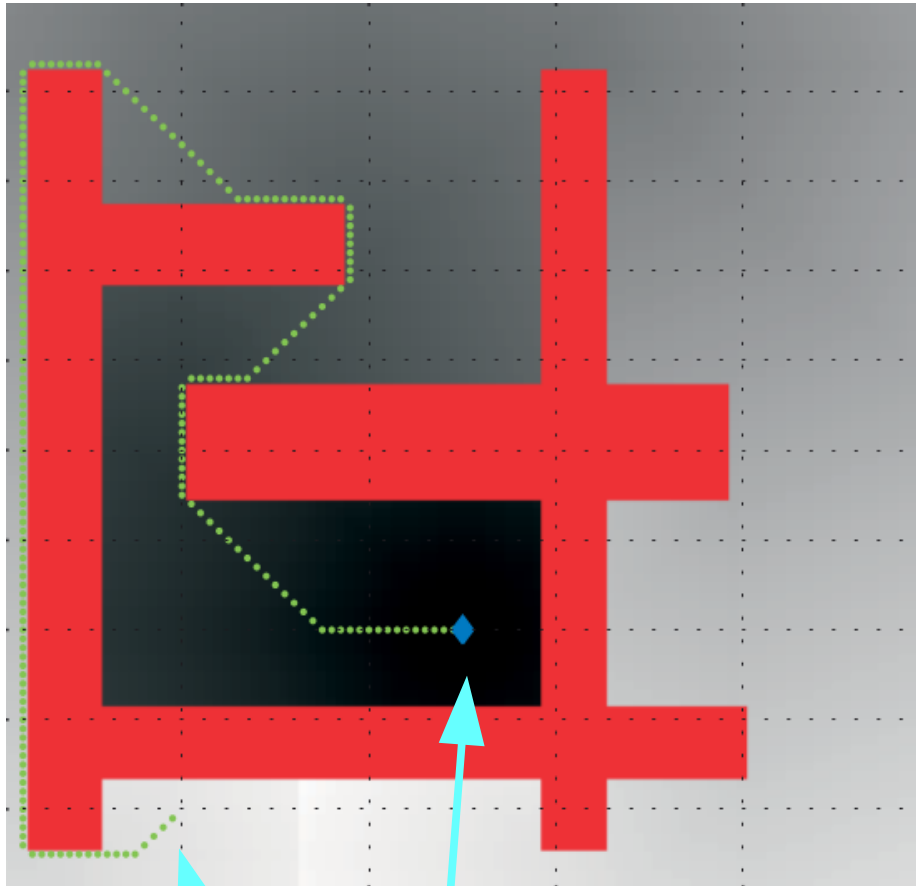


Path planning: BUGs and wavefront



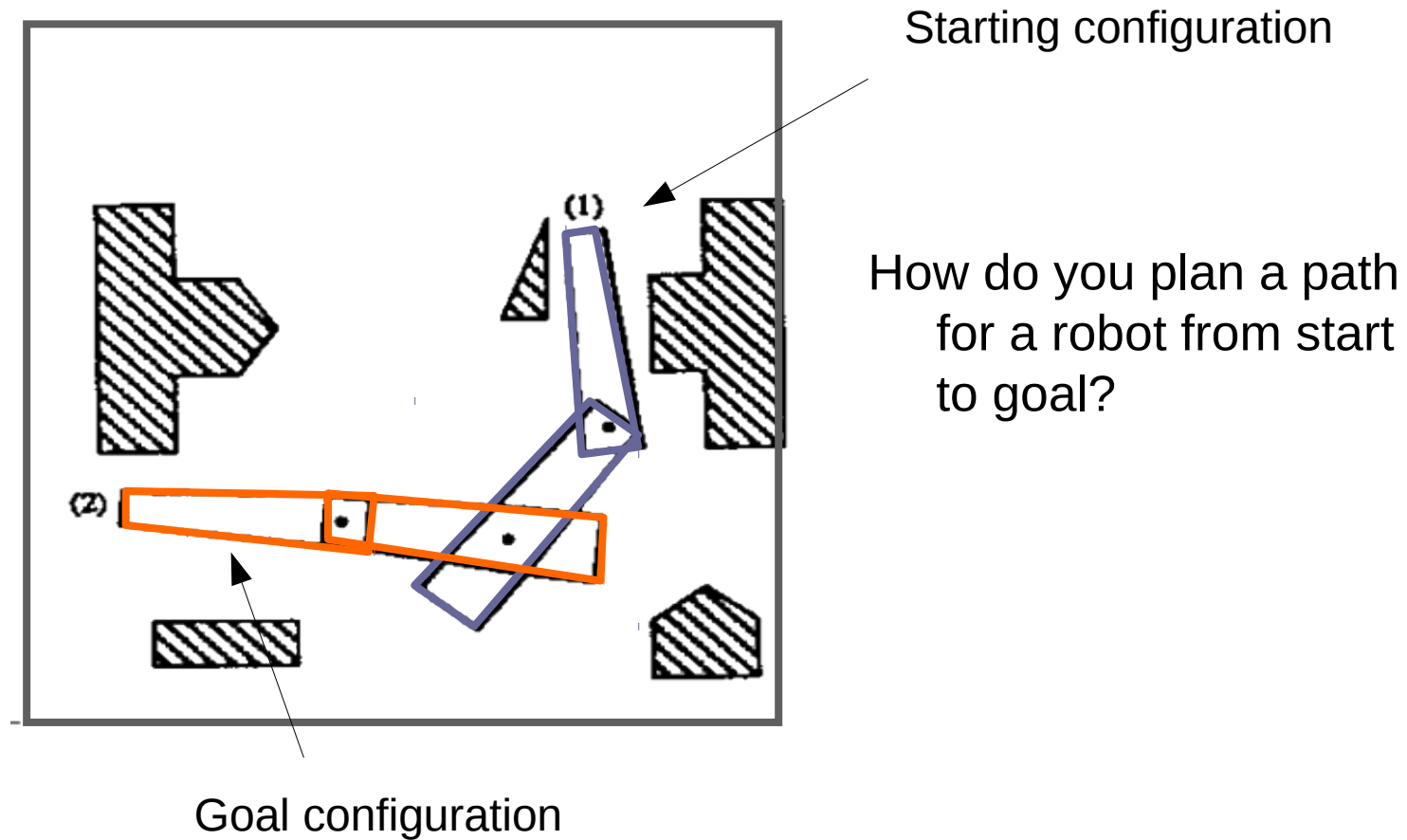
How do you plan a path
for a robot from start
to goal?

Goal

Start

These notes contain materials from Peter Corke's book and
from Howie Choset's lecture materials.

Path planning: BUGs and wavefront



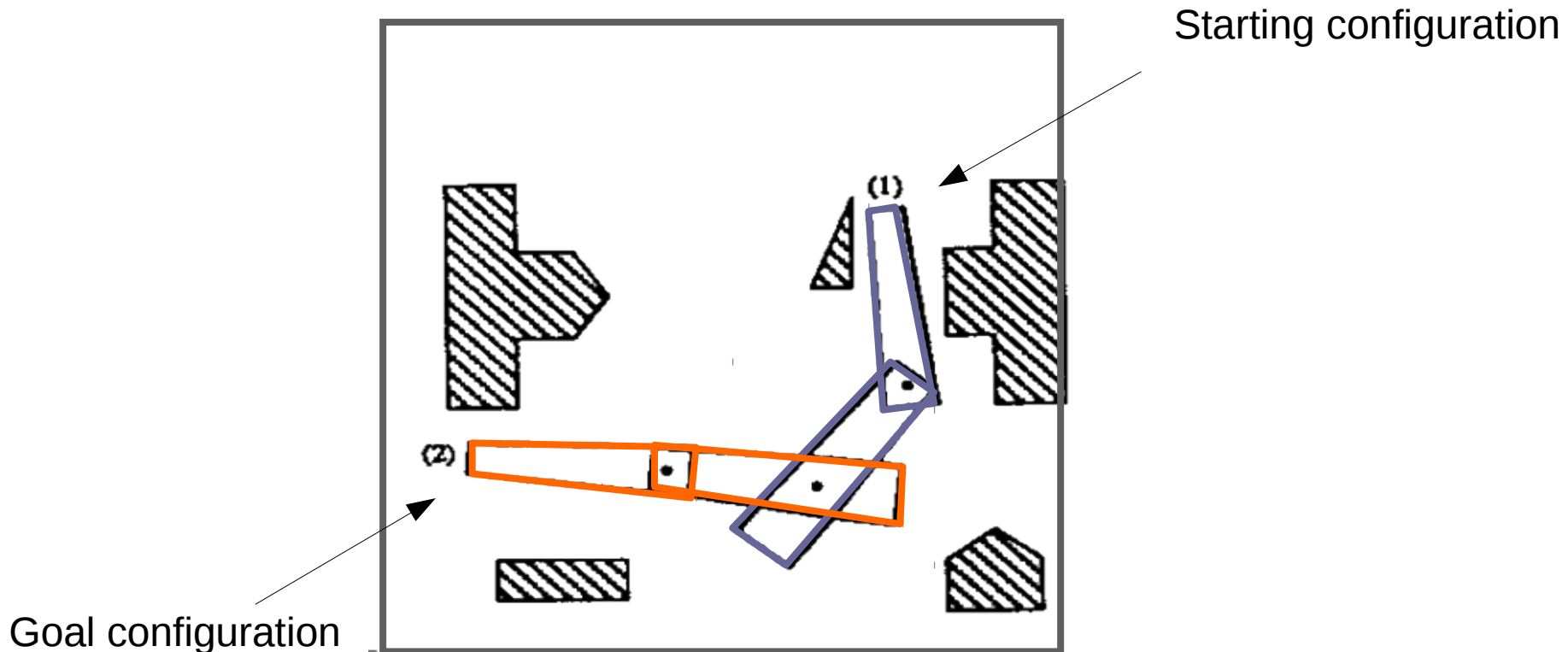
Problem we want to solve

Given:

- a point-robot (robot is a point in space)
- a start and goal configuration

Find:

- path from start to goal that does not result in a collision



Problem we want to solve

Given:

- a point-robot (robot is a point in space)
- a start and goal configuration

Find:

- path from start to goal that does not result in a collision

Assumptions:

- the position of the robot can always be measured perfectly
- the motion of the robot can always be controlled perfectly
- the robot can move in any direction instantaneously

First attempt: BUGs!

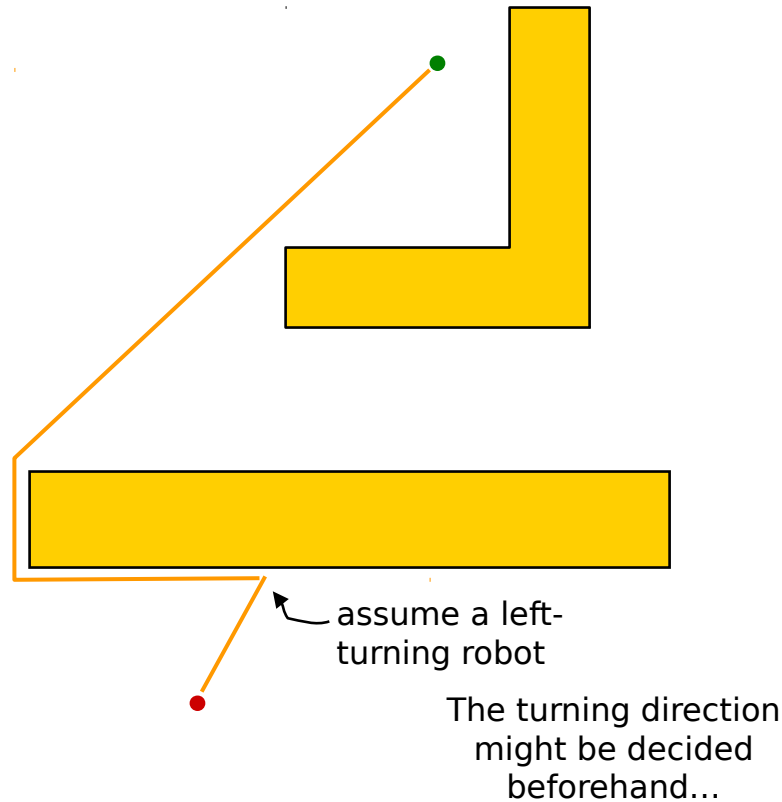


What the heck?

Bug algorithms:

- assume only local knowledge of the environment is available
- simple behaviors: follow a wall; follow straight line toward goal

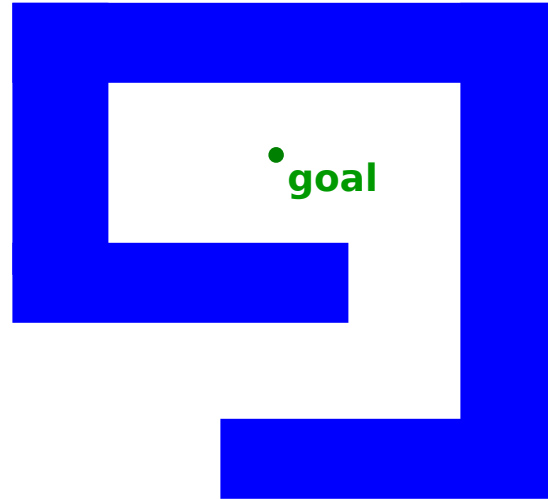
First attempt: BUG 0



BUG 0:

1. head toward goal
2. if hit a wall, turn left
3. follow wall until a line toward goal will move you away from wall.
(assume we only have local sensing – we cannot sense position of walls we are not touching)

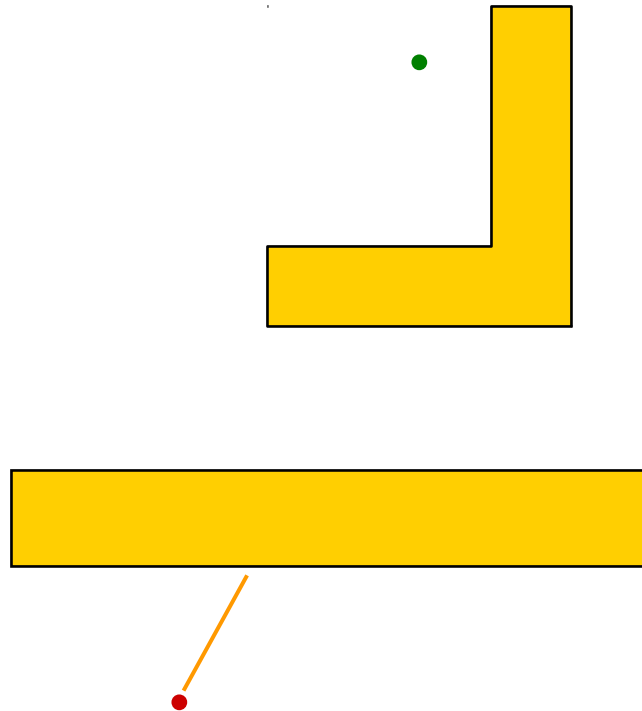
First attempt: BUG 0



• **start**

What happens here?

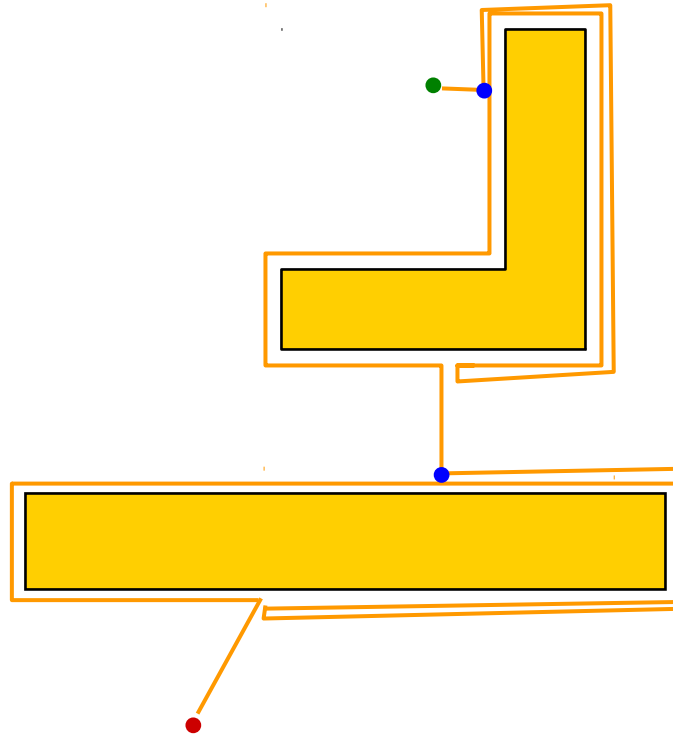
Second attempt: BUG 1



BUG 1:

1. move on straight line toward goal
2. if obstacle encountered, circumnavigate entire obstacle and remember how close bug got to goal
3. return to closest point and continue on a straight line toward goal

Second attempt: BUG 1



BUG 1:

1. move on straight line toward goal
2. if obstacle encountered, circumnavigate entire obstacle and remember how close bug got to goal
3. return to closest point and continue on a straight line toward goal

BUG 1 Performance Analysis

How far does BUG 1 travel before reaching goal?

Best case scenario (lower bound): D

Worst case scenario (upper bound): $D + 1.5 \sum_i P_i$

Where

- D denotes distance from start to goal and
- P_i denotes perimeter of i th obstacle

BUG 1 completeness?

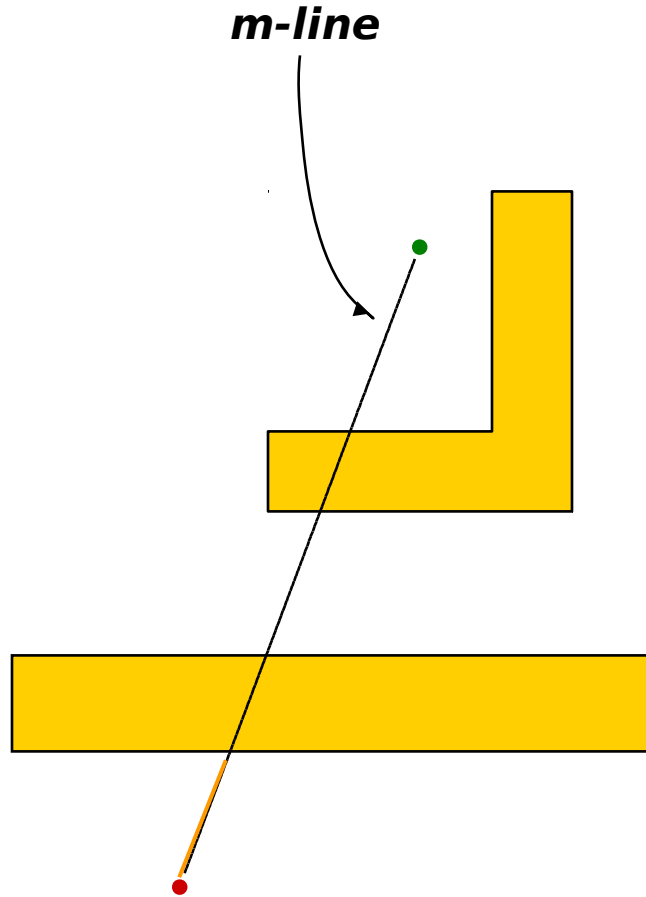
Is BUG 1 *complete*?

- is it guaranteed to find a path if one exists?

Yes? No?

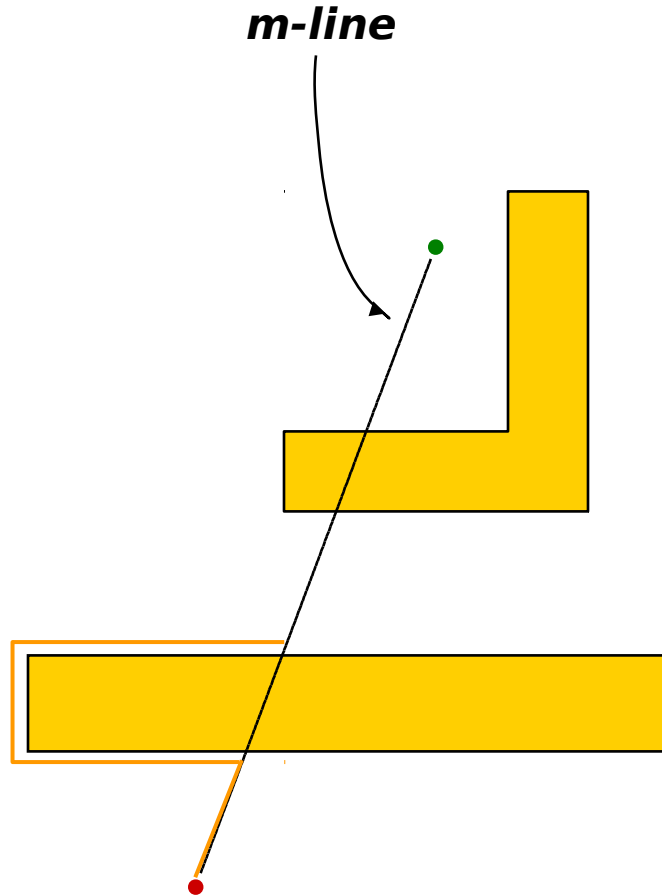
- how would you prove completeness (exercise for class)?

Another bug: BUG 2



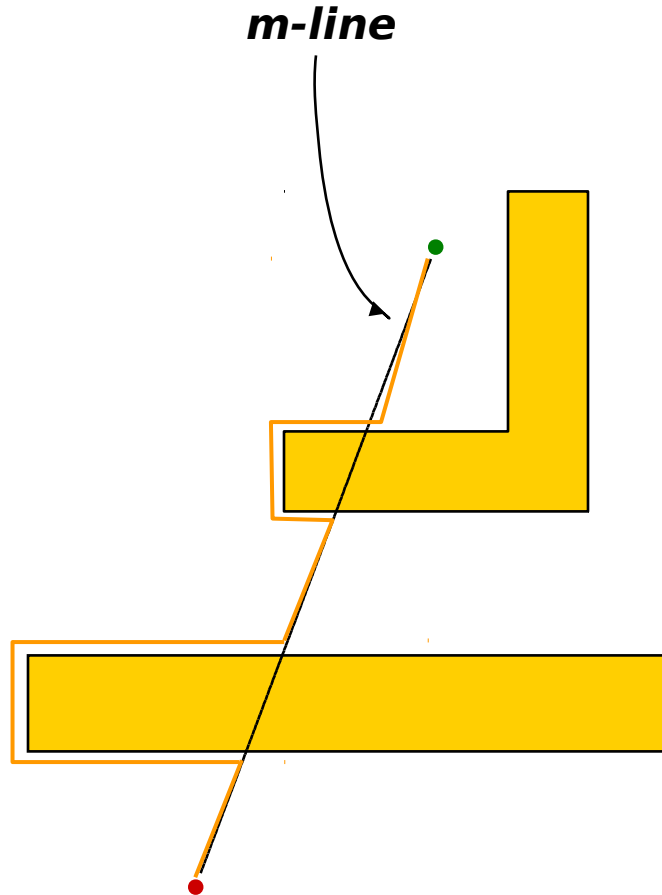
1. head toward goal on m-line

Another bug: BUG 2



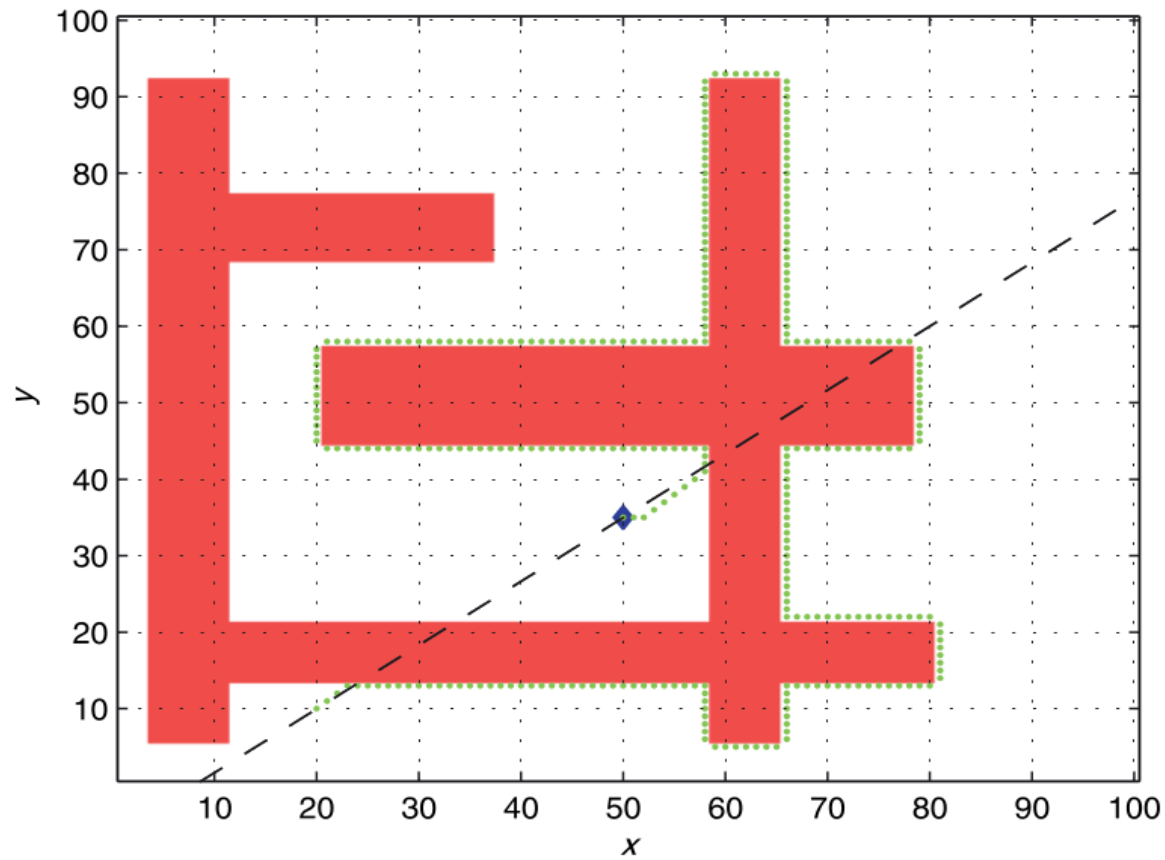
1. head toward goal on m-line
2. if you encounter obstacle, follow it until you encounter m-line again at a point closer to goal

Another bug: BUG 2



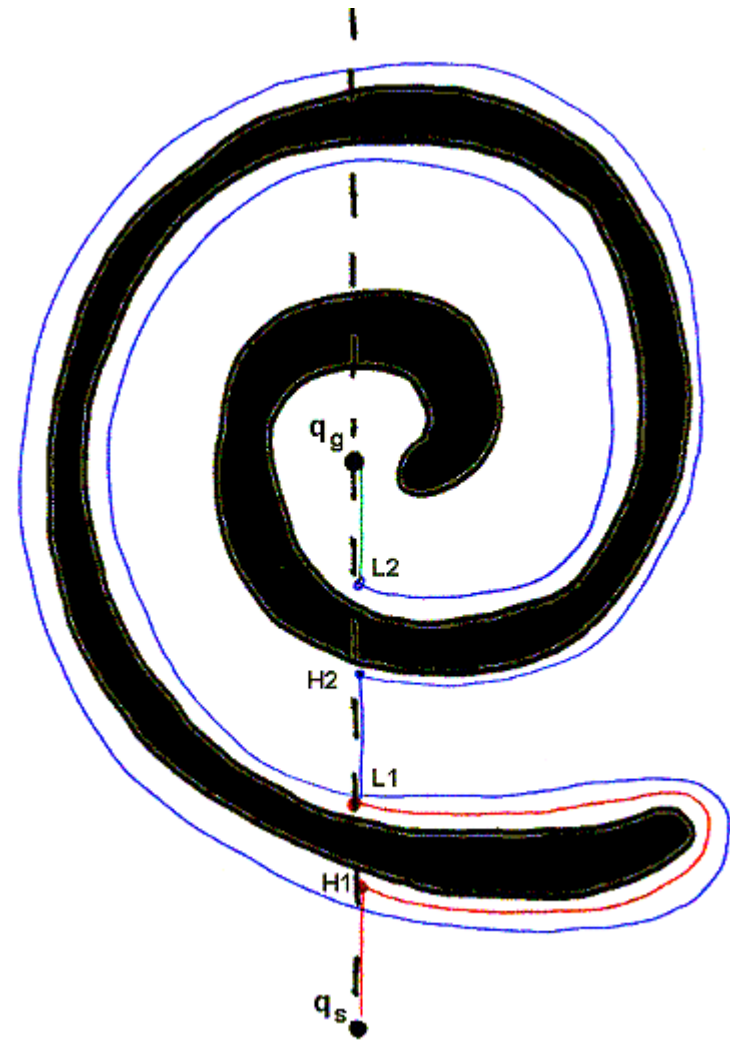
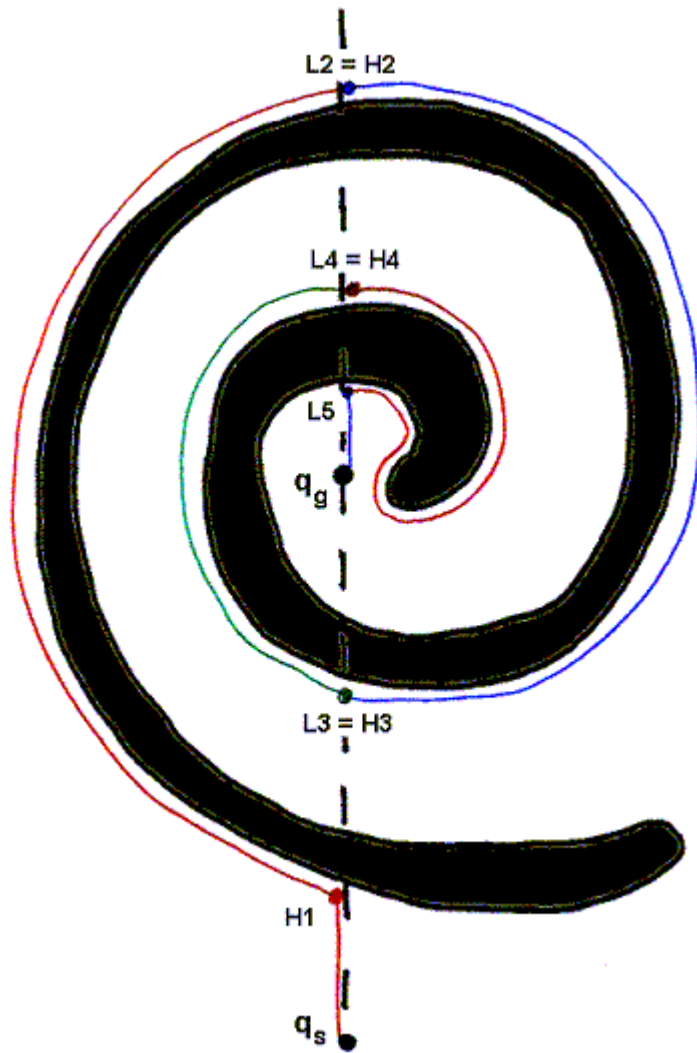
1. head toward goal on m-line
2. if you encounter obstacle, follow it until you encounter m-line again at a point closer to goal
3. leave line and head toward goal again

Another bug: BUG 2

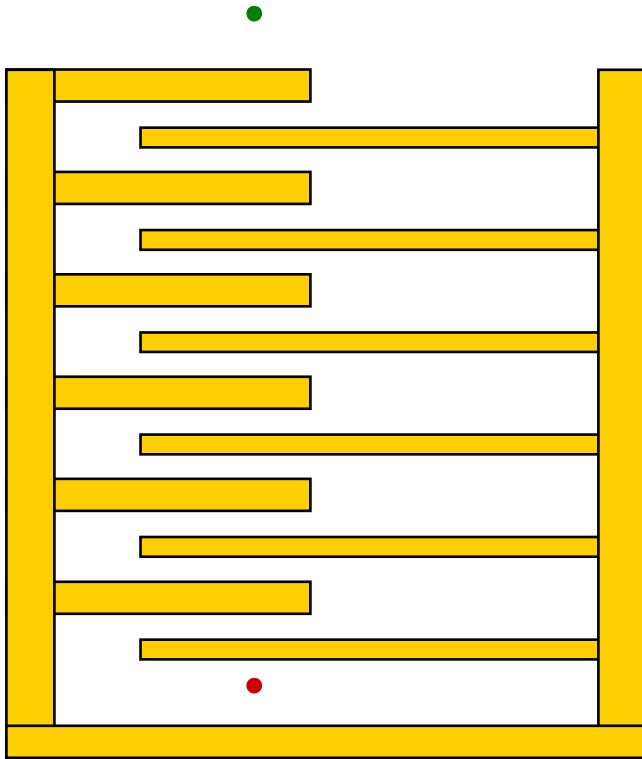


Is BUG 2 complete?
– Why? Why not?

Another bug: BUG 2



Another bug: BUG 2



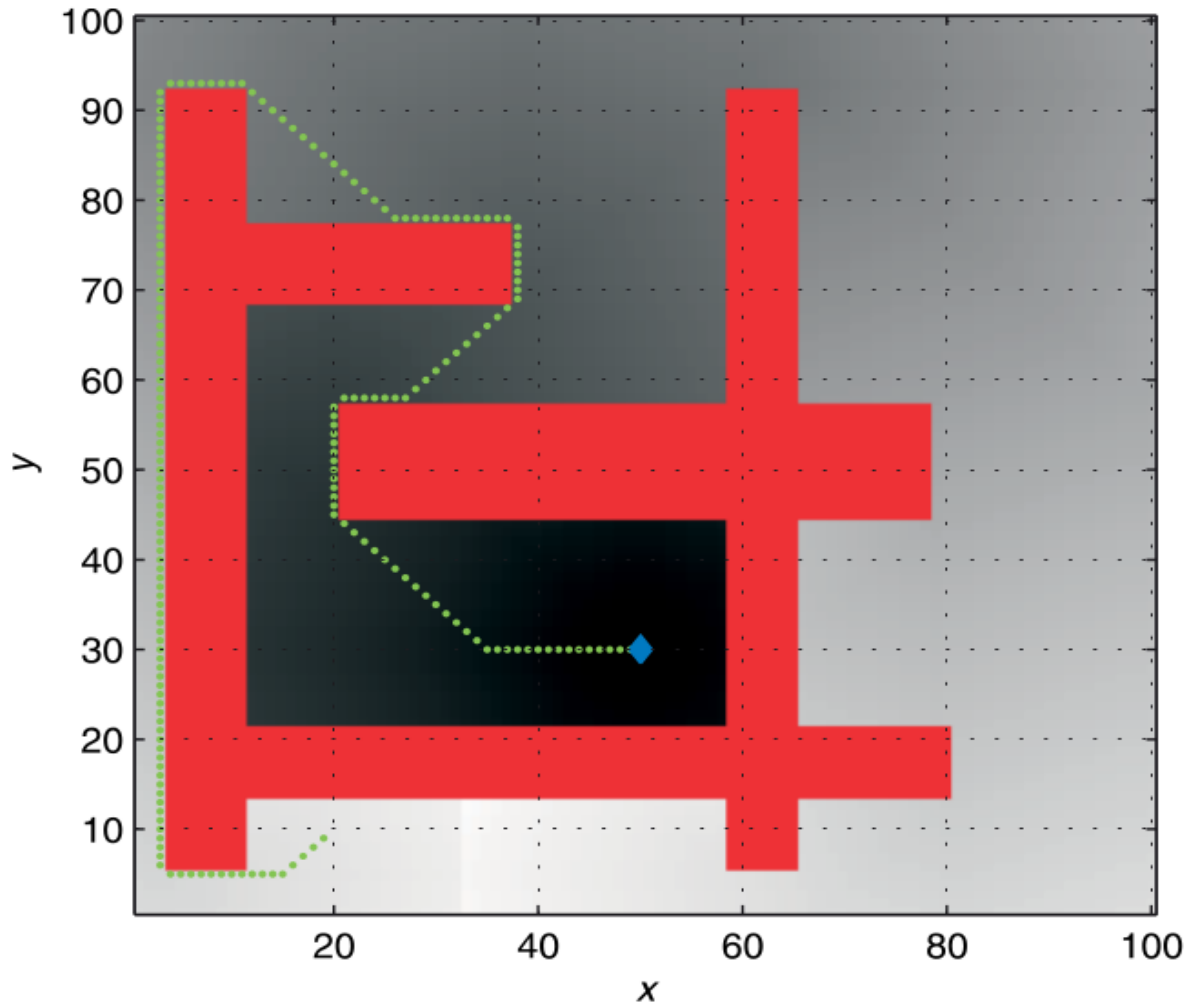
How bad can it get?

Lower bound: D

Upper bound: $D + \sum_i \frac{n_i}{2} P_i$

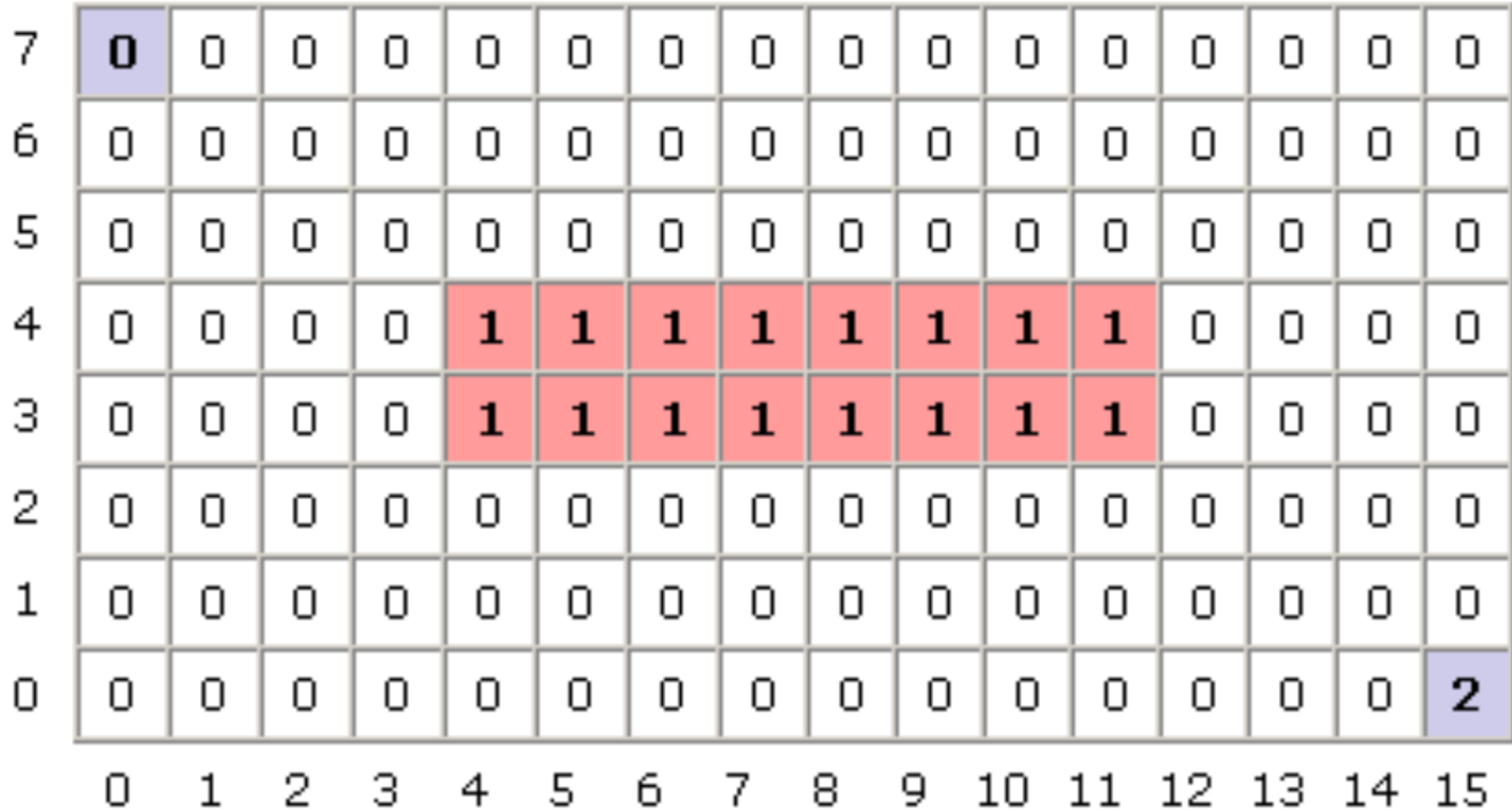
where n_i is the number of s-line intersections
In the i th obstacle.

Wavefront planner (distance transform)



– intensity of a point denotes its (obstacle-respecting) distance from the goal

Wavefront planner (distance transform)



Idea:

- discretize the workspace into cells
- label each cell with distance from goal by expanding a “wavefront”

Wavefront planner (distance transform)

7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4
1	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	2
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5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1	1	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	0	5	4	4
1	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3
0	0	0	0	0	0	0	0	0	0	0	0	0	5	4	2
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6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	1	1	1	1	1	6	6	6
3	0	0	0	0	1	1	1	1	1	1	1	1	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	6	5	4	4
1	0	0	0	0	0	0	0	0	0	0	0	6	5	4	3
0	0	0	0	0	0	0	0	0	0	0	0	6	5	4	2
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	7	7	7	7
4	0	0	0	0	1	1	1	1	1	1	1	1	6	6	6
3	0	0	0	0	1	1	1	1	1	1	1	1	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	7	6	5	4
1	0	0	0	0	0	0	0	0	0	0	0	7	6	5	4
0	0	0	0	0	0	0	0	0	0	0	0	7	6	5	4
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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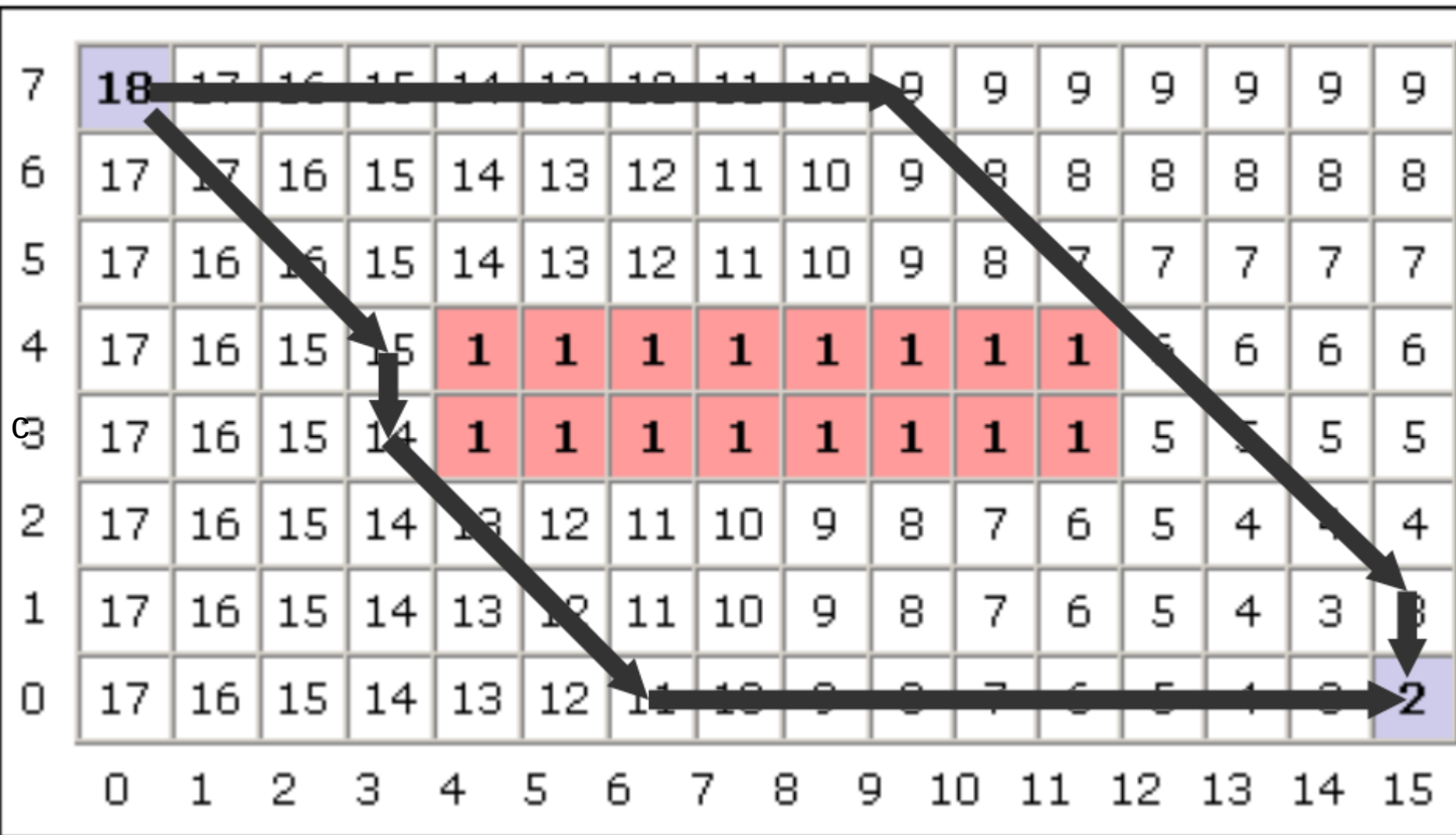
Wavefront planner (distance transform)

7	18	17	16	15	14	13	12	11	10	9	9	9	9	9	9
6	17	17	16	15	14	13	12	11	10	9	8	8	8	8	8
5	17	16	16	15	14	13	12	11	10	9	8	7	7	7	7
4	17	16	15	15	1	1	1	1	1	1	1	1	6	6	6
3	17	16	15	14	1	1	1	1	1	1	1	1	5	5	5
2	17	16	15	14	13	12	11	10	9	8	7	6	5	4	4
1	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3
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Wavefront planner (distance transform)



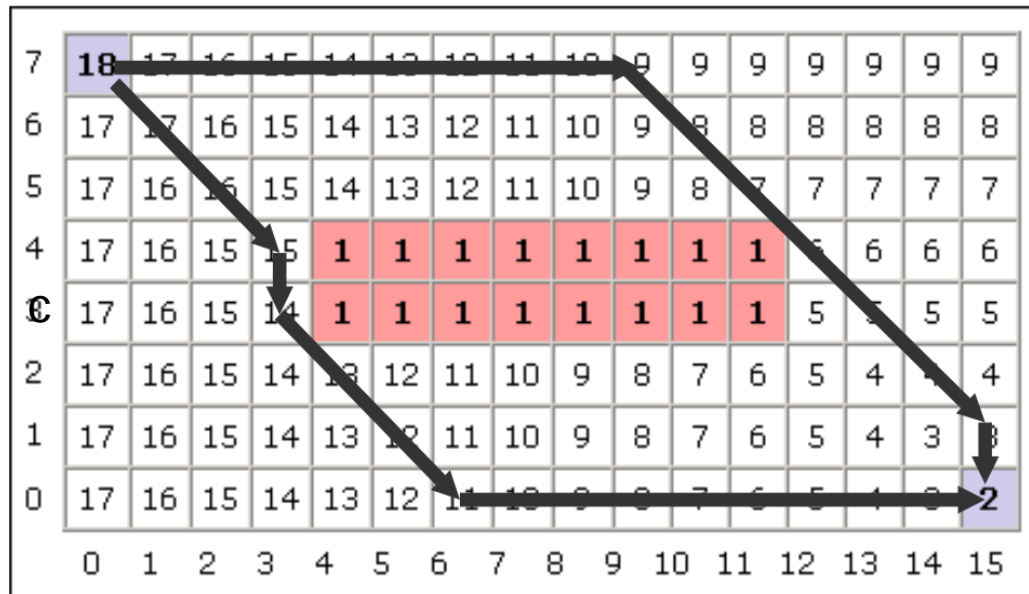
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Wavefront planner (distance transform)

Algorithm:

1. $L = \{\text{goal state}\}$, $d(\text{goal state}) = 2$, $d(\text{obstacle states}) = 1$, $d(\text{rest of states}) = 0$
2. while $L \neq \text{null}$
3. pop item i from L
4. for all neighbors j of i such that $d(j) = 0$
5. $d(j) = d(i) + 1$
6. push j onto L



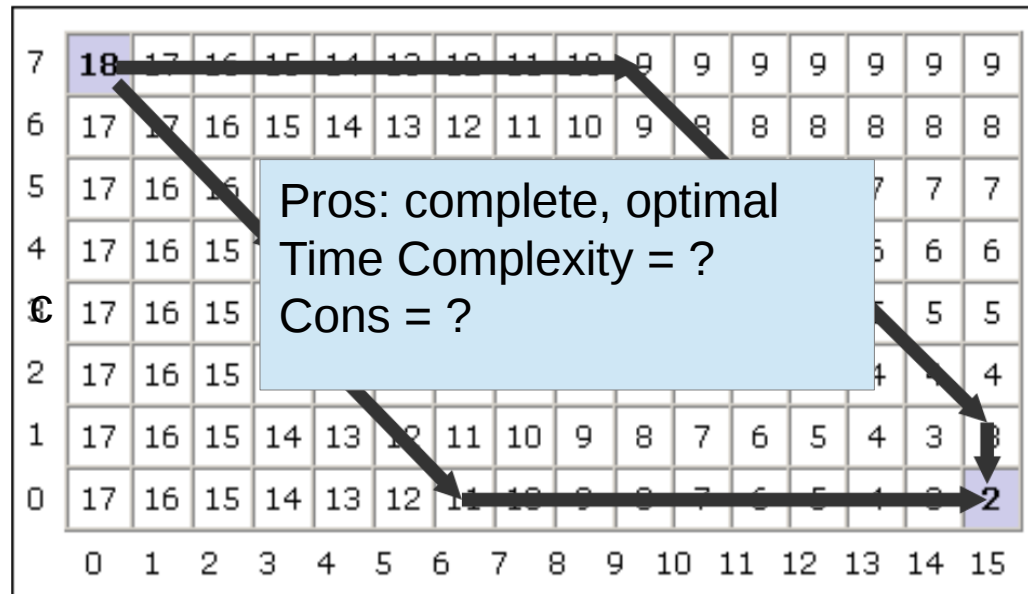
L : list of nodes in wave front; initially just the goal state

d : distance function over nodes; initially zero everywhere except goal state

Wavefront planner (distance transform)

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