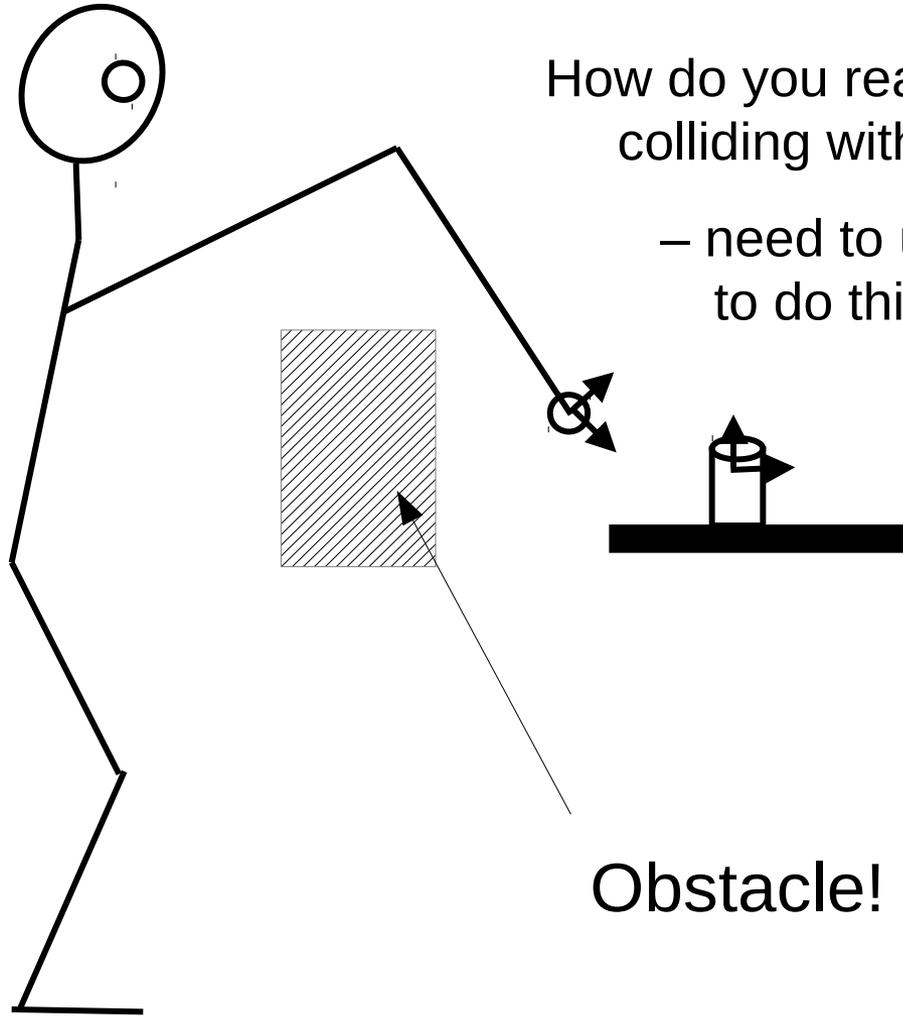


Configuration space



How do you reach toward something without colliding with obstacles in the environment?

– need to understand configuration space to do this!

Obstacle!

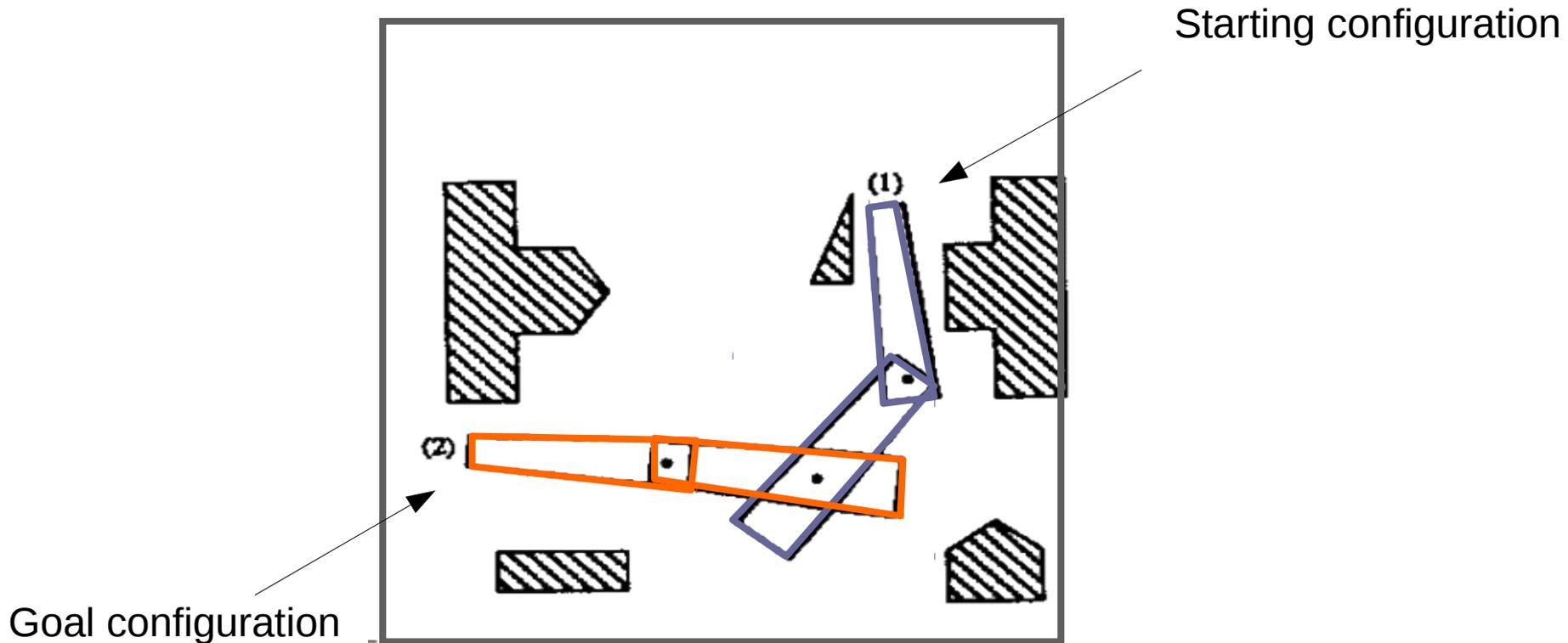
Problem we want to solve

Given:

- description of the robot arm (the manipulator)
- description of the obstacle environment

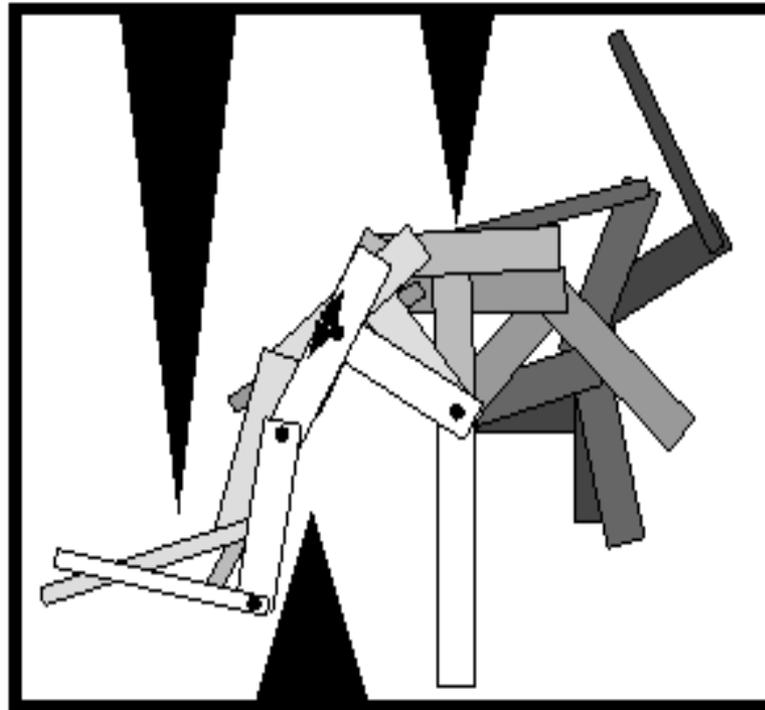
Find:

- path from start to goal that does result in a collision



Problem we want to solve

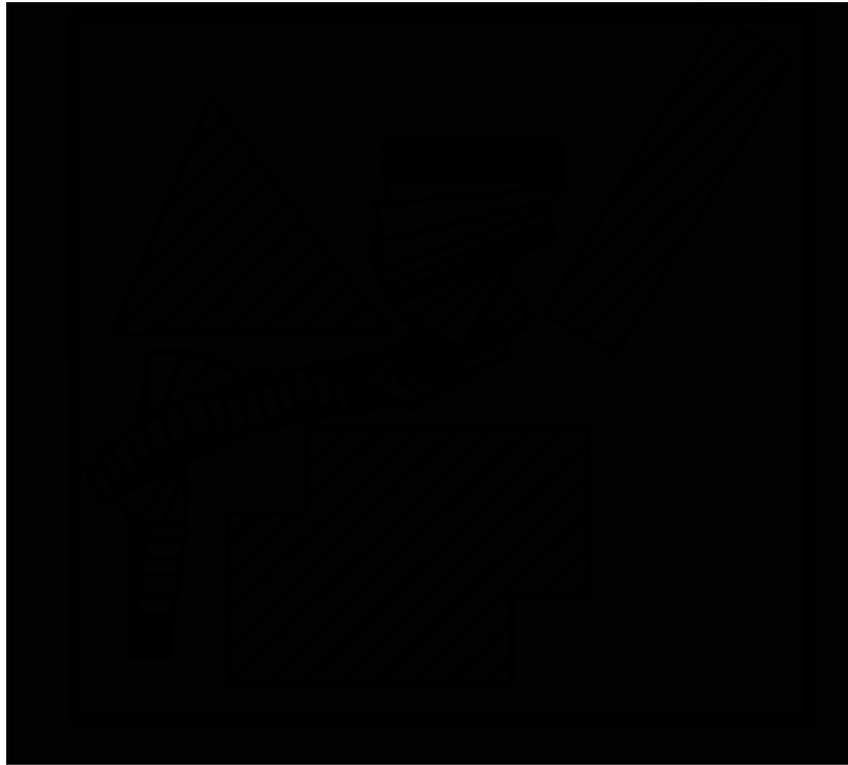
This problem statement is actually very general
– manipulators



Problem we want to solve

This problem statement is actually very general

- manipulators
- mobile robots

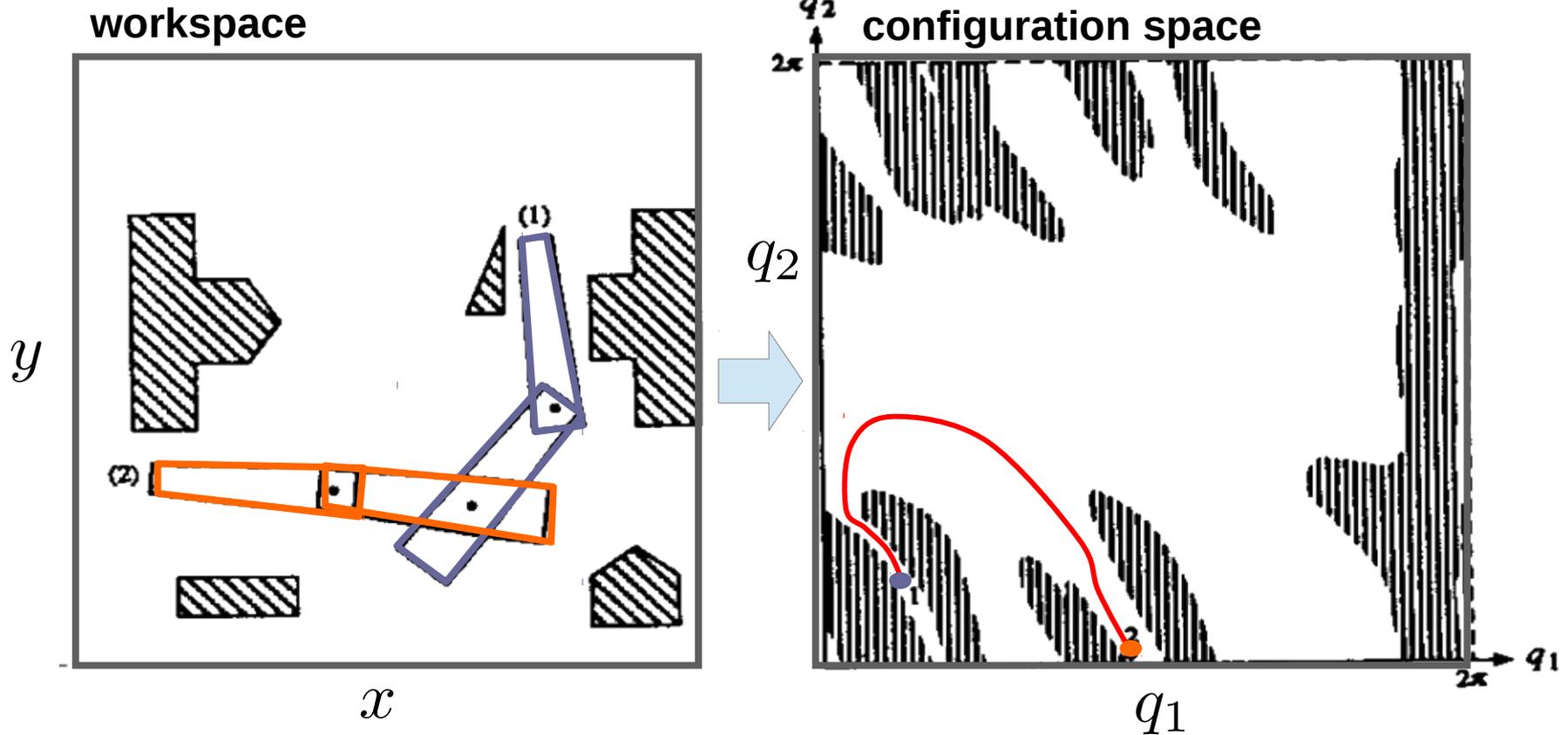


Approach: plan in “configuration space”

Convert the original planning problem into a planning problem for a single point.

Approach: plan in “configuration space”

Convert the original planning problem into a planning problem for a single point.



Original problem

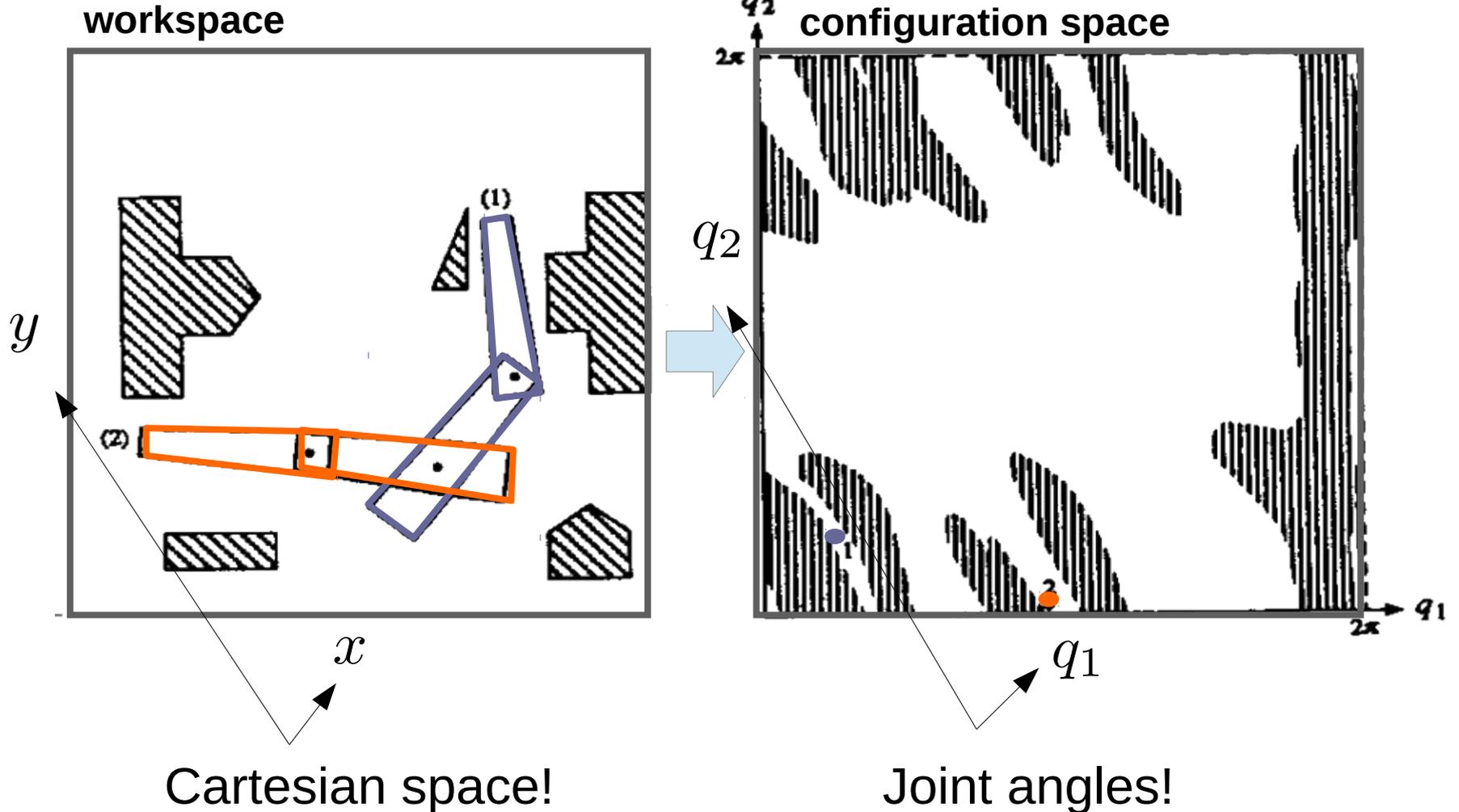
– plan path for robot arm

Equivalent problem:

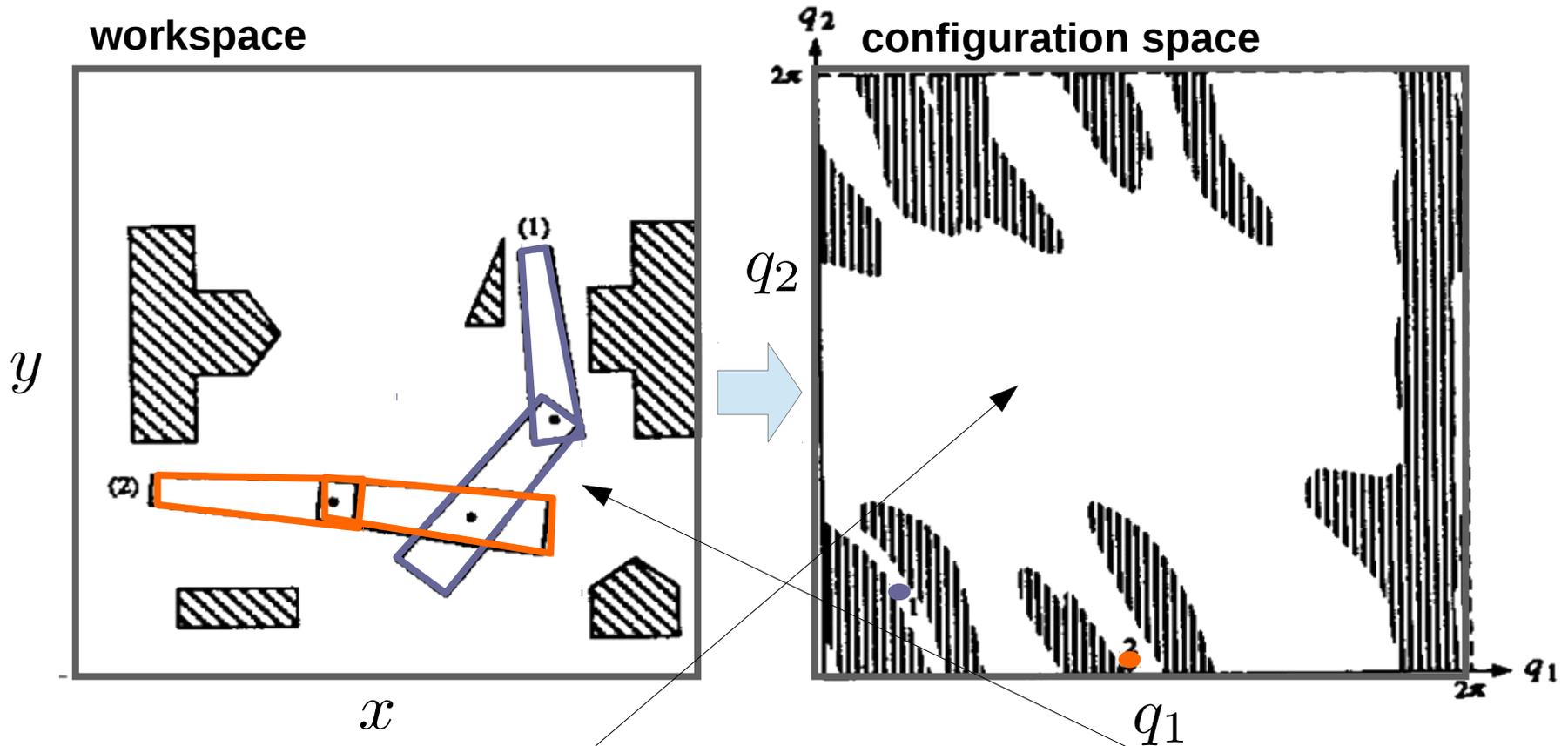
– plan path for a point

Approach: plan in “configuration space”

Notice the axes!

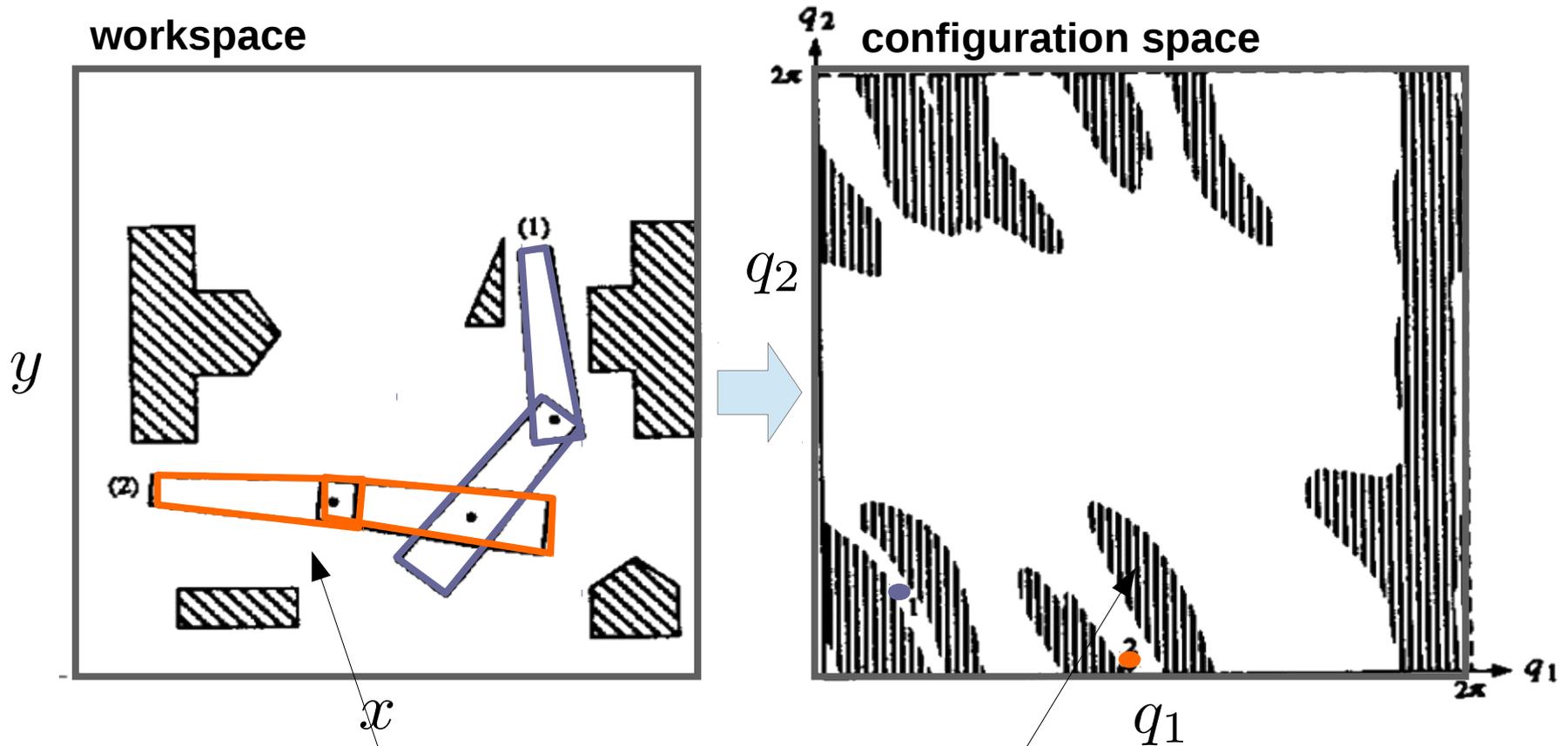


Approach: plan in “configuration space”



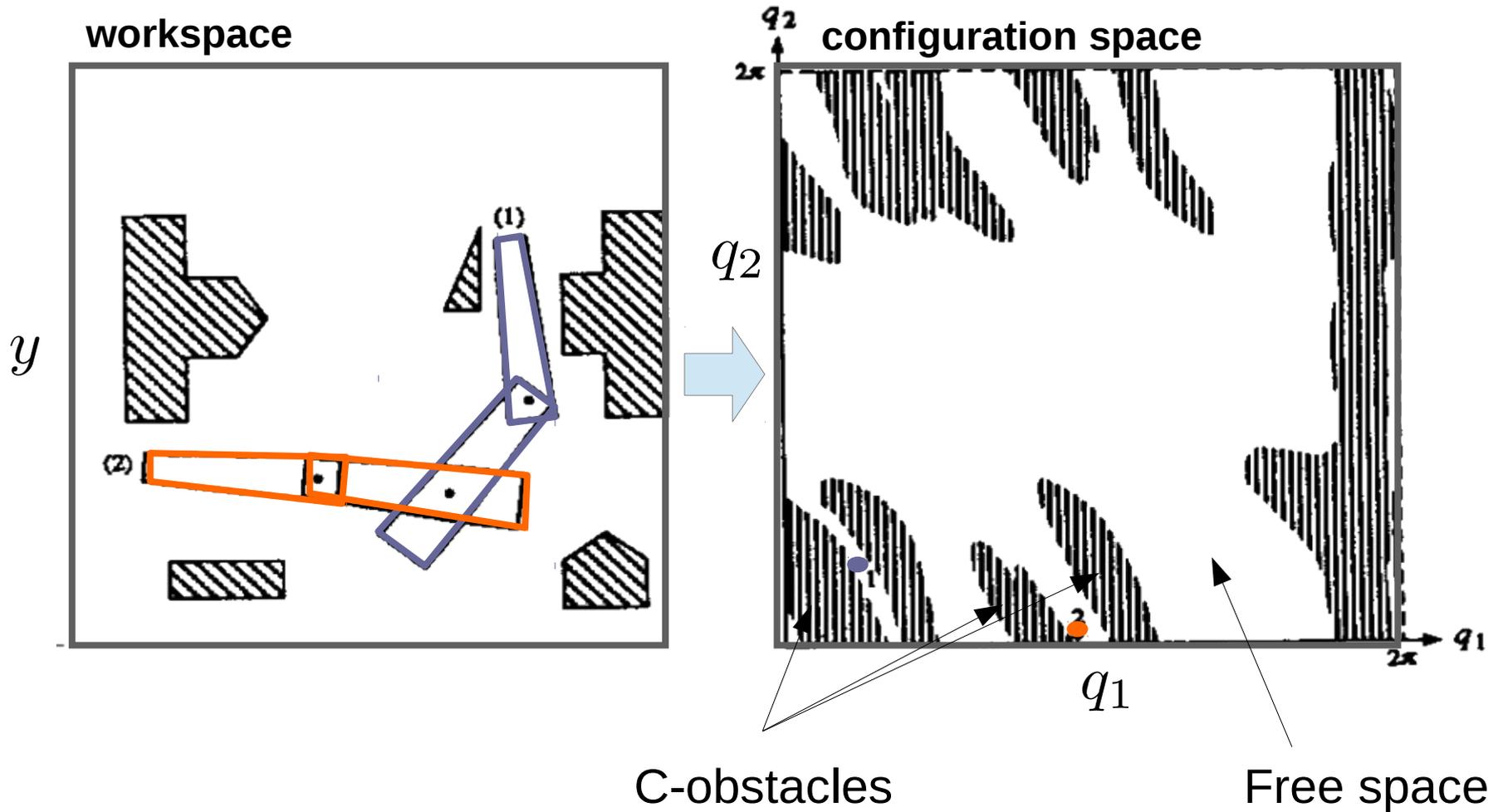
Every point here corresponds to a single robot configuration here

Approach: plan in “configuration space”



Every point that intersects an obstacle here corresponds to an arm configuration that intersects an obstacle

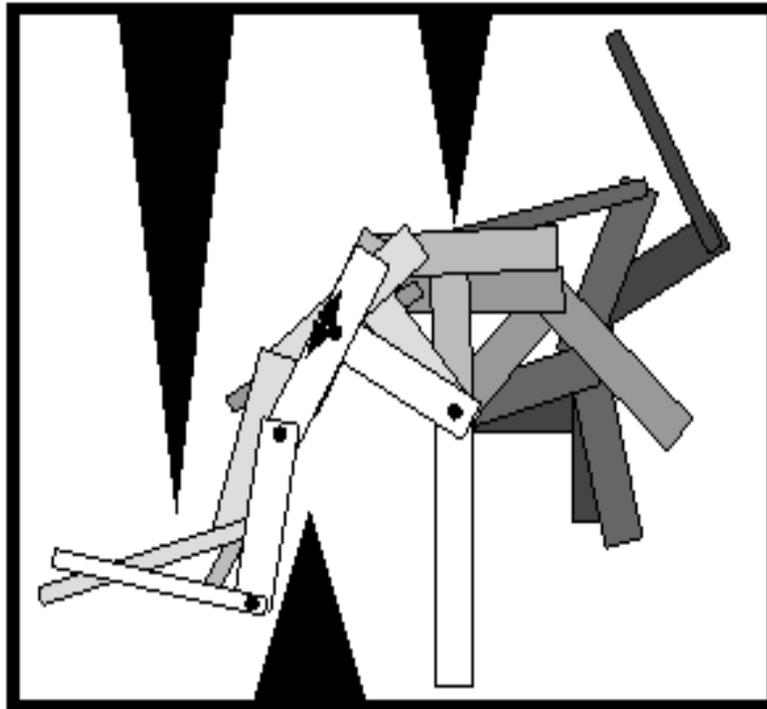
Approach: plan in “configuration space”



Configuration space

The dimension of a configuration space is the minimum number of parameters needed to specify the configuration of the robot completely.

– also called the number of “degrees of freedom” (DOFs)



Dimension = 3

Configuration space

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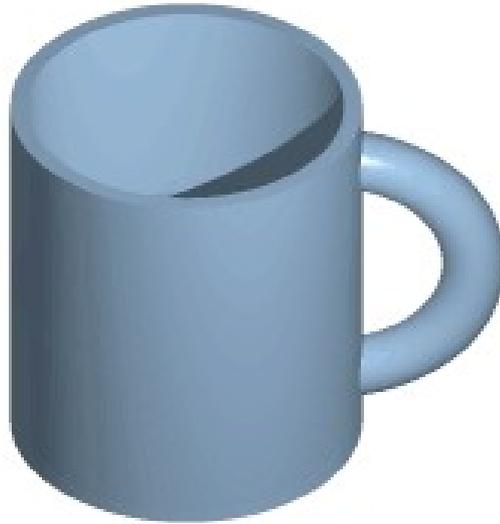


Dimension = ?

Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

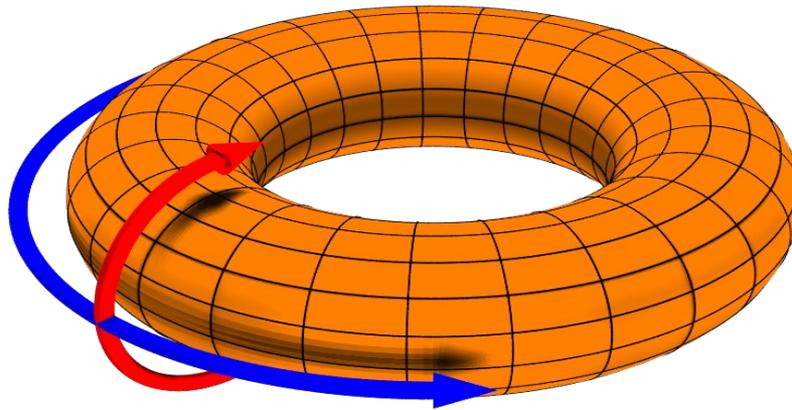


The topology of this mug is a torus

Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

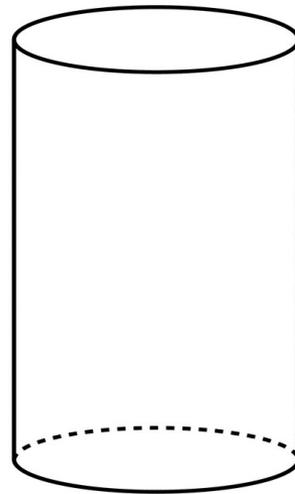


Torus: $C = S^1 \times S^1$

Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

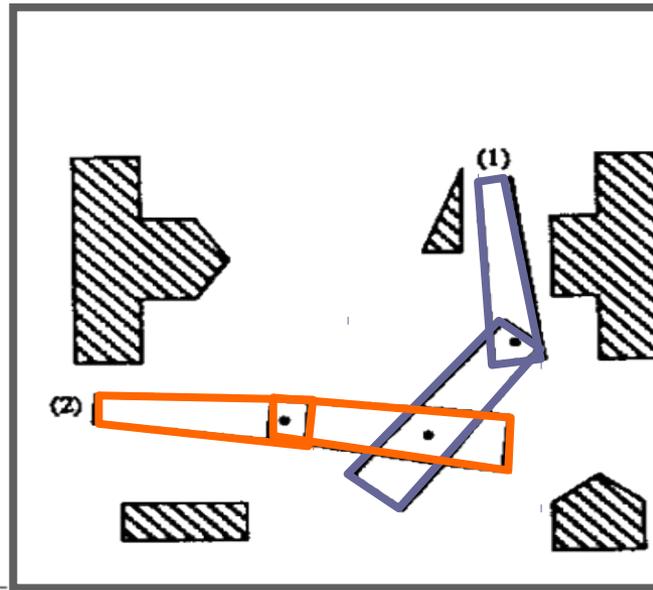


$$\text{Cylinder: } C = \mathbb{R}^2 \times S^1$$

Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing

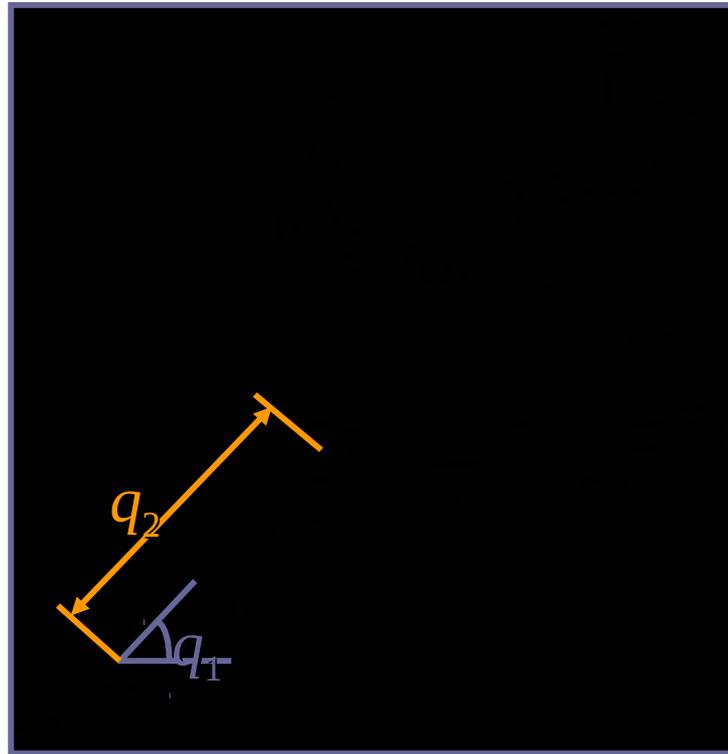


Configuration space: $C = ?$

Topology of configuration space

What is topology?

– the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing



Configuration space: $C = ?$

Paths in c-space

A path is a function from the unit interval onto configuration space:

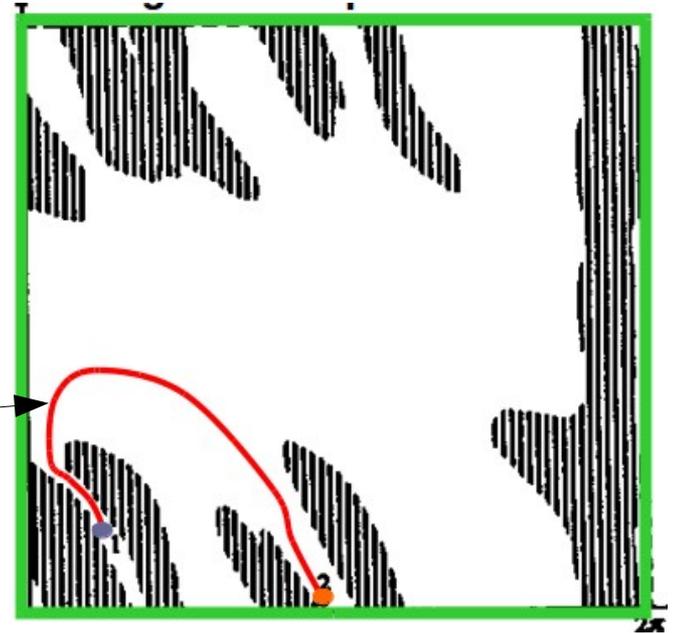
$$\tau : [0, 1] \rightarrow C$$

$\tau(0)$ = start of path

$\tau(1)$ = end of path

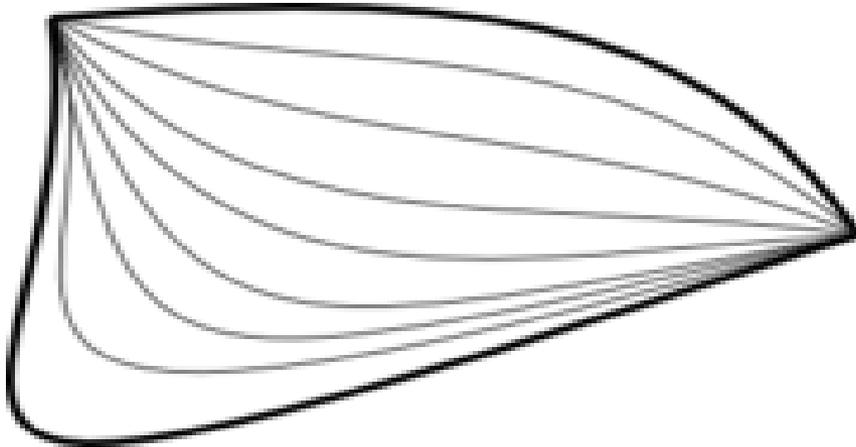
$\tau(0.5)$ = somewhere in between...

A path



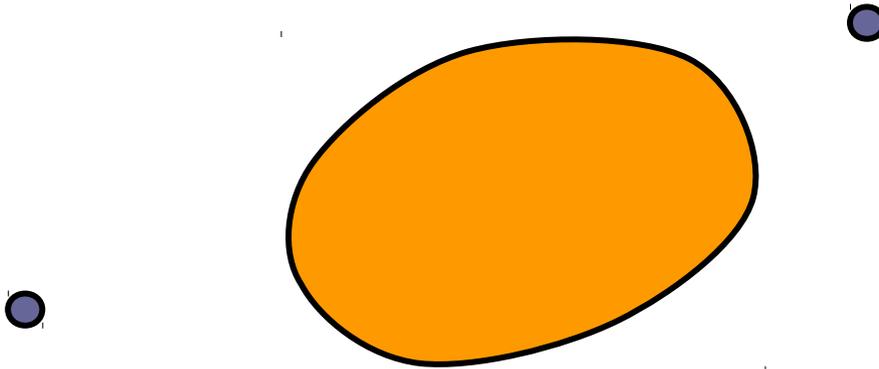
Homotopic paths

Two paths are homotopic if it is possible to continuously deform one into the other



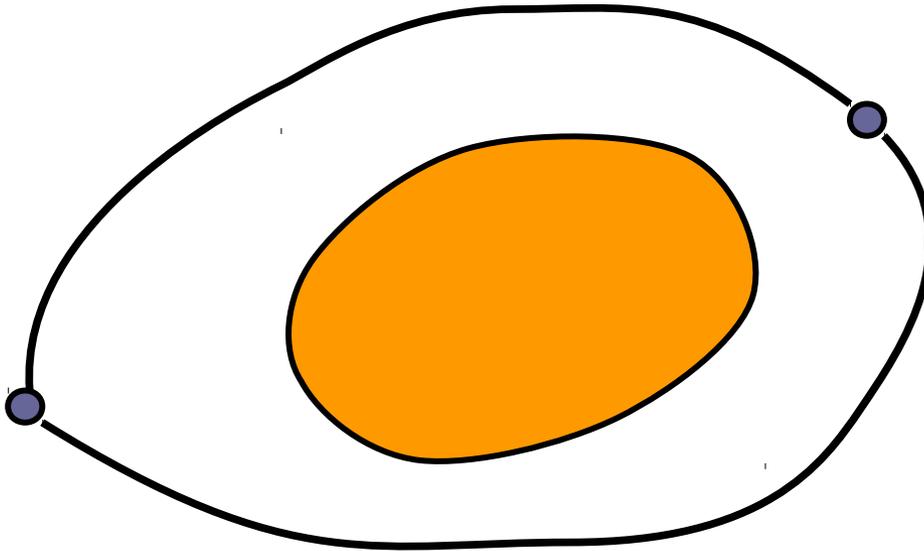
Homotopic paths

How many homotopic paths are there between these two points?



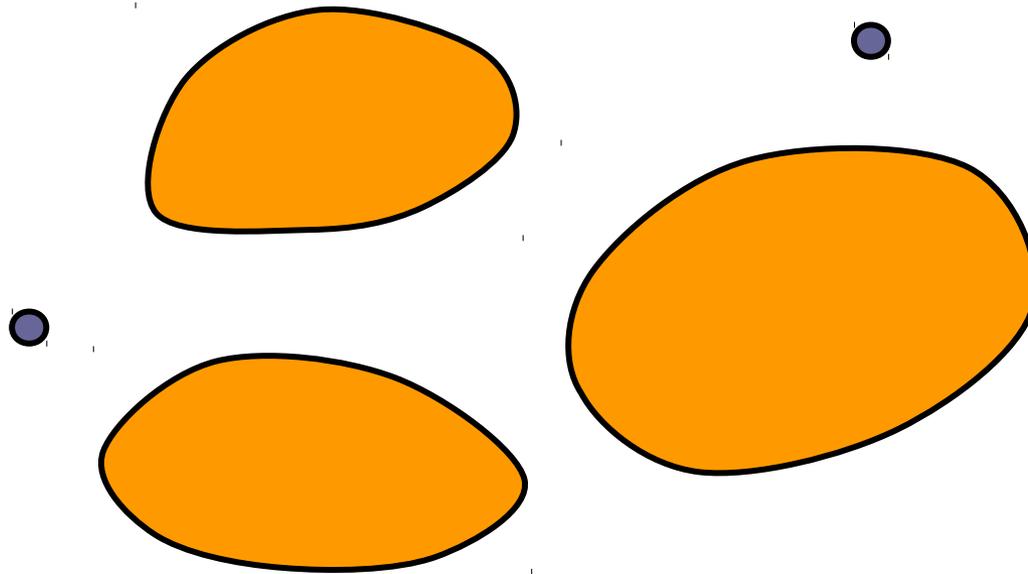
Homotopic paths

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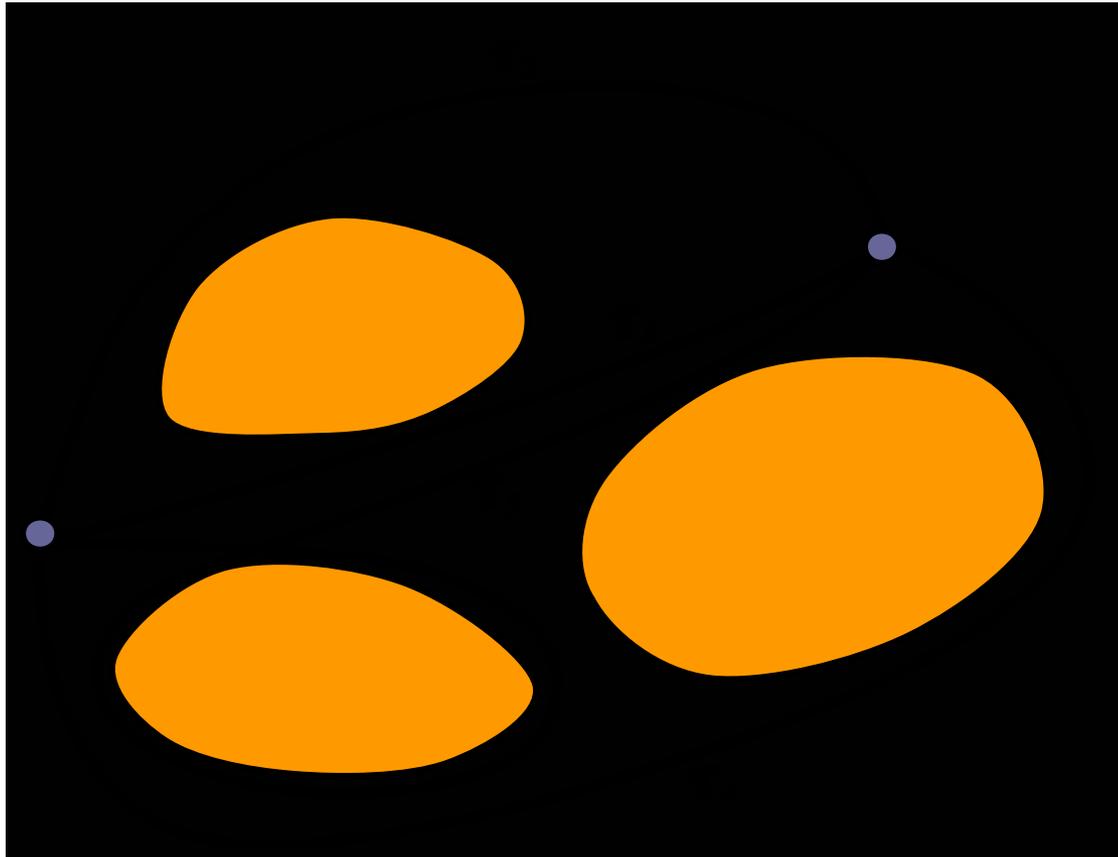
Homotopic paths

How many homotopic paths are there between these two points?



Homotopic paths

Two paths are homotopic if it is possible to continuously deform one into the other



Connectedness of c-space

C is connected if every two configurations can be connected by a path.

C is simply-connected if any two paths connecting the same endpoints are homotopic.

Otherwise C is multiply-connected.