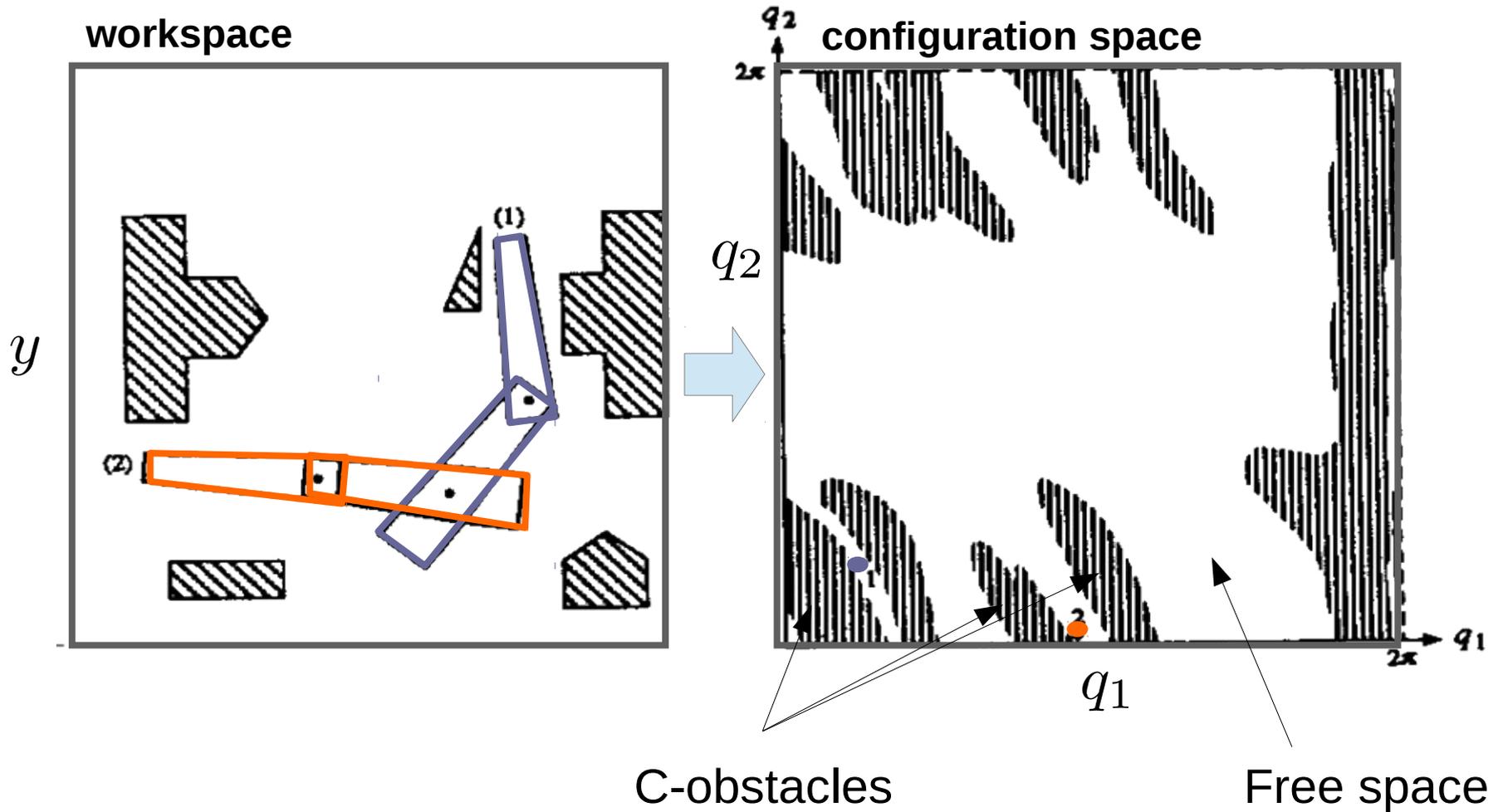
Every point here corresponds to a single robot configuration here

# Approach: plan in “configuration space”



Every point that intersects an obstacle here corresponds to an arm configuration that intersects an obstacle

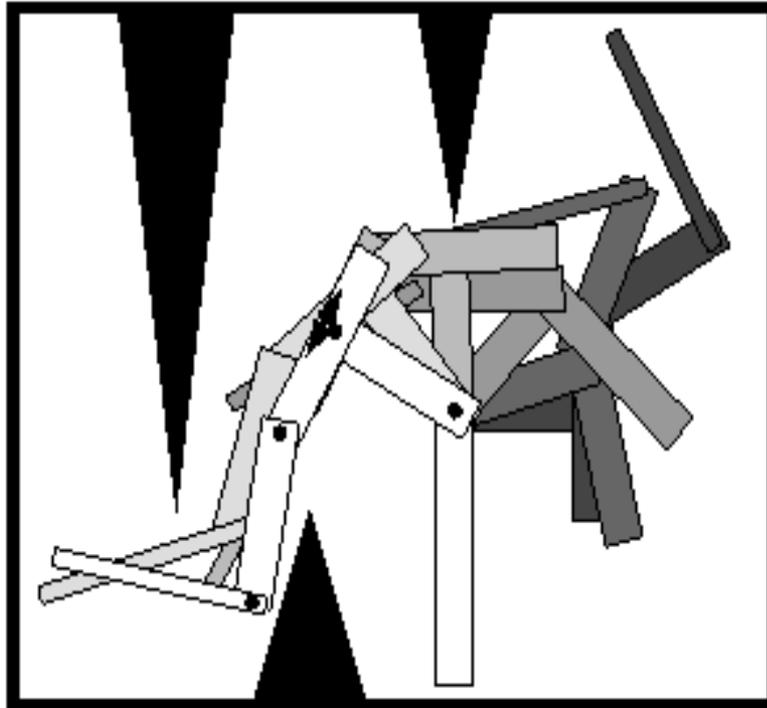
# Approach: plan in “configuration space”



# Configuration space

The dimension of a configuration space is the minimum number of parameters needed to specify the configuration of the robot completely.

– also called the number of “degrees of freedom” (DOFs)

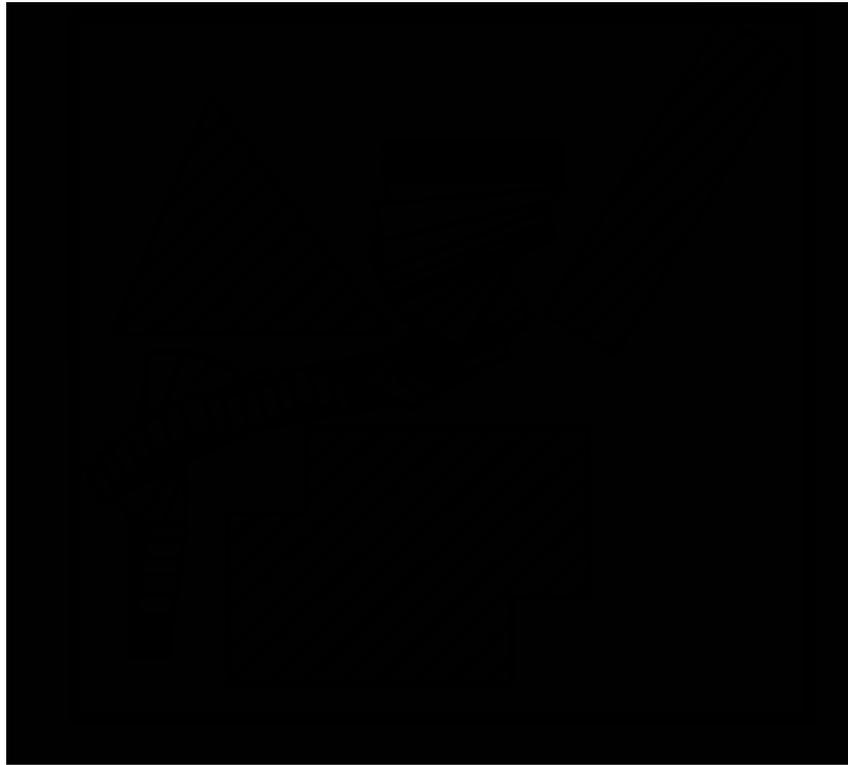


Dimension = 3

# Configuration space

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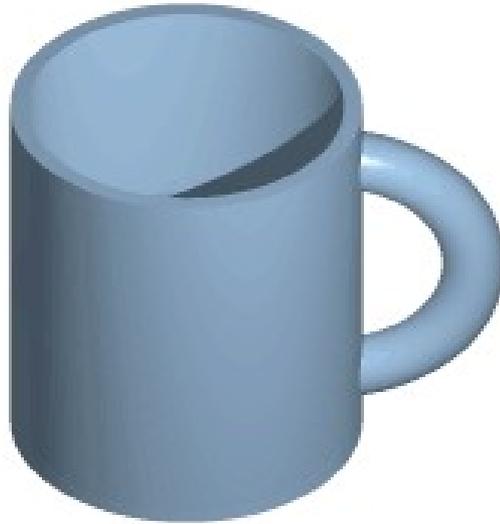


Dimension = ?

# Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

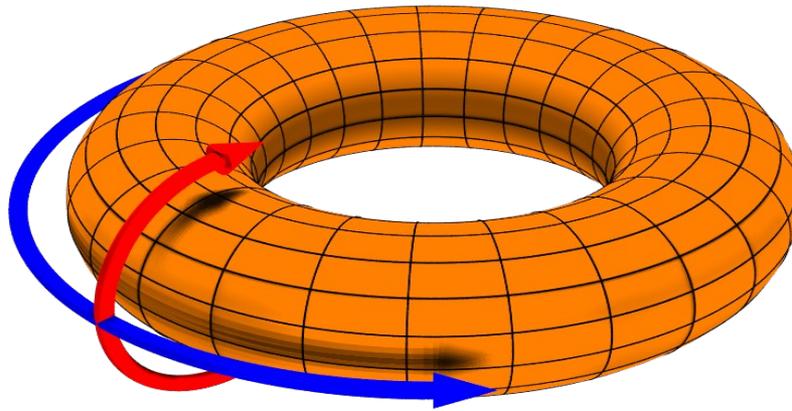


The topology of this mug is a torus

# Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

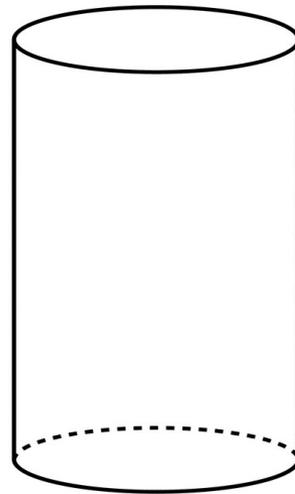


Torus:  $C = S^1 \times S^1$

# Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

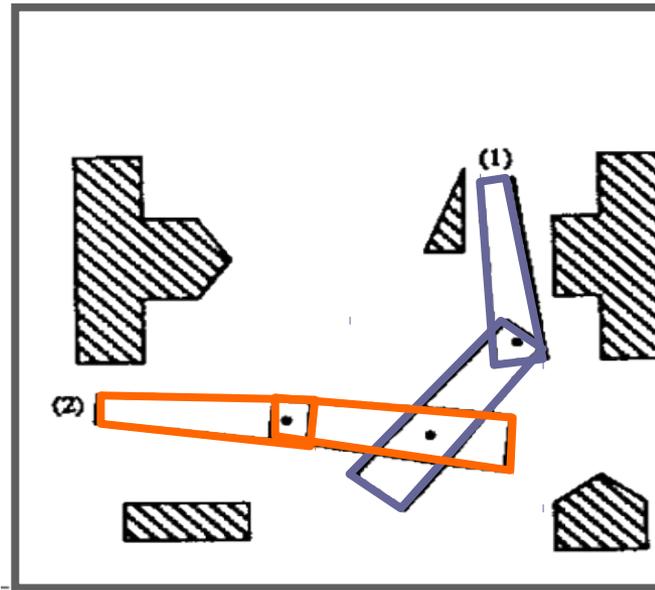


$$\text{Cylinder: } C = R^2 \times S^1$$

# Topology of configuration space

What is topology?

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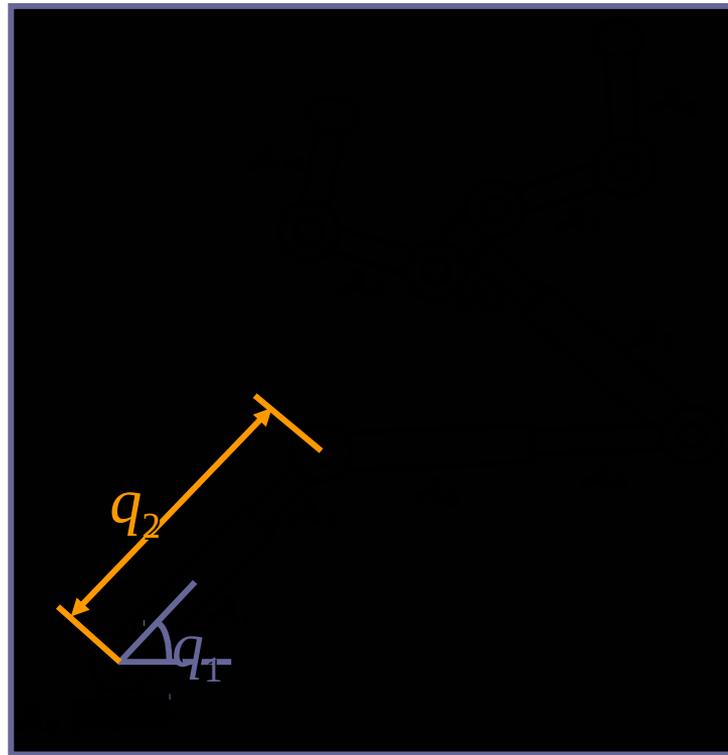


Configuration space:  $C = ?$

# Topology of configuration space

What is topology?

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Configuration space:  $C = ?$

# Paths in c-space

A path is a function from the unit interval onto configuration space:

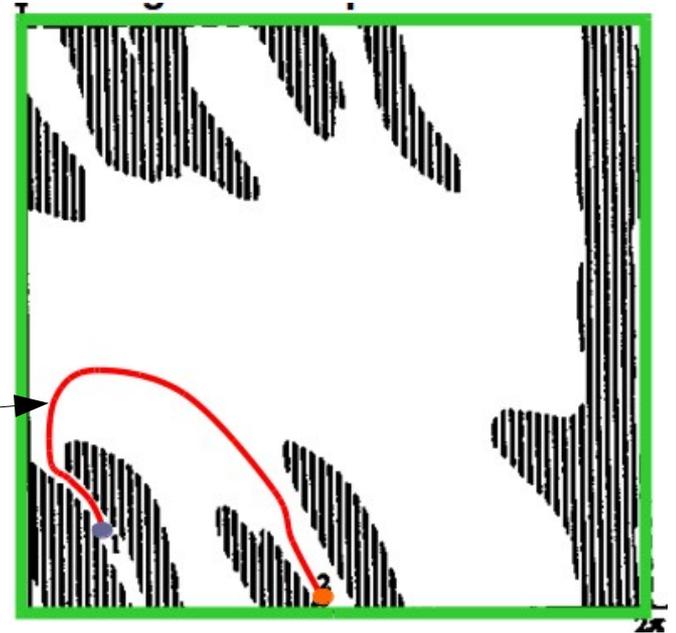
$$\tau : [0, 1] \rightarrow C$$

$\tau(0)$  = start of path

$\tau(1)$  = end of path

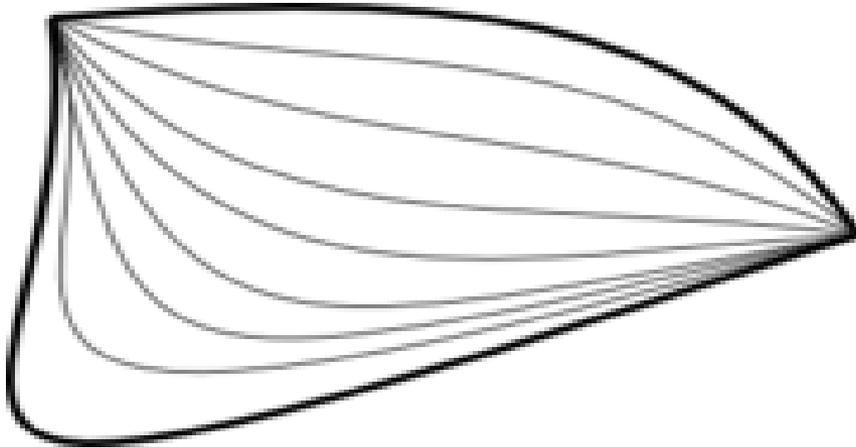
$\tau(0.5)$  = somewhere in between...

A path



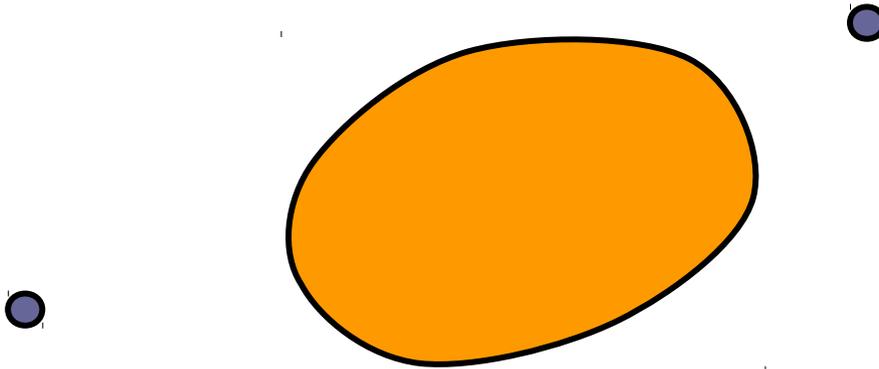
# Homotopic paths

Two paths are homotopic if it is possible to continuously deform one into the other



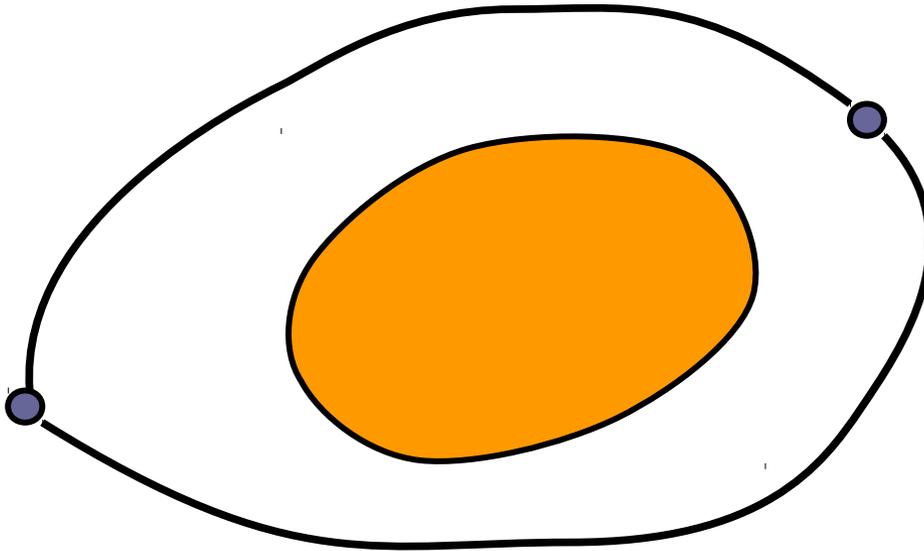
# Homotopic paths

How many homotopic paths are there between these two points?



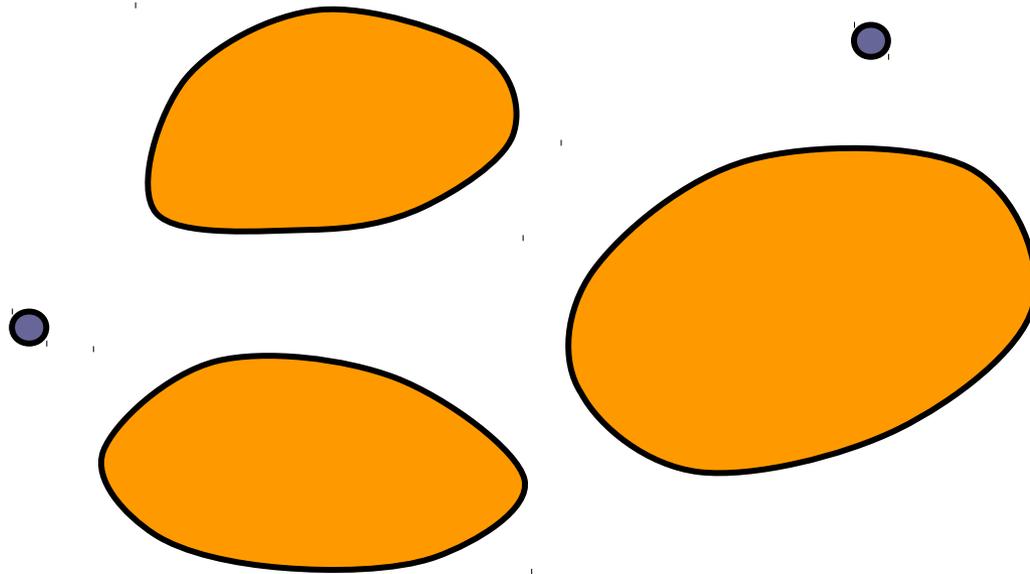
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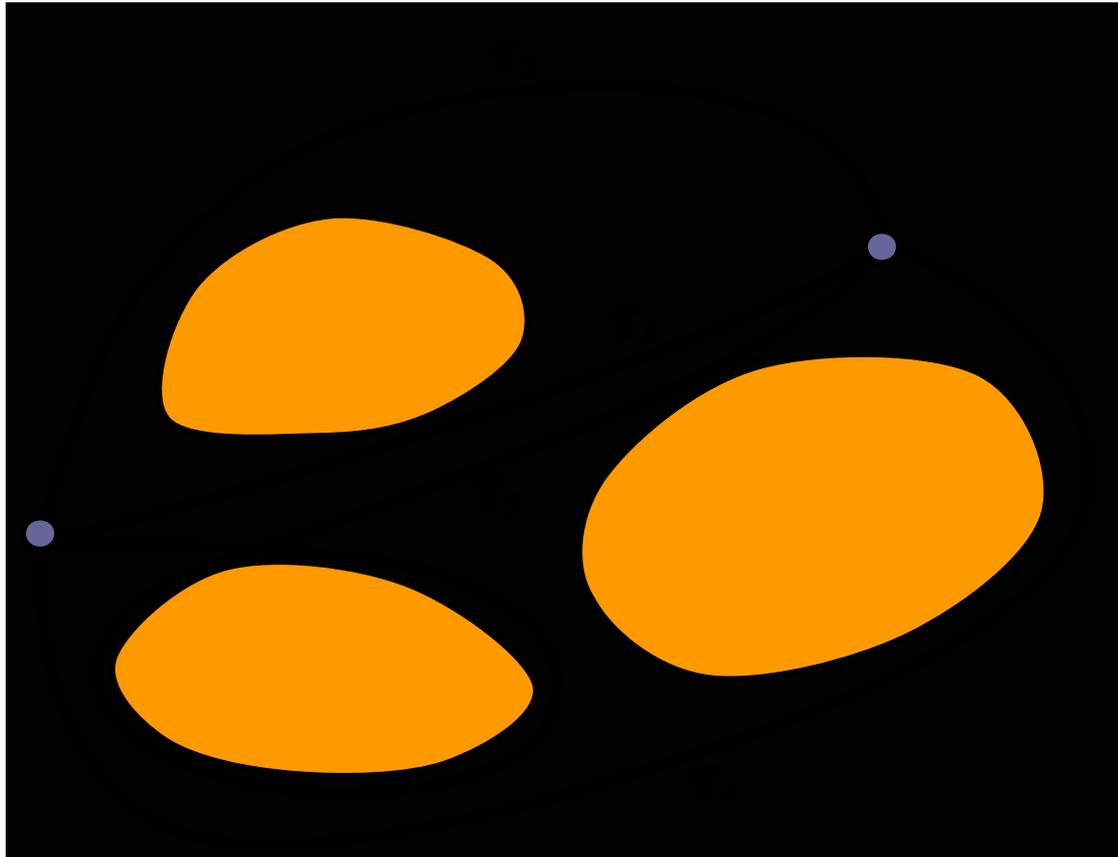
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# Homotopic paths

Two paths are homotopic if it is possible to continuously deform one into the other



# Connectedness of c-space

$C$  is connected if every two configurations can be connected by a path.

$C$  is simply-connected if any two paths connecting the same endpoints are homotopic.

Otherwise  $C$  is multiply-connected.