Neural Networks

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Some images and slides are used from:
1. CS188 UC Berkeley

Problem we want to solve

- The essence of machine learning:
  - A pattern exists
  - We cannot pin it down mathematically
  - We have data on it
- A pattern exists. We don’t know it. We have data to learn it.
- Learning from data to get an information that can make prediction

Problem we want to solve

- Applicant information:

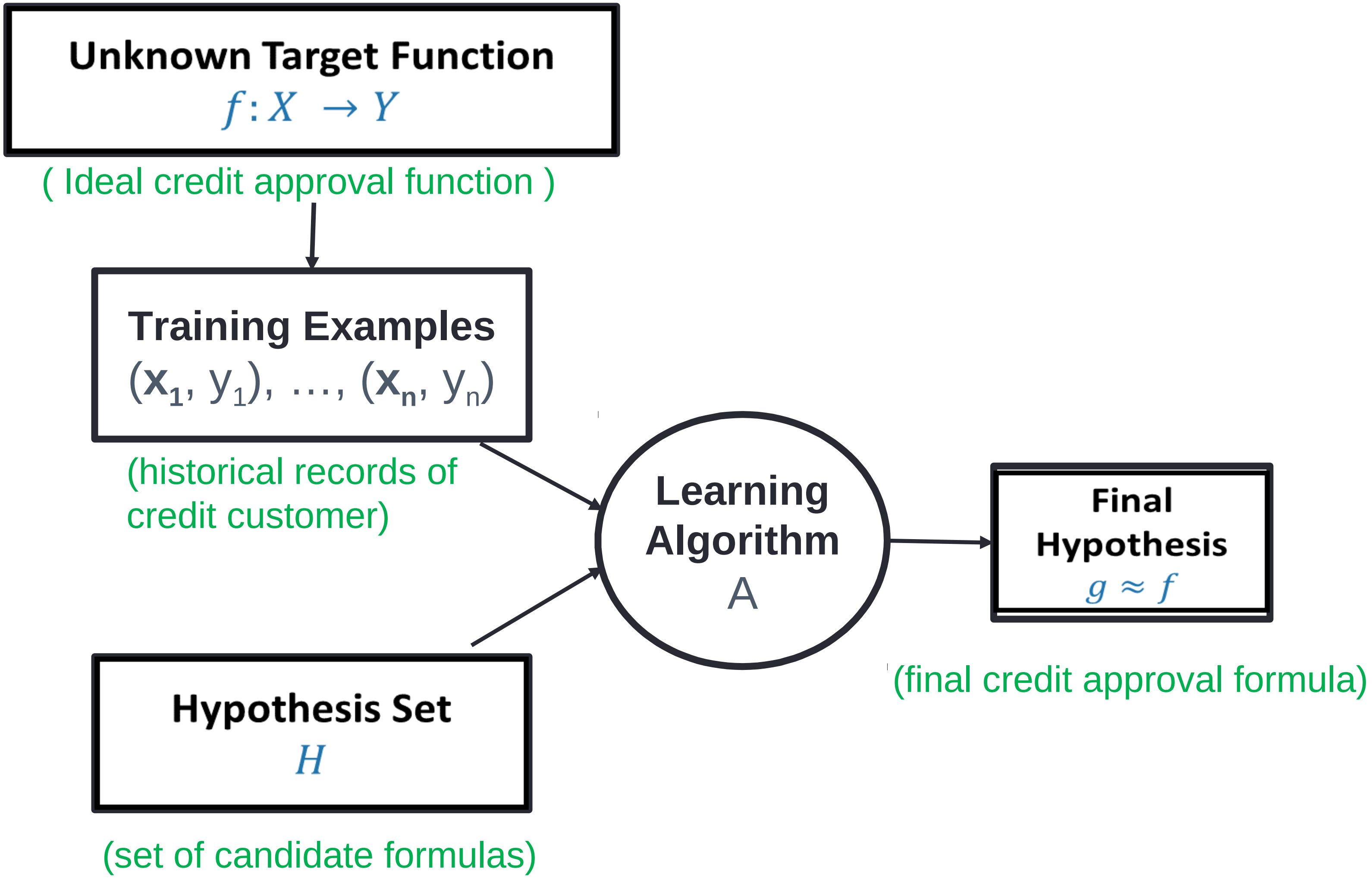
Age	23 years
Gender	male
Annual salary	$30,000
Years in residence	1 year
Years in job	1 year
Current debt	$15,000
…	…

- Approve credit?

Problem we want to solve

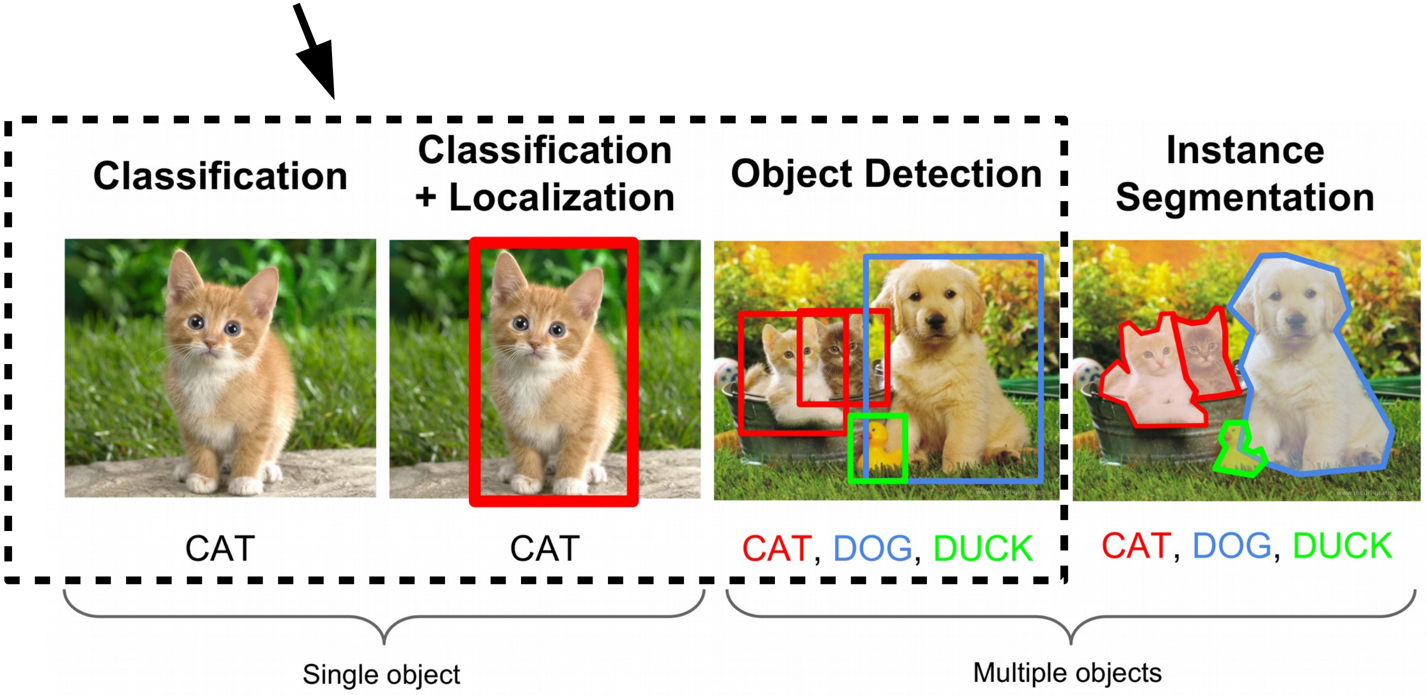
- Formalization:
  - Input: $\mathbf{x}$ (customer application)
  - Output: y (good/bad customer?)
  - Target function: (ideal credit approval formula)
  - Data: $(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \ldots, (\mathbf{x_n}, y_n)$ (historical records)
  -
  - Hypothesis: $g : X \to Y$ (formula/classifier to be used)

Problem we want to solve



Applications

We will focus on these applications

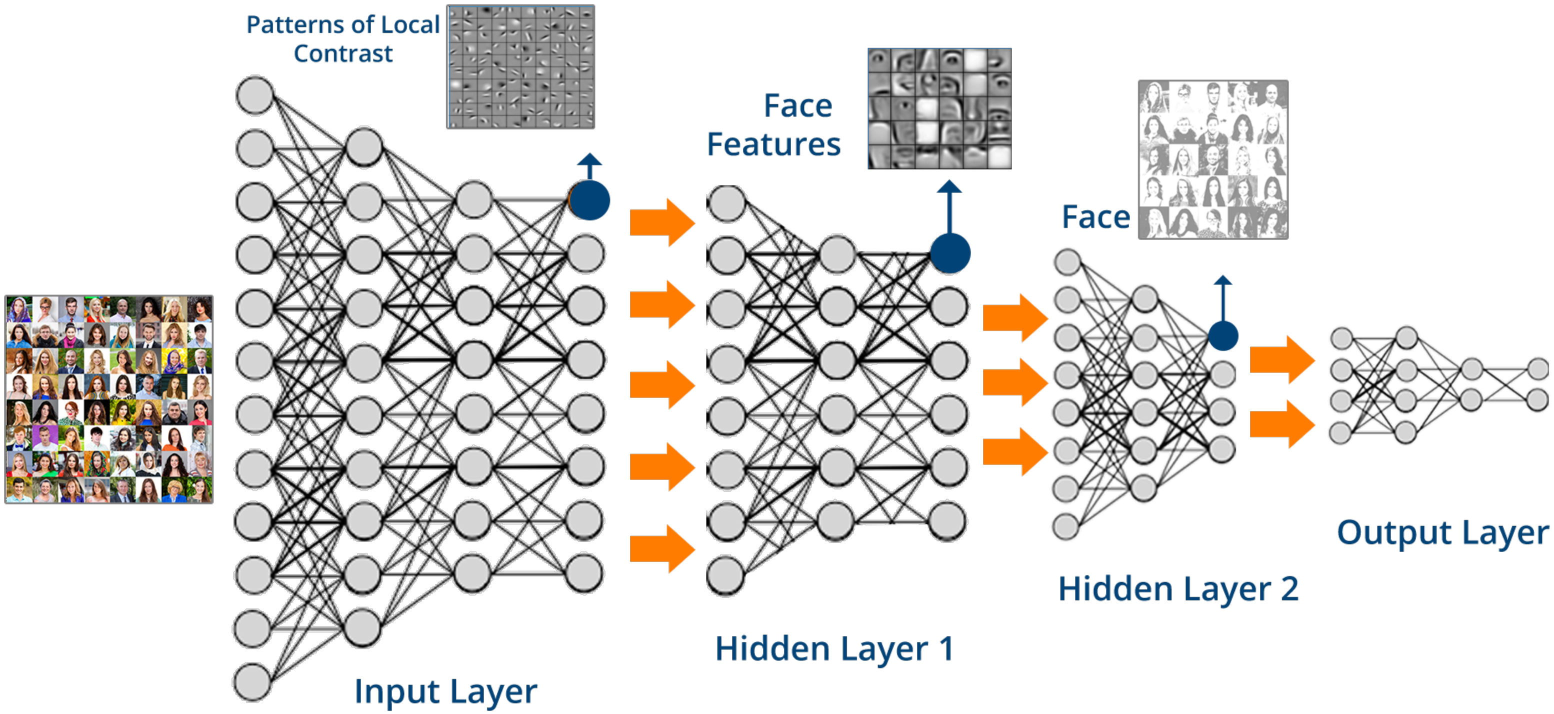


<u>We will ignore these applications</u>

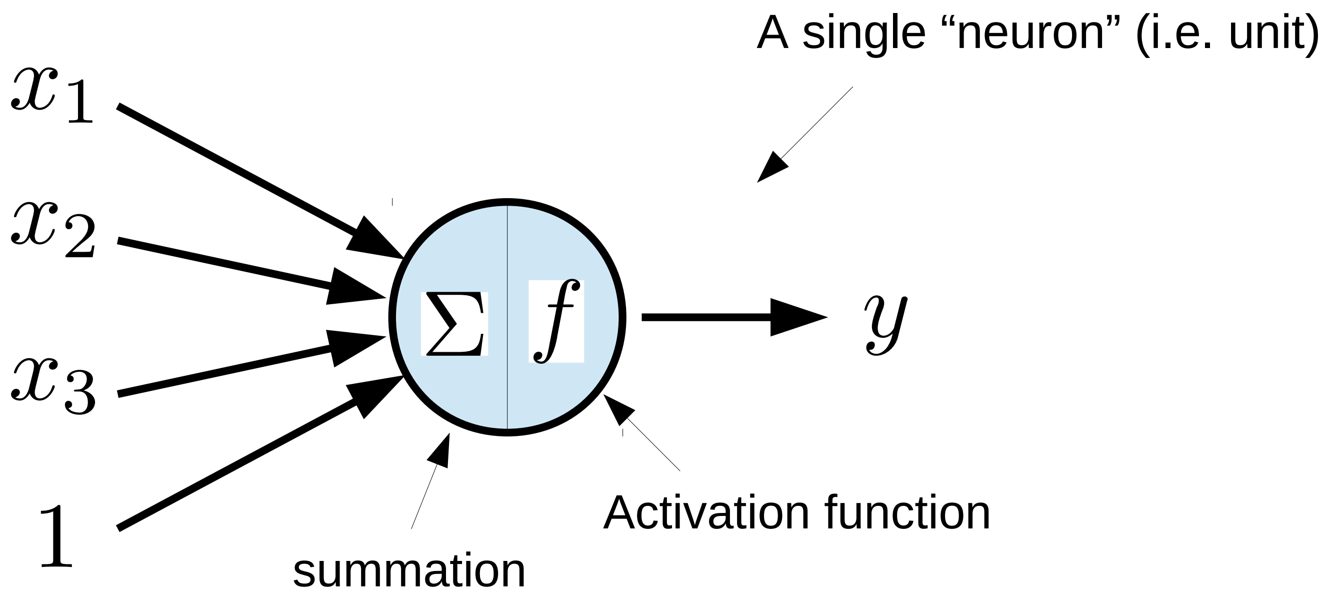
– image segmentation
– speech-to-text
– natural language processing
– …

.. but deep learning has been applied in lots of ways...

Example of a deep neural network



The multi-layer perceptron

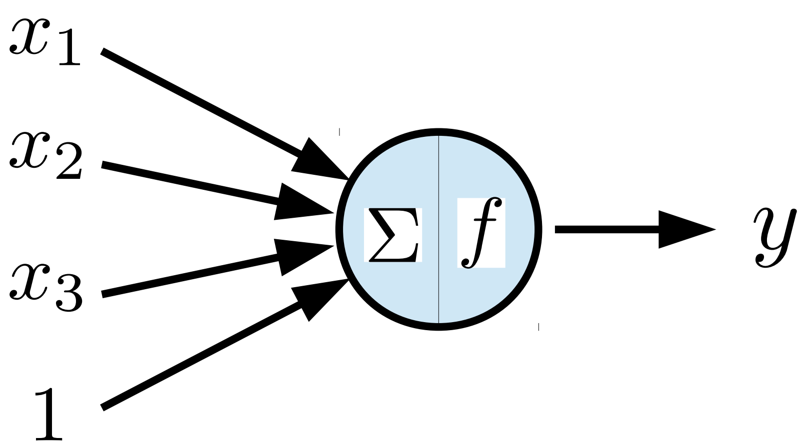


$$y = f(w_1 x_1 + w_2 x_2 + w_3 x_3 + \cdots + b)$$

$$= f(w^T x + b)$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

The multi-layer perceptron

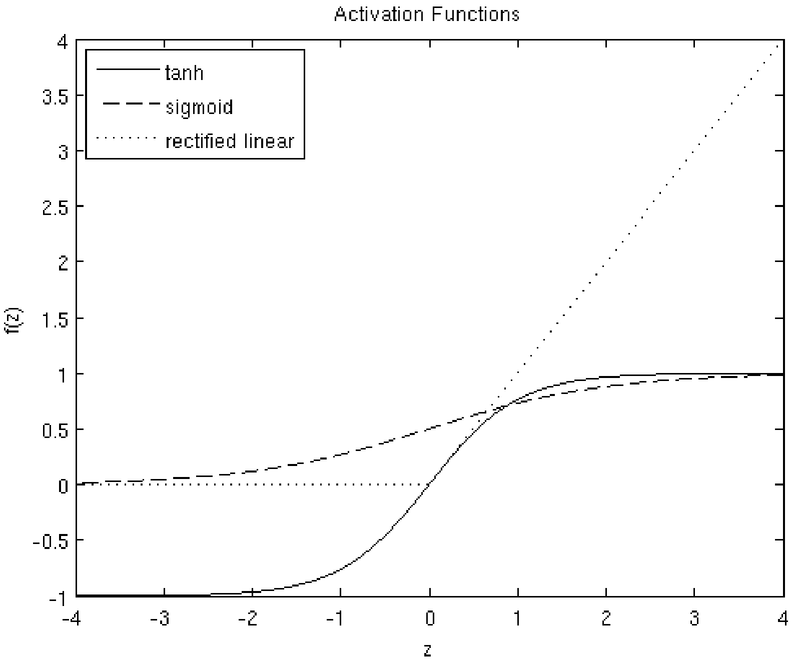


Different activation functions:

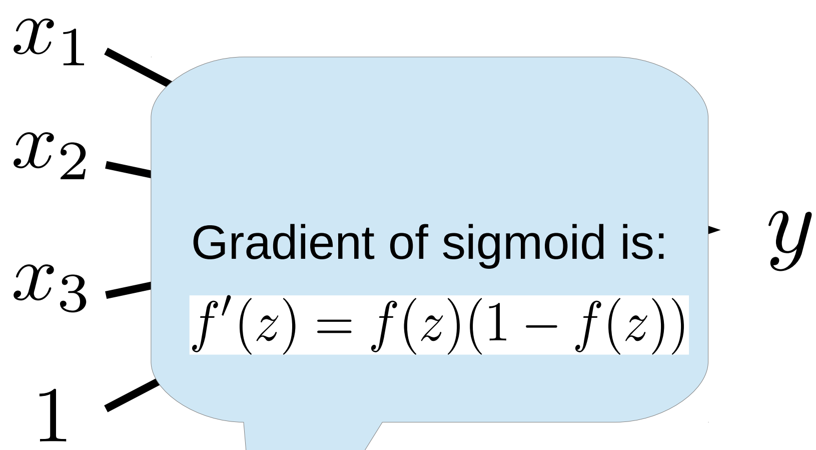
– sigmoid $$f(z) = \frac{1}{1+e^{-z}}$$

– tanh $$f(z) = tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

– rectified linear unit (ReLU) $$f(z) = \max(0, z)$$



The multi-layer perceptron



Gradient of sigmoid is:

$$f'(z) = f(z)(1 - f(z))$$

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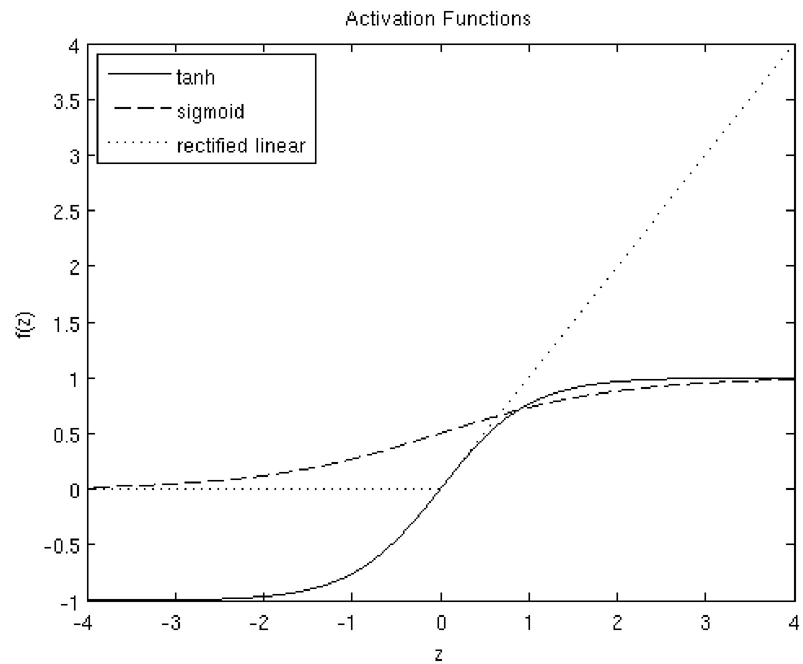
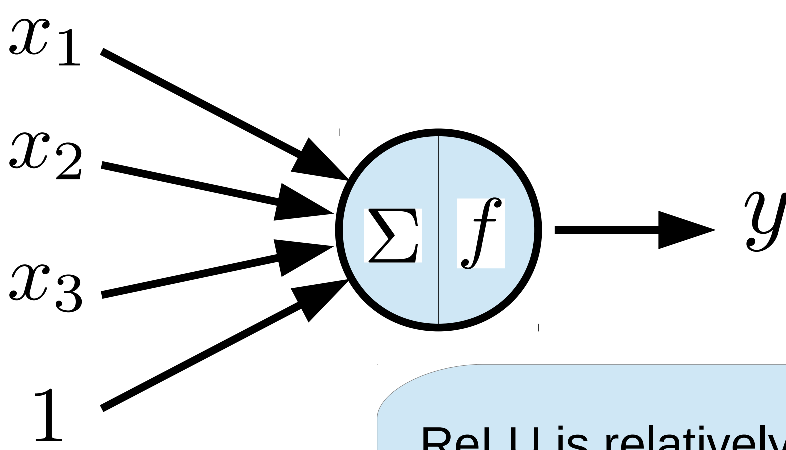


The multi-layer perceptron



ReLU is relatively new
– efficient to evaluate
– enables more layers b/c
attenuates gradient less

Different activation functions:

– sigmoid $$f(z) = \frac{1}{1+e^{-z}}$$

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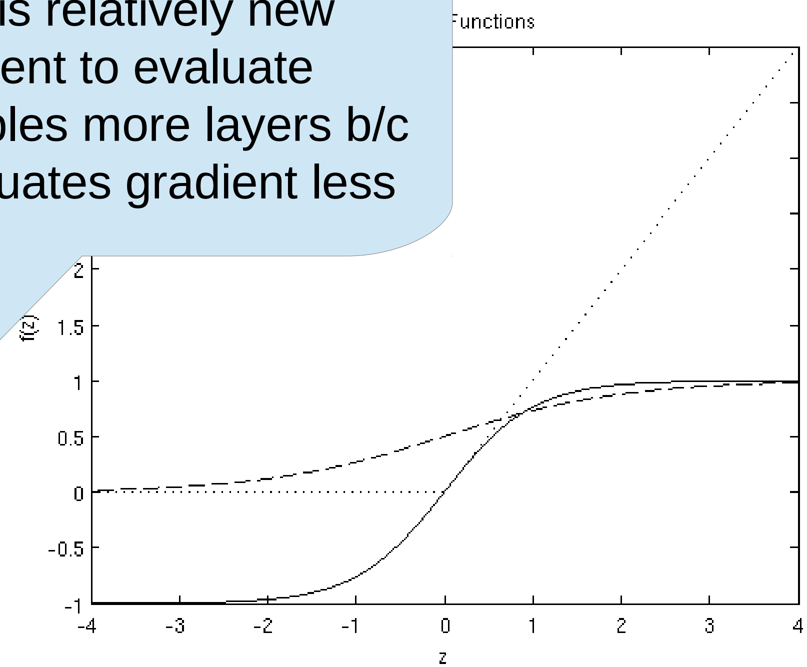
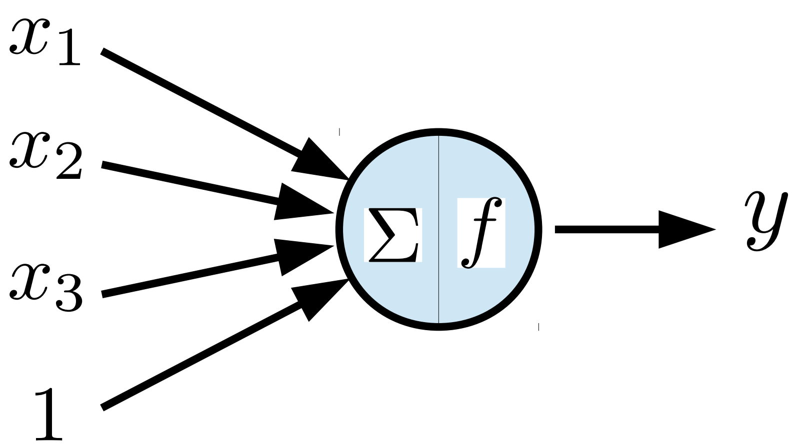


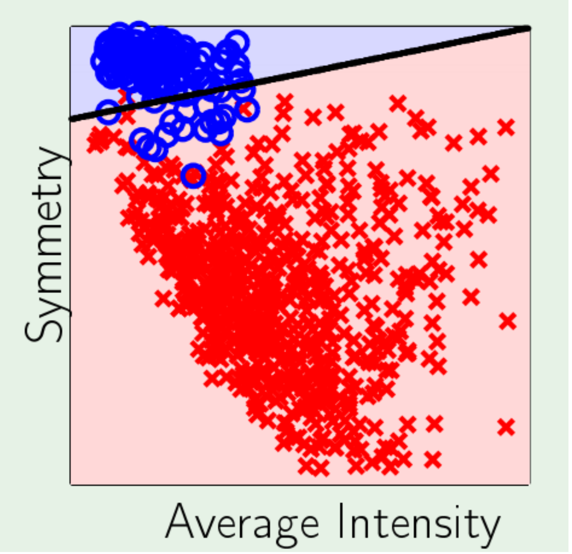
The multi-layer perceptron



One layer neural network has a simple interpretation: linear classification.

$$y = f(w^T x + b)$$

X_1 == symmetry
X_2 == avg intensity
Y == class label (binary)



What do *w* and *b* correspond to in this picture?

Training

Given a dataset: $$D = \{(x^1, y^1), (x^2, y^2), \ldots, (x^n, y^n)\}$$

Define loss function: $$L(x^i, y^i; w, b) = (y^i - f(w^T x^i + b))^2$$

Training

Given a dataset: $D = \{(x^1, y^1), (x^2, y^2), \ldots, (x^n, y^n)\}$

Define loss function: $L(x^i, y^i; w, b) = \frac{1}{2}(y^i - f(w^T x^i + b))^2$

Loss function tells us how well the network classified x^i

Training

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Loss function tells us how well the network classified x^i

Method of training: adjust *w, b* so as to minimize the net loss over the dataset

i.e.: adjust *w, b* so as to minimize: $$\sum_{(x^i, y^i) \in D} L(x^i, y^i; w, b)$$

If the sum of losses is zero, then the network has classified the dataset perfectly

Training

Method of training: adjust *w, b* so as to minimize the net loss over the dataset

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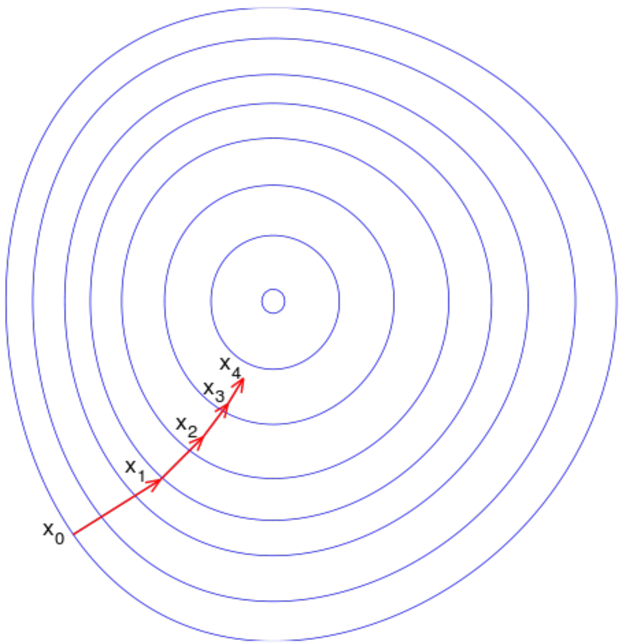
How?

Time out for gradient descent

Suppose someone gives you an unknown function F(x)
- you want to find a minimum for F
- but, you do not have an analytical description of F(x)

Use gradient descent!
- all you need is the ability to evaluate F(x) and its gradient at any point x

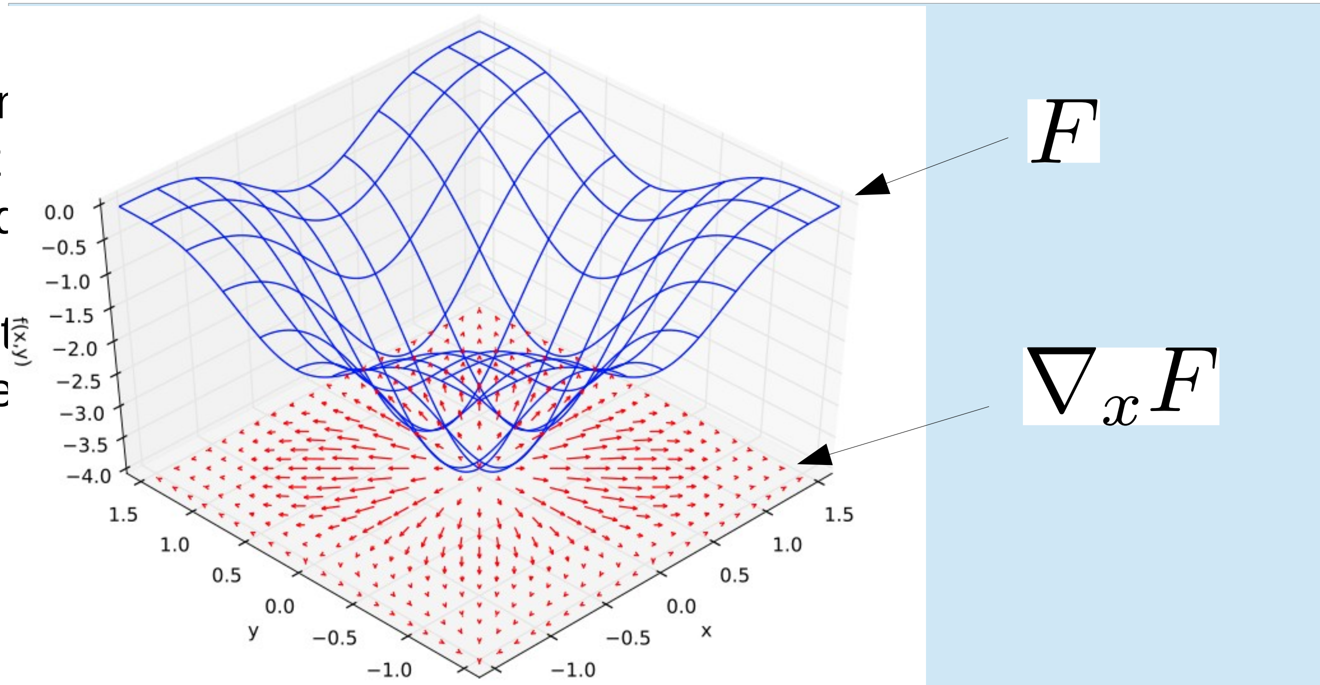
1. pick x_0 at random
2. $x_1 = x_0 - \alpha \nabla_x F(x_0)$
3. $x_2 = x_1 - \alpha \nabla_x F(x_1)$
4. $x_3 = x_2 - \alpha \nabla_x F(x_2)$
5. ...



Time out for gradient descent

Suppose sor
– you want
– but, you c

Use gradient
– all you ne



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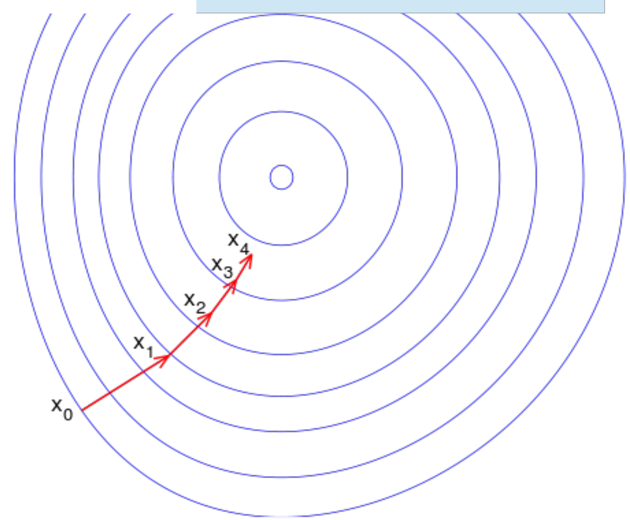

Training

Method of training: adjust *w, b* so as to minimize the net loss over the dataset

i.e.: adjust *w, b* so as to minimize: $$\sum_{(x^i,y^i)\in D} L(x^i, y^i; w, b)$$

Do gradient descent on dataset:

1. repeat

2. $$w \leftarrow w - \alpha \frac{1}{n} \sum_{(x^i,y^i)\in D} \nabla_w L(x^i, y^i; w, b)$$

3. $$b \leftarrow b - \alpha \frac{1}{n} \sum_{(x^i,y^i)\in D} \nabla_b L(x^i, y^i; w, b)$$

4. until converged

Where:

$$\nabla_w L(x^i, y^i; w, b) = -(y^i - f(w^T x^i + b)) f'(w^T x^i + b) x^i$$

$$\nabla_b L(x^i, y^i; w, b) = -(y^i - f(w^T x^i + b)) f'(w^T x^i + b)$$

Training

Method of training: adjust *w, b* so

i.e.: adjust *w, b* so as to minimize:

This is the similar to logistic regression
– logistic regression uses a cross entropy loss
– we are using a quadratic loss

<u>Do gradient descent on dataset:</u>

1. repeat

2. $$w \leftarrow w - \alpha \frac{1}{n} \sum_{(x^i, y^i) \in D} \nabla_w L(x^i, y^i; w, b)$$

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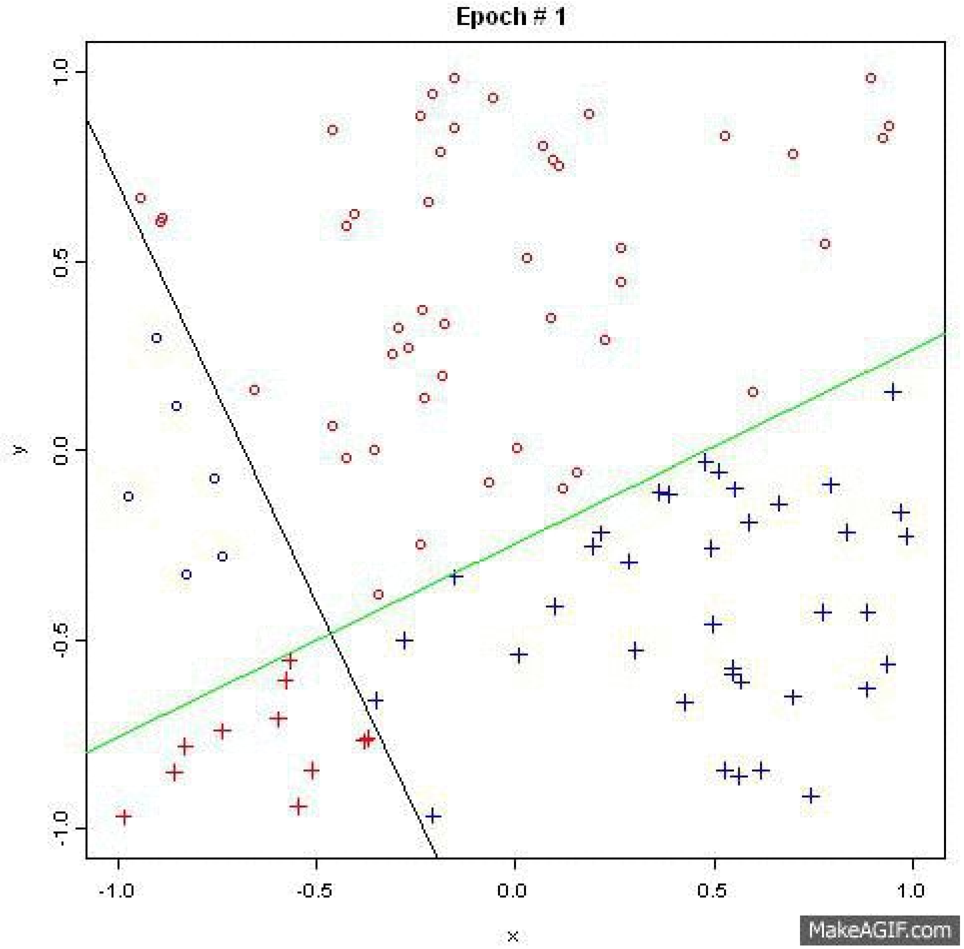
Where:

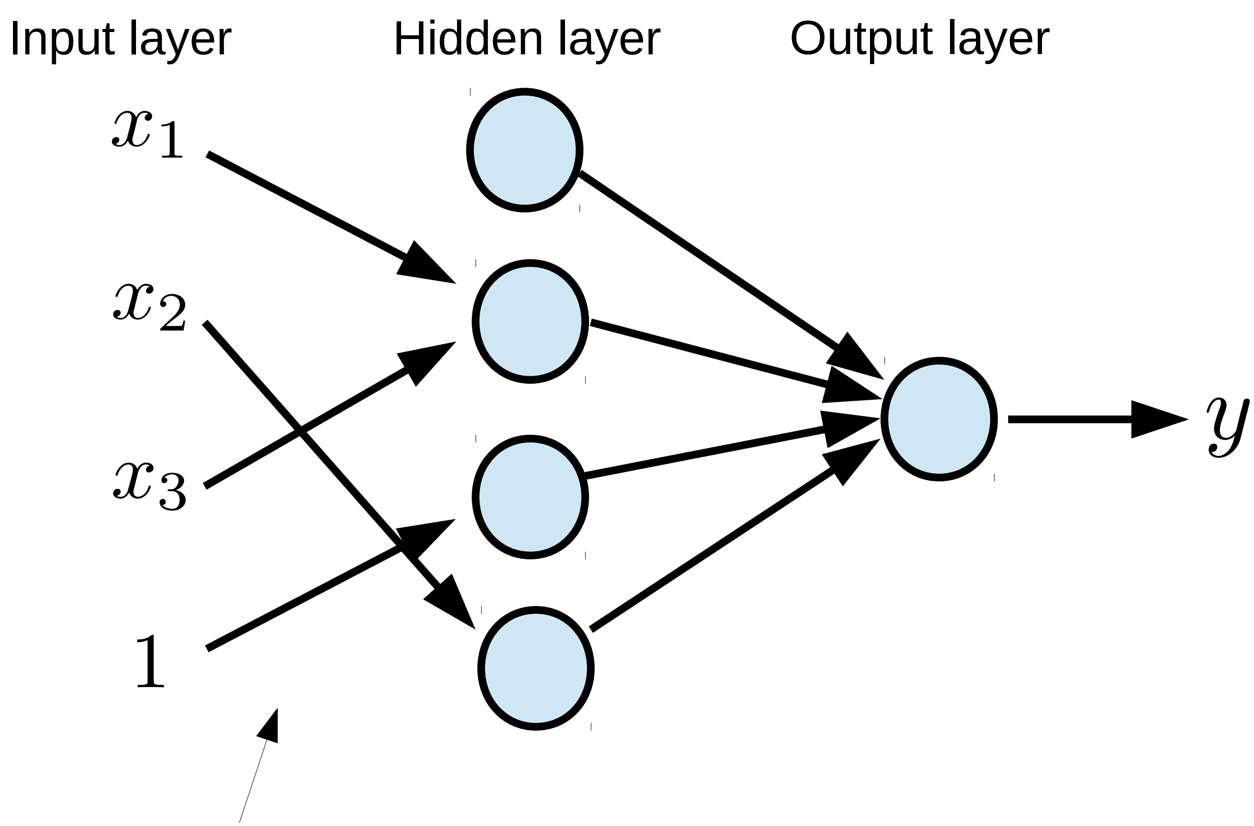
$$\nabla_w L(x^i, y^i; w, b) = -(y^i - f(w^T x^i + b)) f'(w^T x^i + b) x^i$$

$$\nabla_b L(x^i, y^i; w, b) = -(y^i - f(w^T x^i + b)) f'(w^T x^i + b)$$

Training example

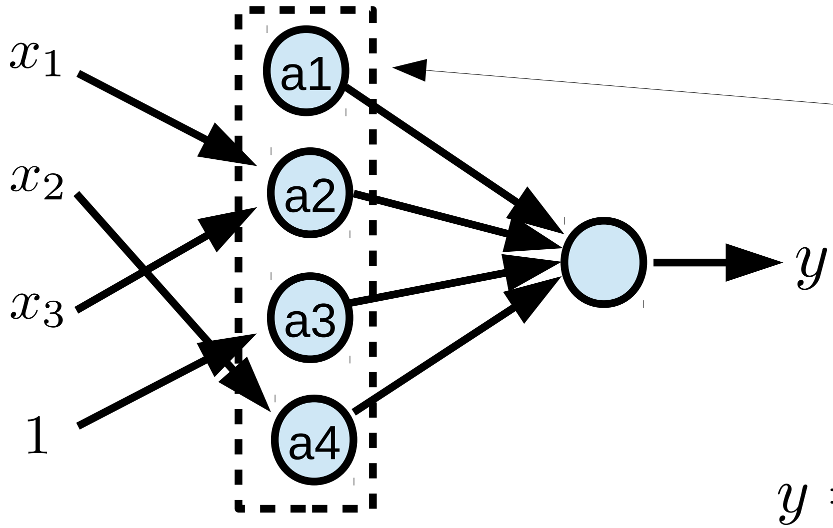


Going deeper: a one layer network



Each hidden node is
connected to every input

Multi-layer evaluation works similarly



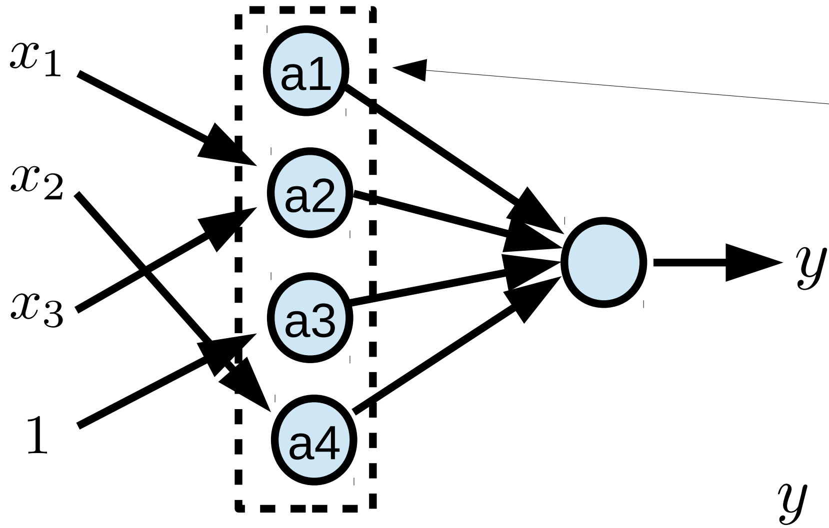
$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Vector of hidden layer activations

$$y = f({w^{(2)}}^T a + b)$$

Single activation: $a_i = f({w_i^{(1)}}^T x + b_i^{(1)})$

Multi-layer evaluation works similarly



$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Vector of hidden layer activations

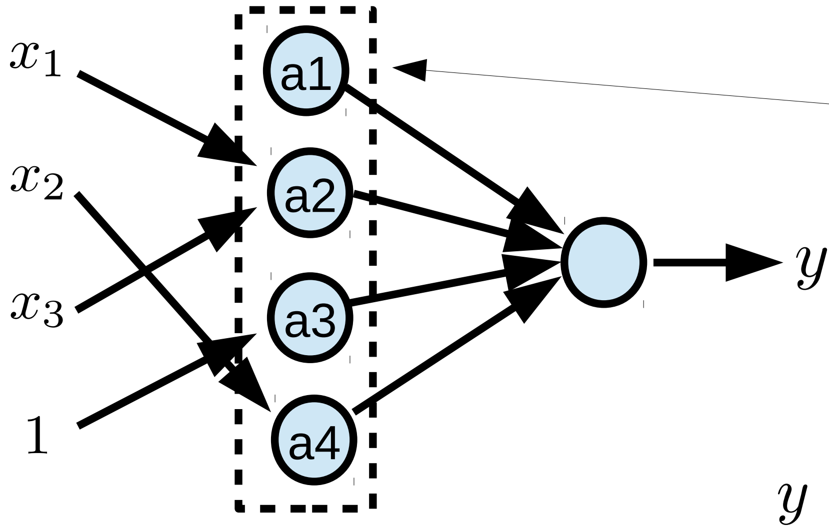
$$y = f({w^{(2)}}^T a + b)$$

Single activation: $a_i = f({w_i^{(1)}}^T x + b_i^{(1)})$

Vector of activations: $a = f(W^{(1)} x + b^{(1)})$

where $W^{(1)} = \begin{pmatrix} {w_1^{(1)}}^T \\ \vdots \\ {w_4^{(1)}}^T \end{pmatrix} \quad b^{(1)} = \begin{pmatrix} b_1 \\ \vdots \\ b_4 \end{pmatrix}$

Multi-layer evaluation works similarly



$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Vector of hidden layer activations

$$y = f({w^{(2)}}^T a + b)$$

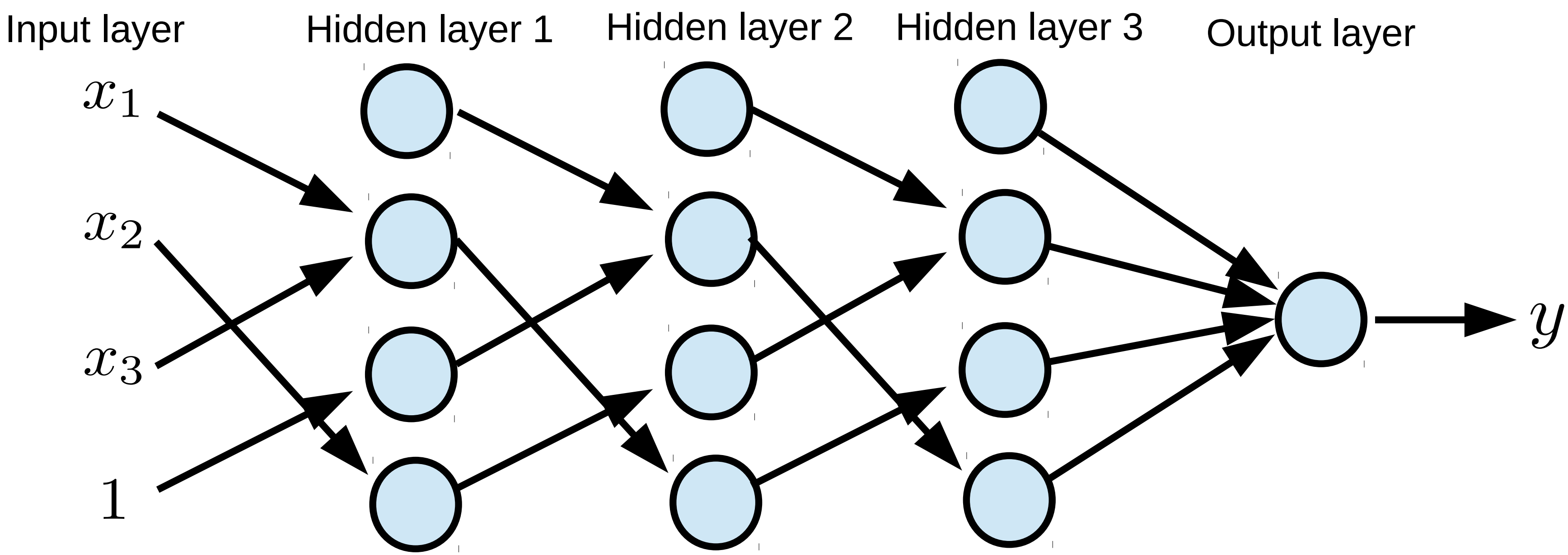
Called “forward propagation” – b/c the activations are propogated forward...

Single activation: $a_i = f({w_i^{(1)}}^T x + b_i^{(1)})$

Vector of activations: $a = f(W^{(1)} x + b^{(1)})$

where $W^{(1)} = \begin{pmatrix} {w_1^{(1)}}^T \\ \vdots \\ {w_4^{(1)}}^T \end{pmatrix}$ $b^{(1)} = \begin{pmatrix} b_1 \\ \vdots \\ b_4 \end{pmatrix}$

Can create networks of arbitrary depth...



– Forward propagation works the same for any depth network.

– Whereas a single output node corresponds to linear classification, adding hidden nodes makes classification non-linear

How do we train multi-layer networks?

Almost the same as in the single-node case...

Do gradient descent on dataset:

1. repeat
2. $$w \leftarrow w - \alpha \frac{1}{n} \sum_{(x^i, y^i) \in D} \nabla_w L(x^i, y^i; w, b)$$
3. $$b \leftarrow b - \alpha \frac{1}{n} \sum_{(x^i, y^i) \in D} \nabla_b L(x^i, y^i; w, b)$$
4. until converged

Now, we're doing gradient descent on all weights/biases in the network – not just a single layer

– this is called *backpropagation*

Backpropagation

1. Perform a feedforward pass, computing the activations for layers L_2, L_3, and so on up to the output layer L_{n_l}.

2. For each output unit i in layer n_l (the output layer), set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \|y - h_{W,b}(x)\|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

3. For $l = n_l - 1, n_l - 2, n_l - 3, \ldots, 2$

   For each node i in layer l, set

$$\delta_i^{(l)} = \left(\sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$$

4. Compute the desired partial derivatives, which are given as:

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W, b; x, y) = a_j^{(l)} \delta_i^{(l+1)}$$

$$\frac{\partial}{\partial b_i^{(l)}} J(W, b; x, y) = \delta_i^{(l+1)}.$$

http://ufldl.stanford.edu/tutorial/supervised/MultiLayerNeuralNetworks/

Training in mini-batches

A batch is typically between 32 and 128 samples

1. repeat
2. randomly sample a mini-batch: $B \subset D$
3. $$w \leftarrow w - \alpha \frac{1}{n} \sum_{(x^i,y^i) \in B} \nabla_w L(x^i, y^i; w, b)$$
3. $$b \leftarrow b - \alpha \frac{1}{n} \sum_{(x^i,y^i) \in B} \nabla_b L(x^i, y^i; w, b)$$
4. until converged

Training in mini-batches helps b/c:
– don't have to load the entire dataset into memory
– training is still relatively stable
– random sampling of batches helps avoid local minima

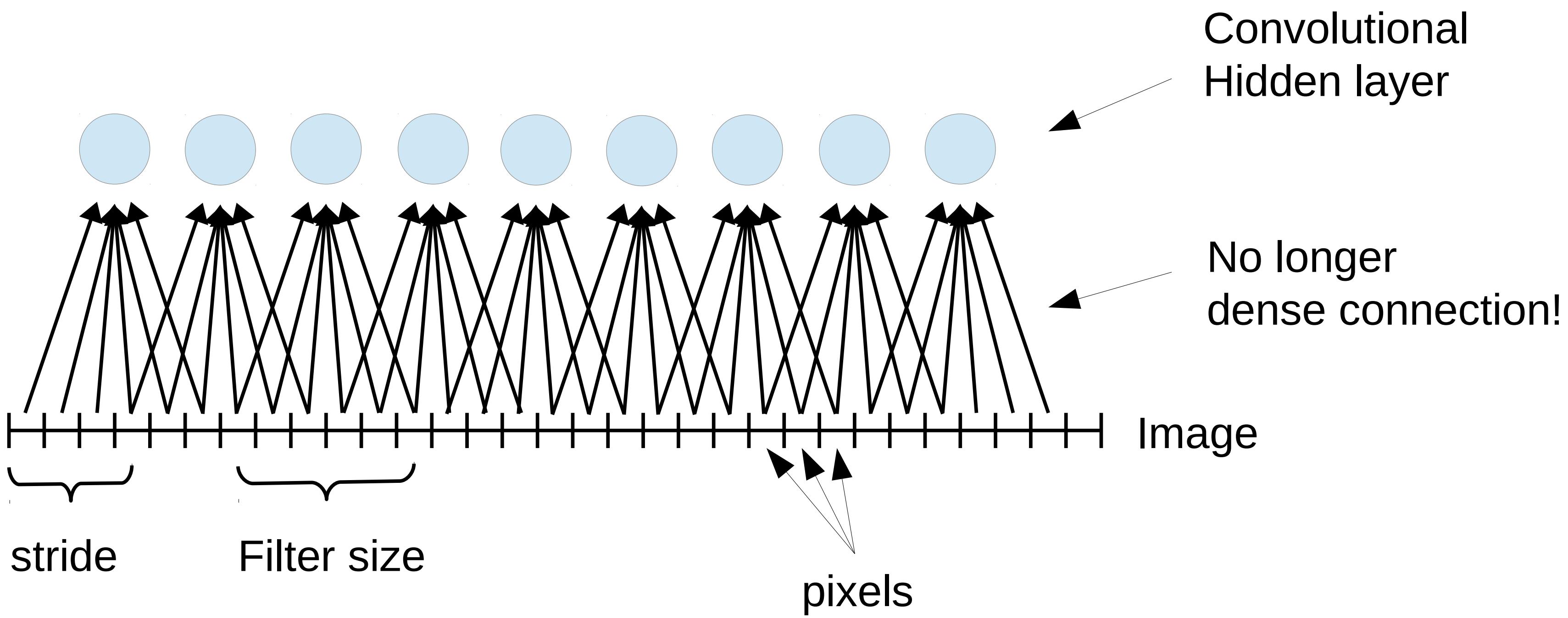
Convolutional layers

Deep multi-layer perceptron networks
– general purpose
– involve huge numbers of weights

We want:
– special purpose network for image and NLP data
– fewer parameters
– fewer local minima

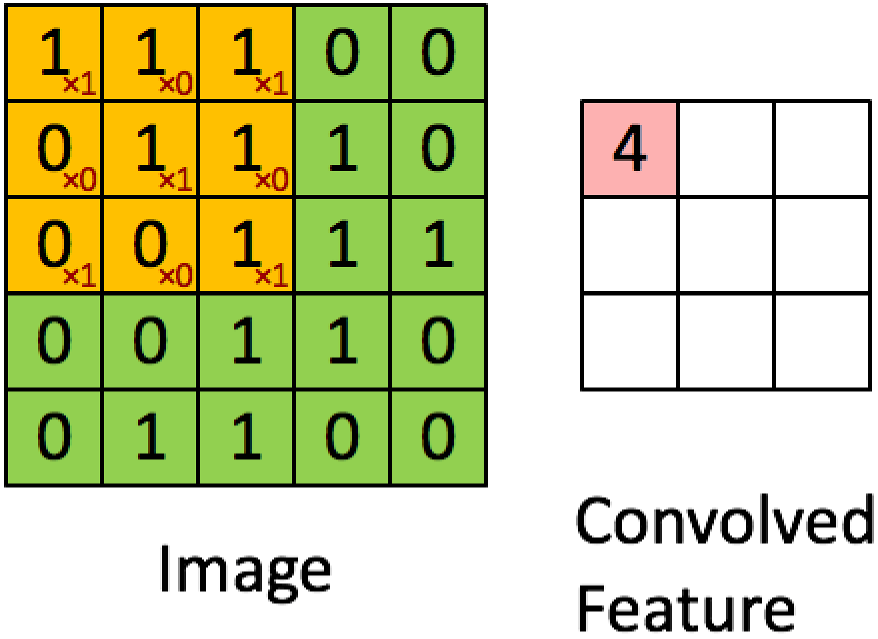
Answer: convolutional layers!

Convolutional layers



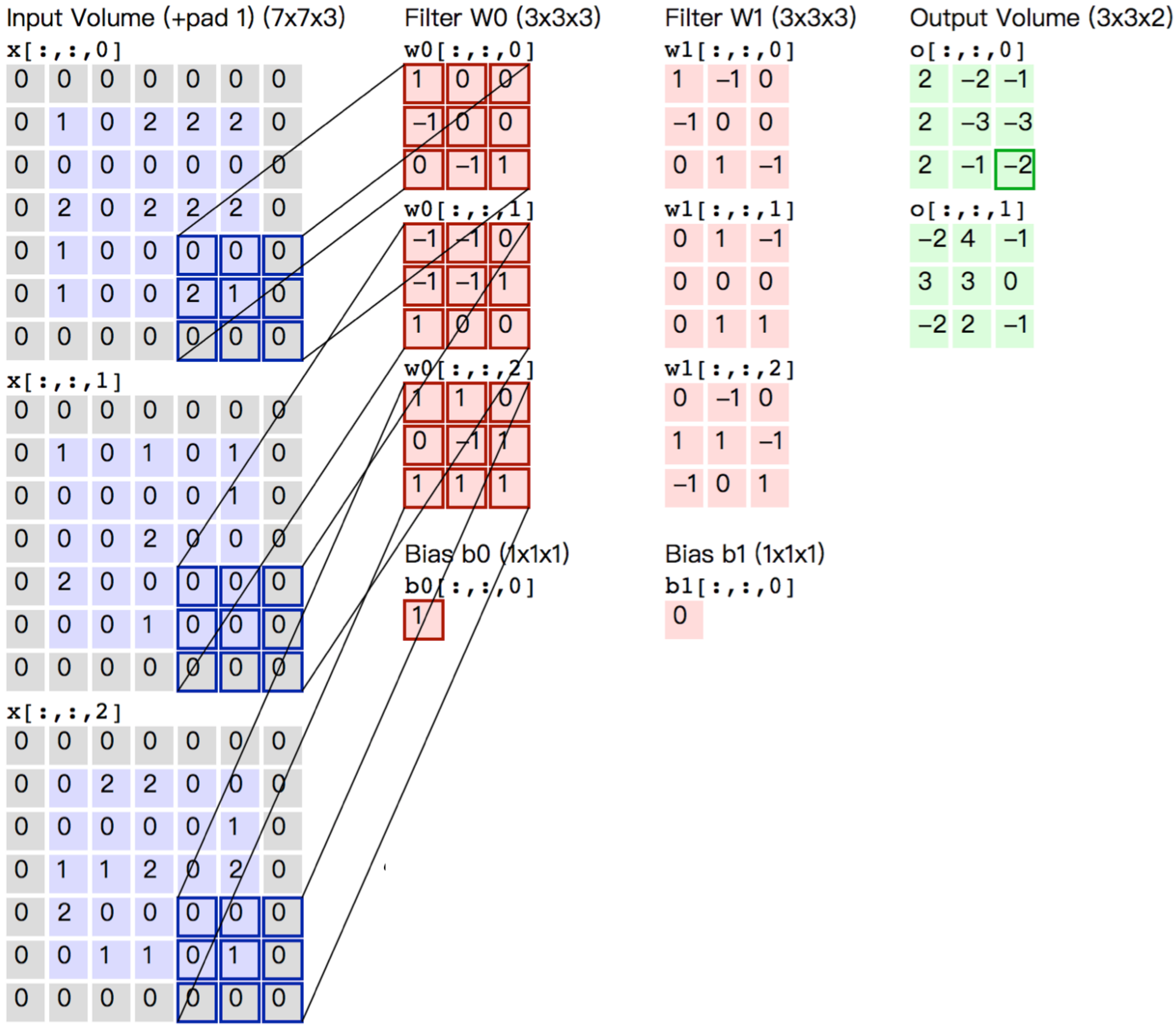
Convolutional layers

Two dimensional example:

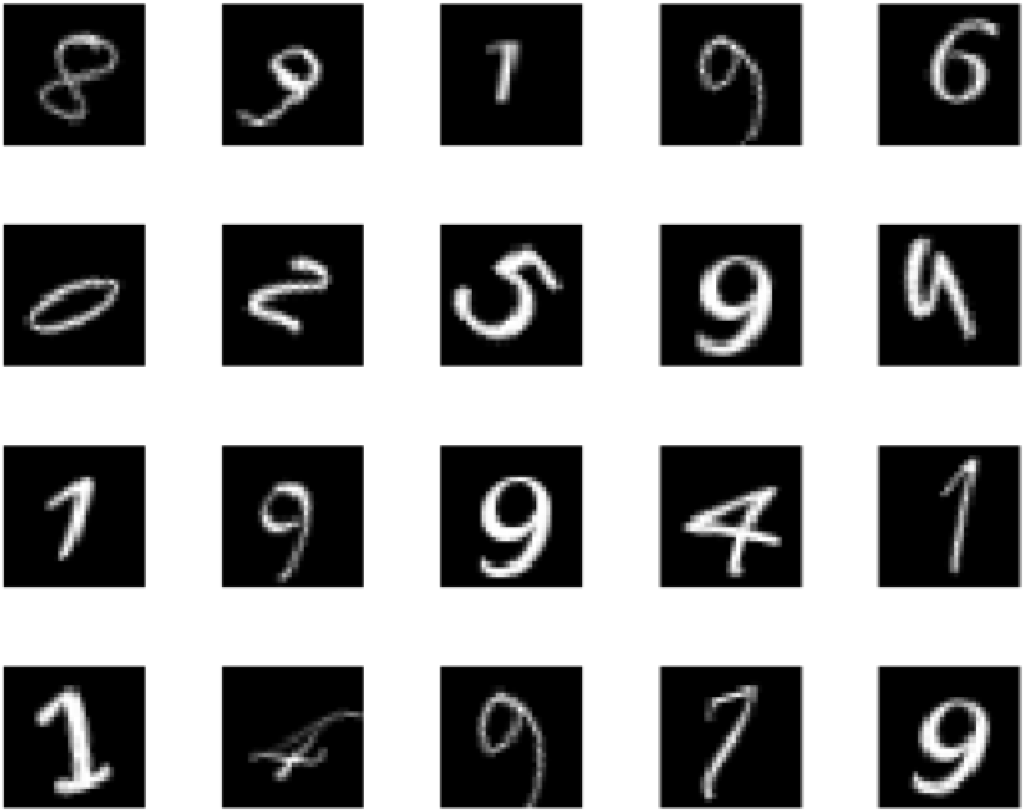


Why do you think they call this “convolution”?

Convolutional layers



Example: MNIST digit classification with LeNet

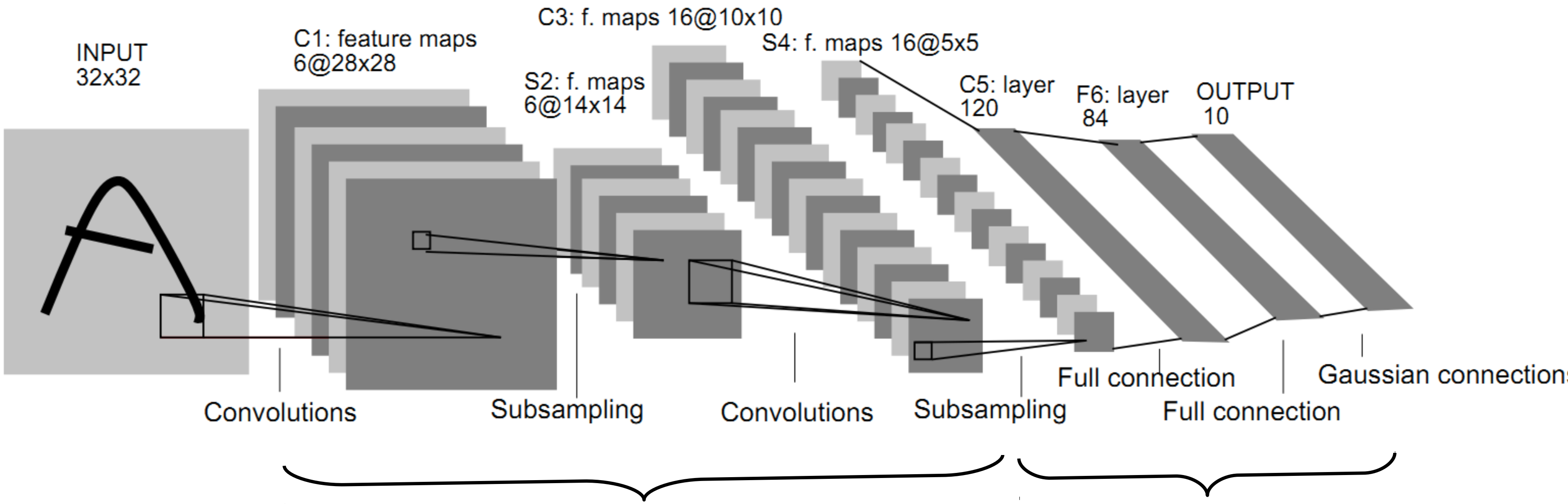


MNIST dataset: images of 10,000 handwritten digits

Objective: classify each image as the corresponding digit

Example: MNIST digit classification with LeNet

LeNet:



two convolutional layers
– conv, relu, pooling

two fully connected layers
– relu
– last layer has logistic
activation function

Example: MNIST digit classification with LeNet

Load dataset, create train/test splits

```
digitDatasetPath = fullfile(matlabroot,'toolbox','nnet','nndemos',...
    'nndatasets','DigitDataset');
digitData = imageDatastore(digitDatasetPath,...
    'IncludeSubfolders',true,'LabelSource','foldernames');

trainNumFiles = 750;
[trainDigitData,valDigitData] = splitEachLabel(digitData,trainNumFiles,'randomize');
```

Example: MNIST digit classification with LeNet

Define the neural network structure:

```
layers = [
    imageInputLayer([28 28 1])
    convolution2dLayer(3,16,'Padding',1)
    batchNormalizationLayer
    reluLayer
    maxPooling2dLayer(2,'Stride',2)
    convolution2dLayer(3,32,'Padding',1)
    batchNormalizationLayer
    reluLayer
    maxPooling2dLayer(2,'Stride',2)
    convolution2dLayer(3,64,'Padding',1)
    batchNormalizationLayer
    reluLayer
    fullyConnectedLayer(50)
    reluLayer
    fullyConnectedLayer(10)
    softmaxLayer
    classificationLayer];

options = trainingOptions('sgdm',...
'MaxEpochs',3, ...
'ValidationData',valDigitData,...
'ValidationFrequency',30,...
'Verbose',true,...
'ExecutionEnvironment','gpu',...
'Plots','training-progress');
```

Input

Conv1

Conv2

FC1

FC2

Example: MNIST digit classification with LeNet

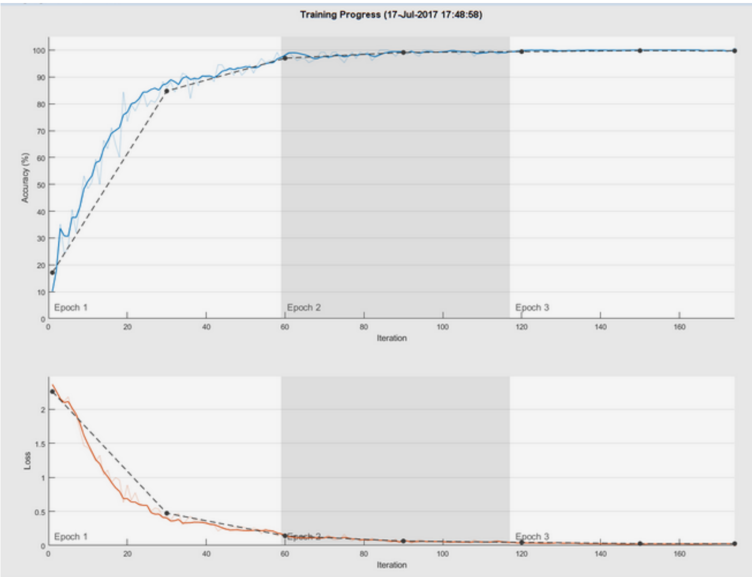
Train network, classify test set, measure accuracy

– notice we test on a different set (a holdout set) than we trained on

```
net = trainNetwork(trainDigitData,layers,options);

predictedLabels = classify(net,valDigitData);
valLabels = valDigitData.Labels;

accuracy = sum(predictedLabels == valLabels)/numel(valLabels);
```



Using the GPU makes
a huge differece...

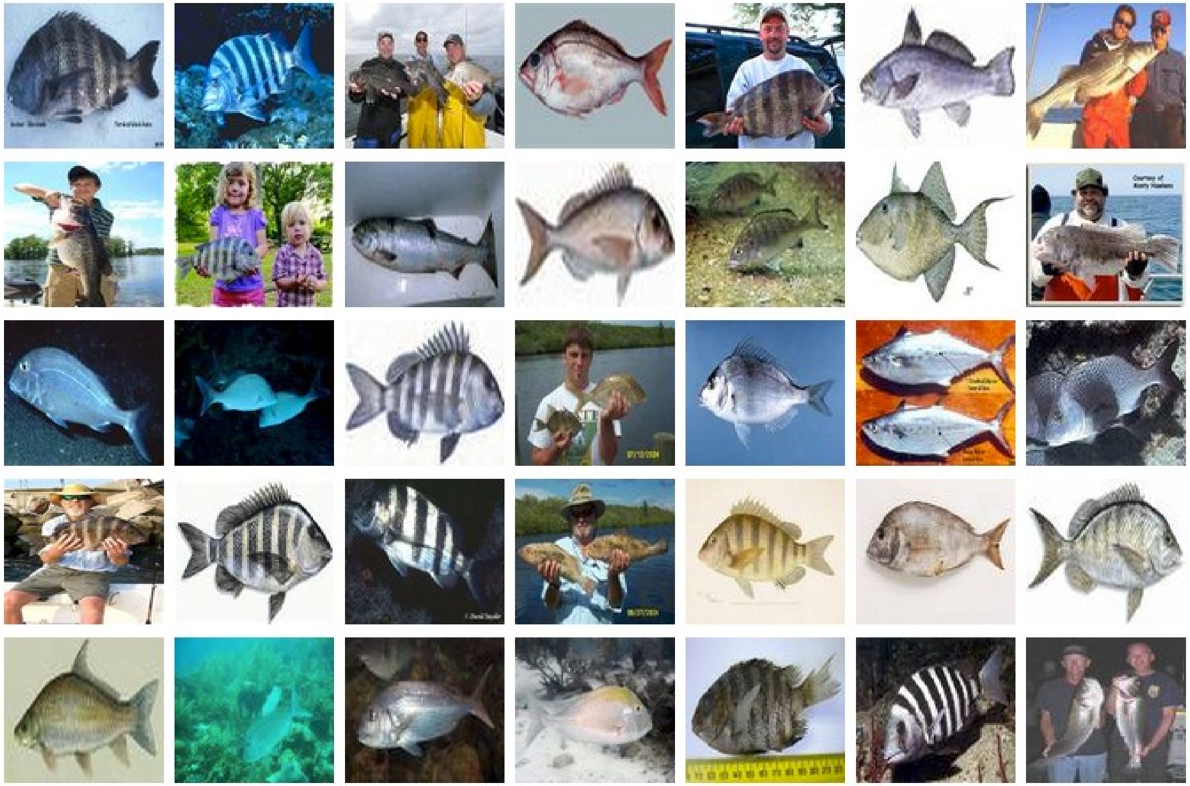
Deep learning packages

You don’t need to use Matlab (obviously)

Tensorflow is probably the most popular platform

Caffe and Theano are also big

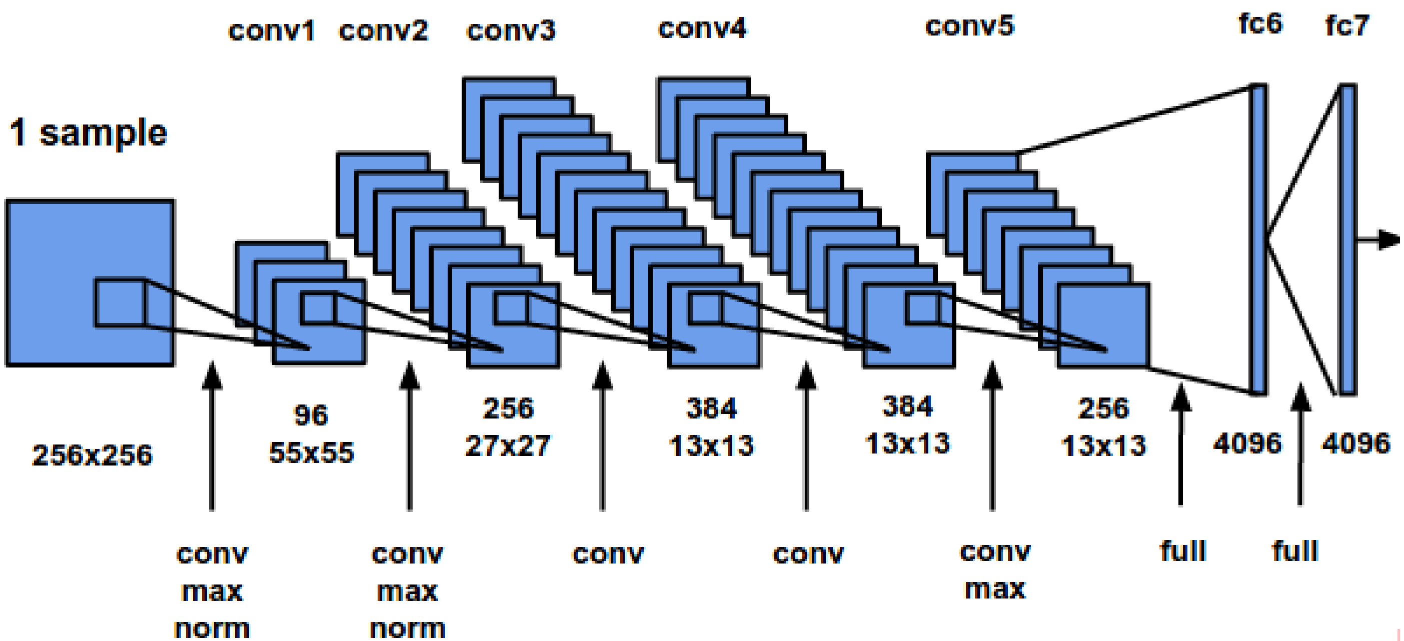
Another example: image classification w/ AlexNet



ImageNet dataset: millions of images of objects

Objective: classify each image as the corresponding object (1k categories in ILSVRC)

Another example: image classification w/ AlexNet



AlexNet has 8 layers: five conv followed by three fully connected

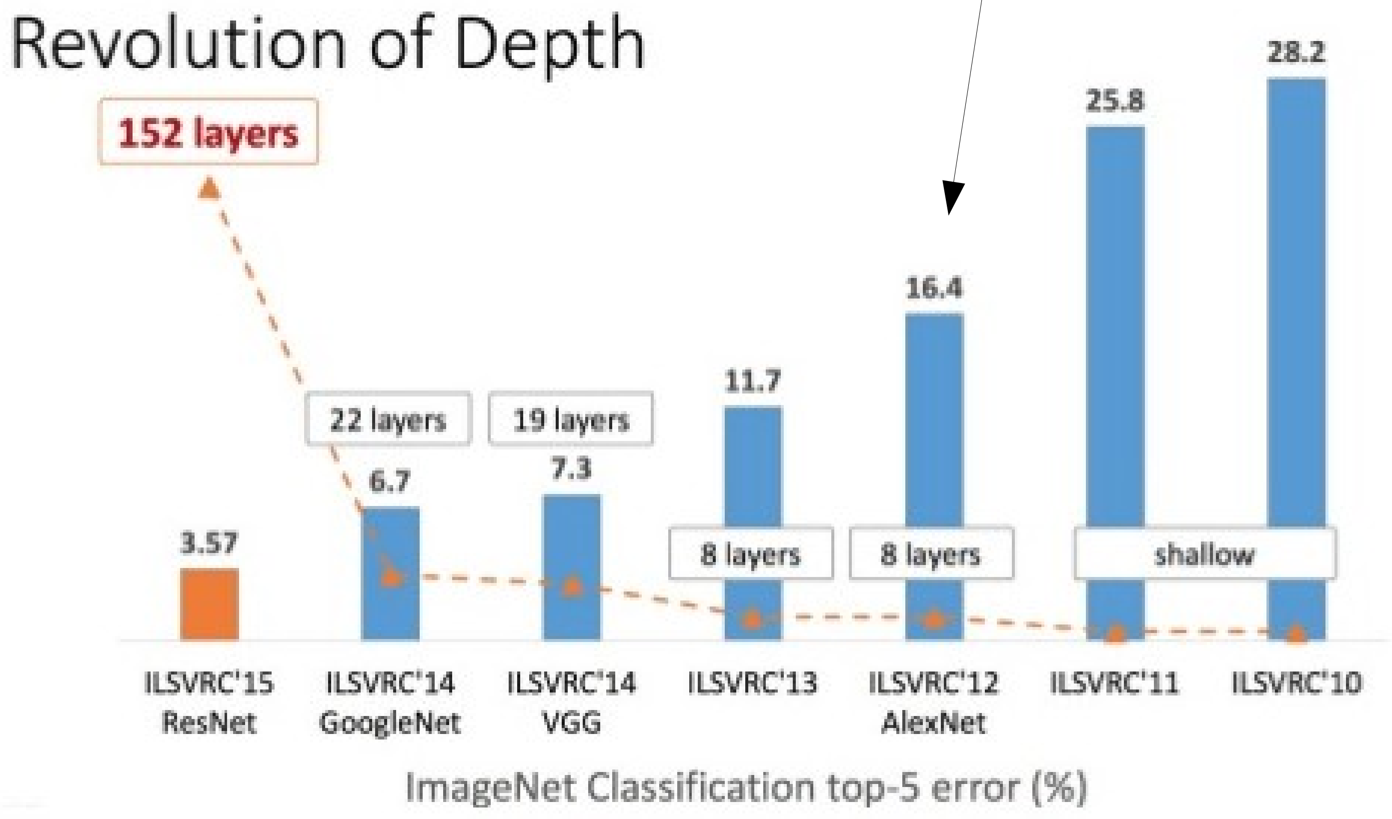
Another example: image classification w/ AlexNet

1	'data'	Image Input	227x227x3 images with 'zerocenter' normalization
2	'conv1'	Convolution	96 11x11x3 convolutions with stride [4 4] and padding [0 0]
3	'relu1'	ReLU	ReLU
4	'norm1'	Cross Channel Normalization	cross channel normalization with 5 channels per element
5	'pool1'	Max Pooling	3x3 max pooling with stride [2 2] and padding [0 0]
6	'conv2'	Convolution	256 5x5x48 convolutions with stride [1 1] and padding [2 2]
7	'relu2'	ReLU	ReLU
8	'norm2'	Cross Channel Normalization	cross channel normalization with 5 channels per element
9	'pool2'	Max Pooling	3x3 max pooling with stride [2 2] and padding [0 0]
10	'conv3'	Convolution	384 3x3x256 convolutions with stride [1 1] and padding [1 1]
11	'relu3'	ReLU	ReLU
12	'conv4'	Convolution	384 3x3x192 convolutions with stride [1 1] and padding [1 1]
13	'relu4'	ReLU	ReLU
14	'conv5'	Convolution	256 3x3x192 convolutions with stride [1 1] and padding [1 1]
15	'relu5'	ReLU	ReLU
16	'pool5'	Max Pooling	3x3 max pooling with stride [2 2] and padding [0 0]
17	'fc6'	Fully Connected	4096 fully connected layer
18	'relu6'	ReLU	ReLU
19	'drop6'	Dropout	50% dropout
20	'fc7'	Fully Connected	4096 fully connected layer
21	'relu7'	ReLU	ReLU
22	'drop7'	Dropout	50% dropout
23	'fc8'	Fully Connected	1000 fully connected layer
24	'prob'	Softmax	softmax
25	'output'	Classification Output	crossentropyex with 'tench', 'goldfish', and 998 other classes

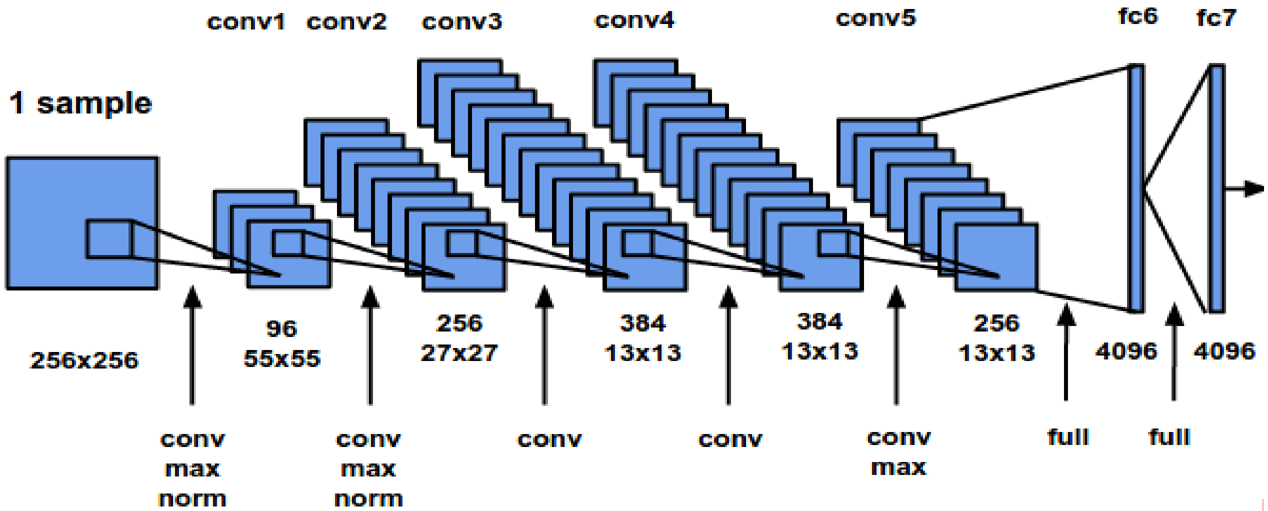
AlexNet has 8 layers: five conv followed by three fully connected

Another example: image classification w/ AlexNet

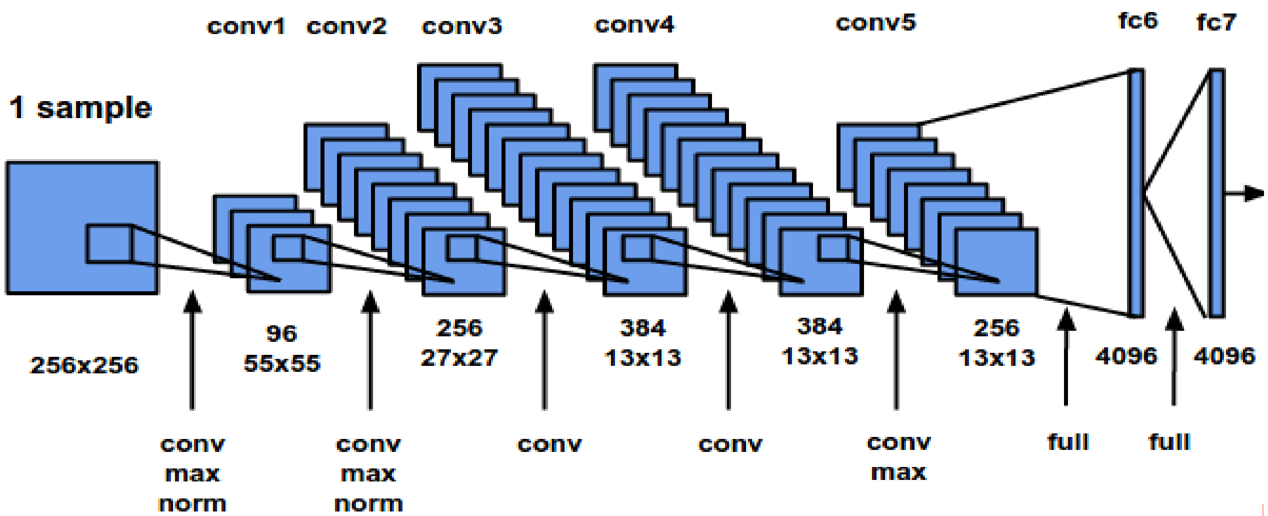
AlexNet won the 2012 ILSVRC challenge
– sparked the deep learning craze



What exactly are deep conv networks learning?



What exactly are deep conv networks learning?



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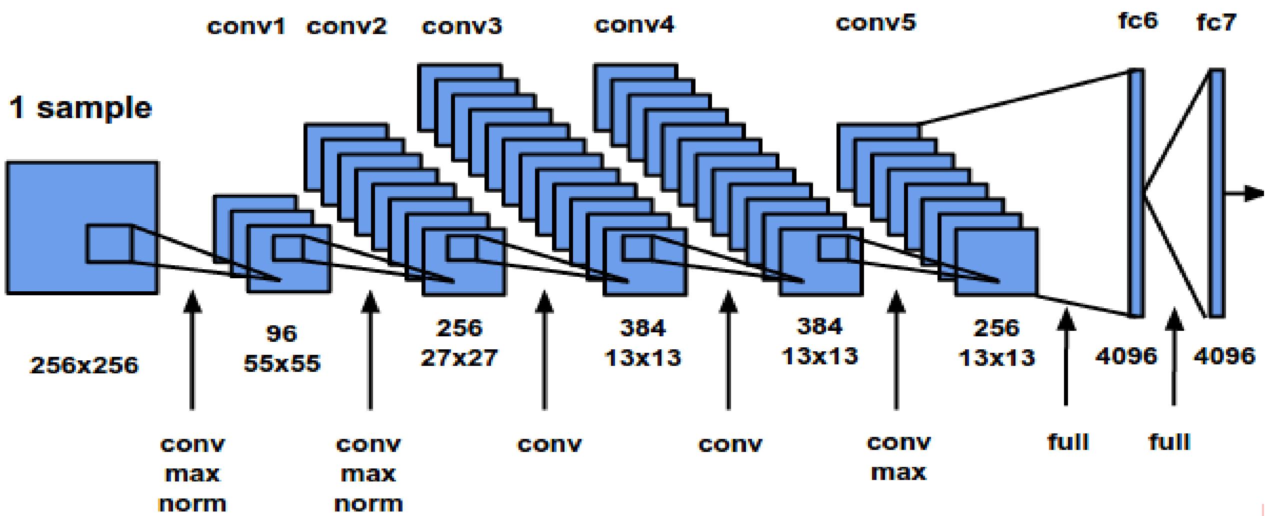

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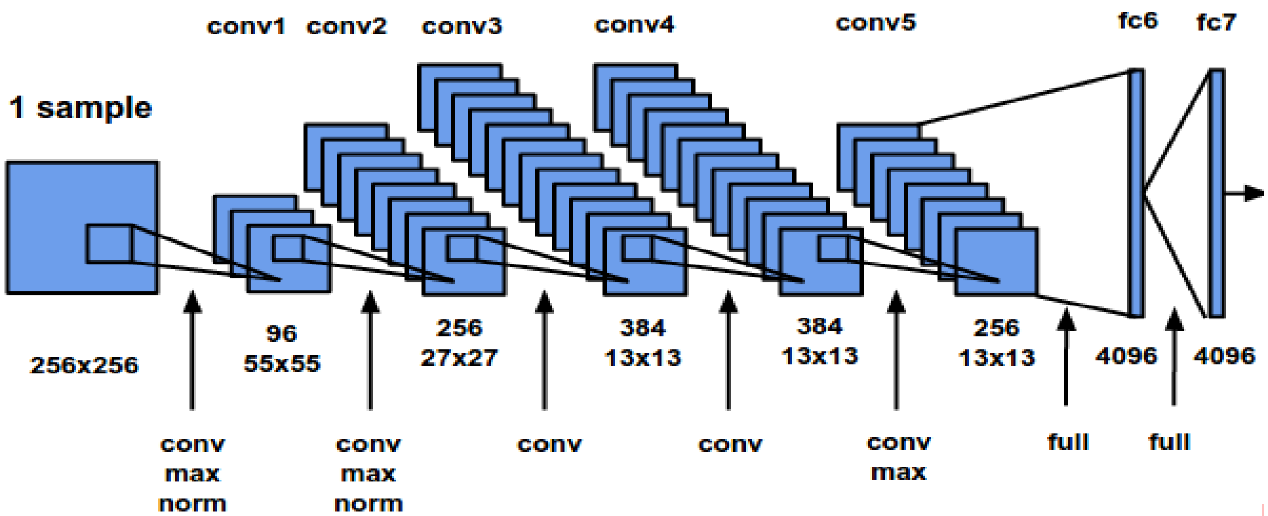
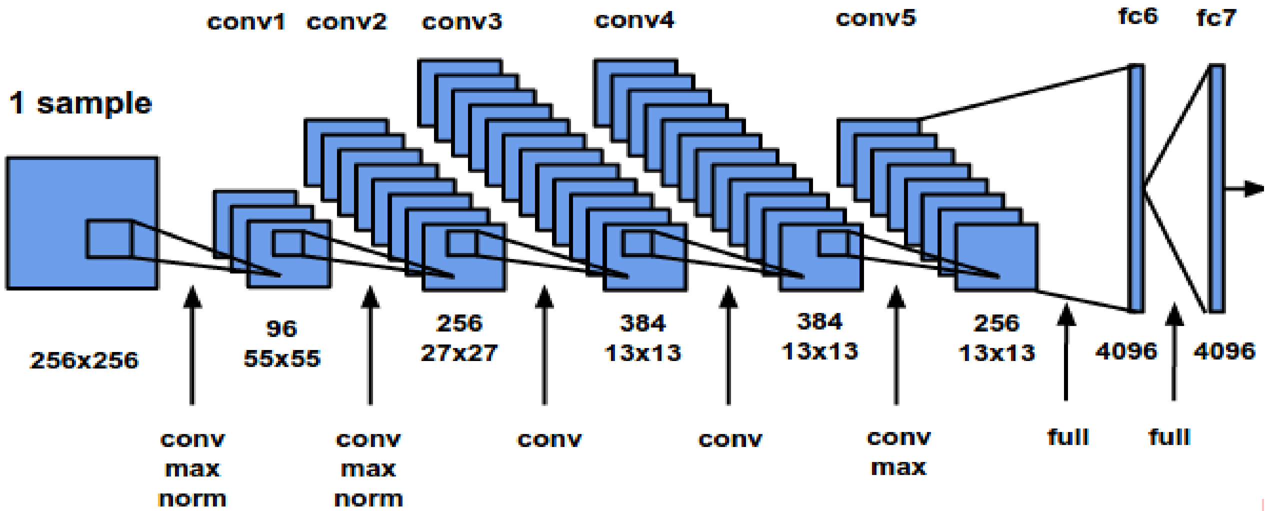


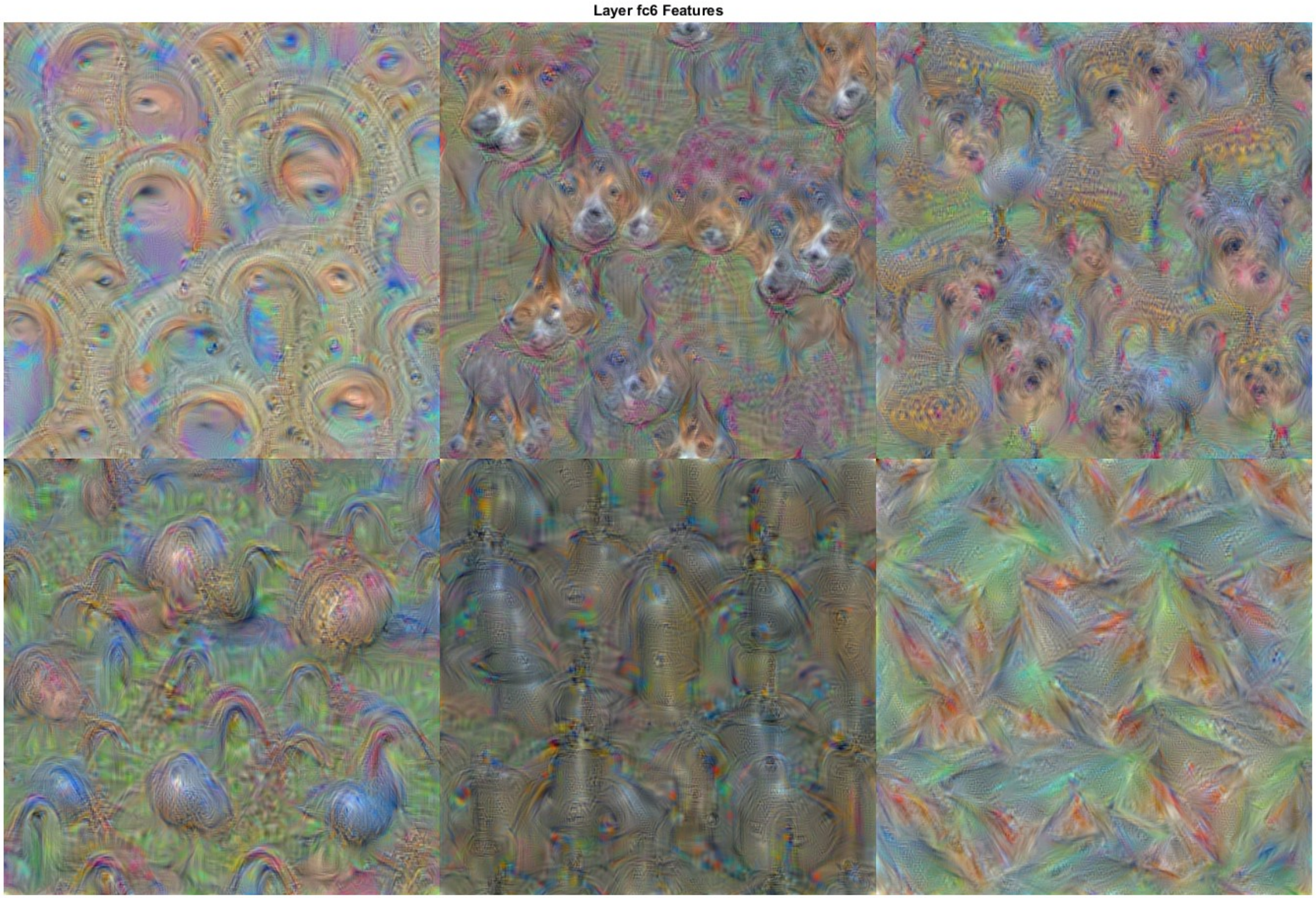
Layer conv4 Features

What exactly are deep conv networks learning?



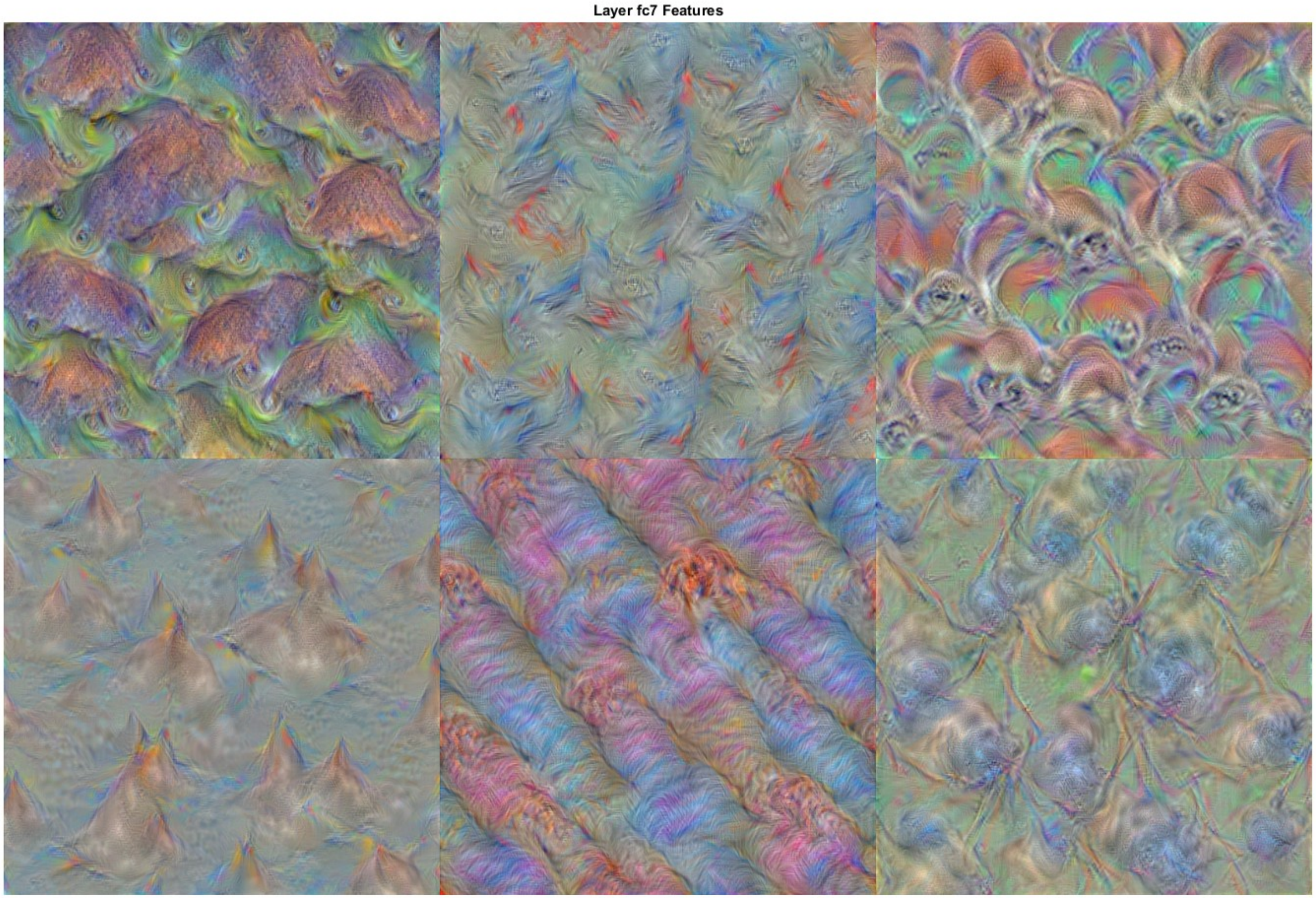
What exactly are deep conv networks learning?

FC layer 6



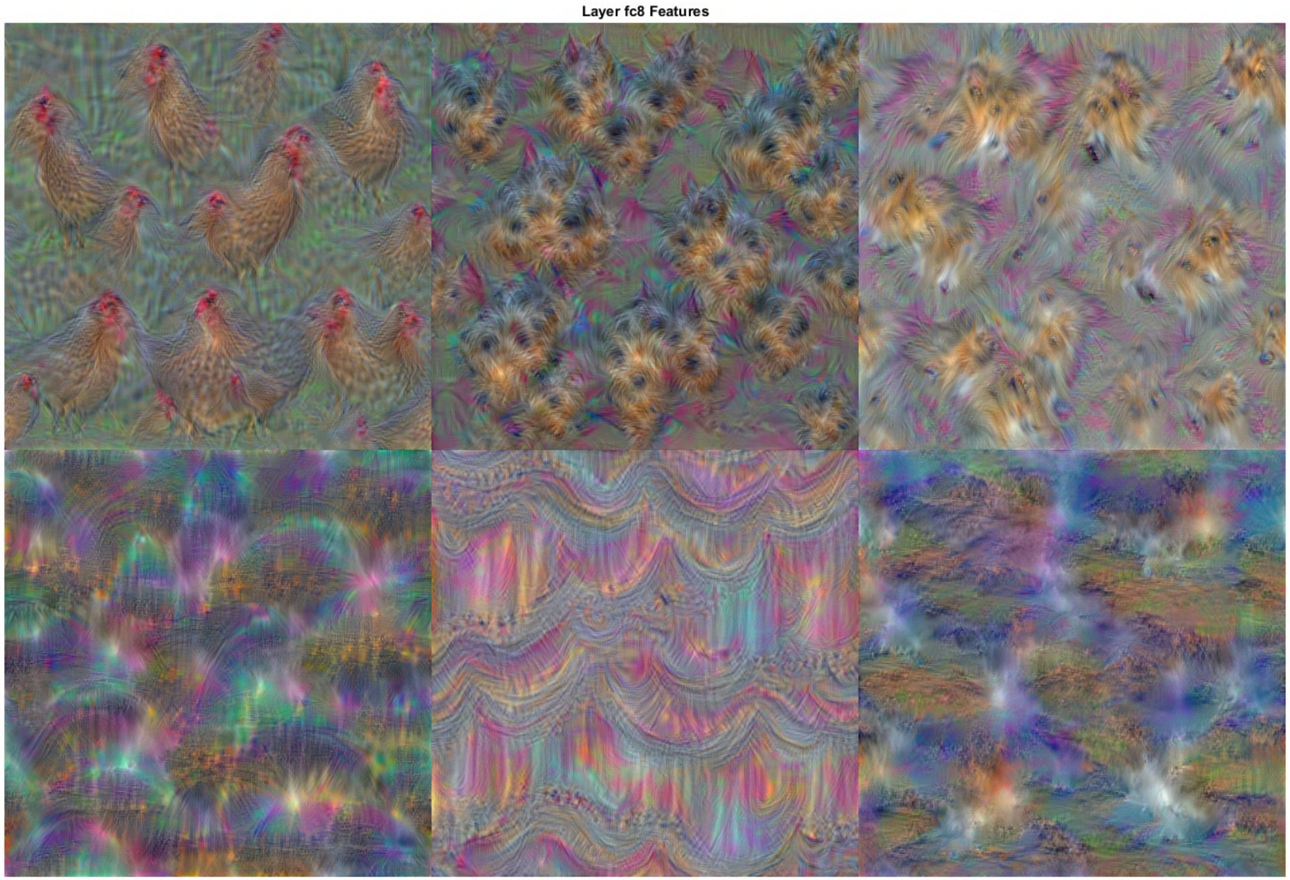
What exactly are deep conv networks learning?

FC layer 7



What exactly are deep conv networks learning?

Output layer



Finetuning

AlexNet has 60M parameters
– therefore, you need a very large training set (like imagenet)

Suppose we want to train on our own images, but we only have a few hundred?
– AlexNet will drastically overfit such a small dataset… (won’t generalize at all)

Finetuning

Idea:

1. pretrain on imagenet
2. finetune on your own dataset

AlexNet has 60M parameters
– therefore, you need a very large training set (like imagenet)

Suppose we want to train on our own images, but we only have a few hundred?
– AlexNet will drastically overfit such a small dataset… (won't generalize at all)