Adaptively secure MPC in sublinear communication

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BU & Northeastern

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Northeastern

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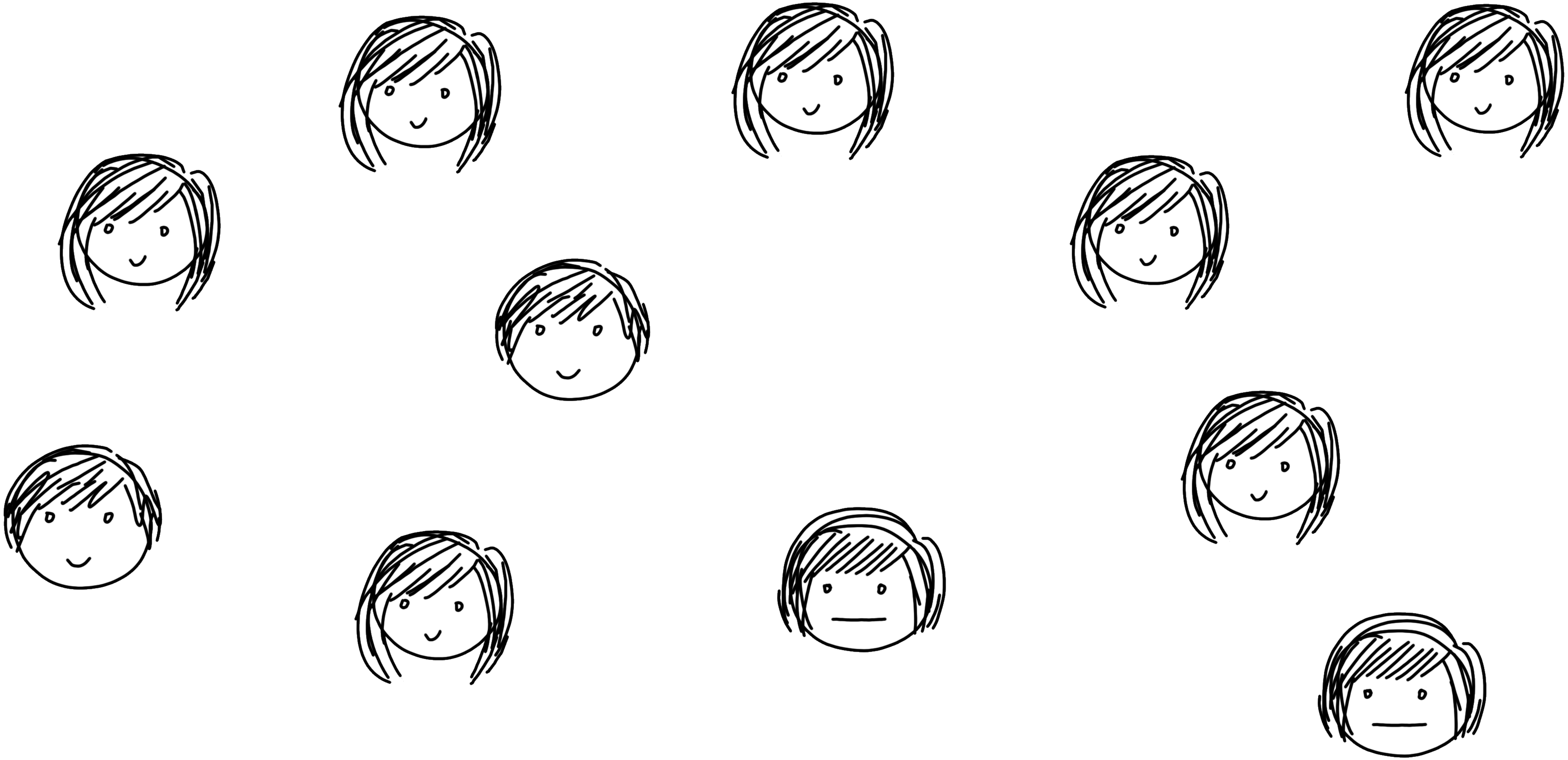
Northeastern

Static corruptions

Adv picks corrupted parties before protocol begins.

Adv



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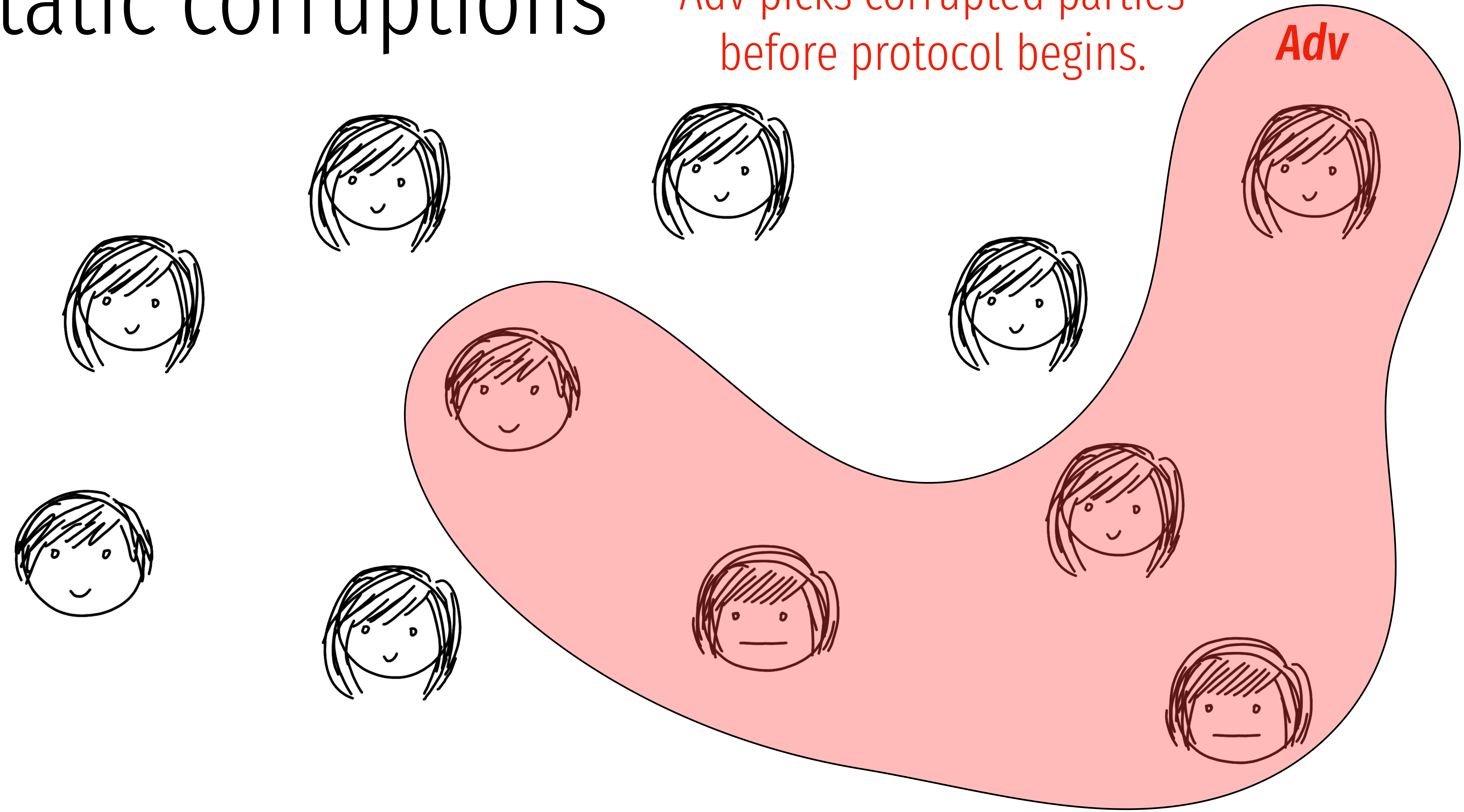
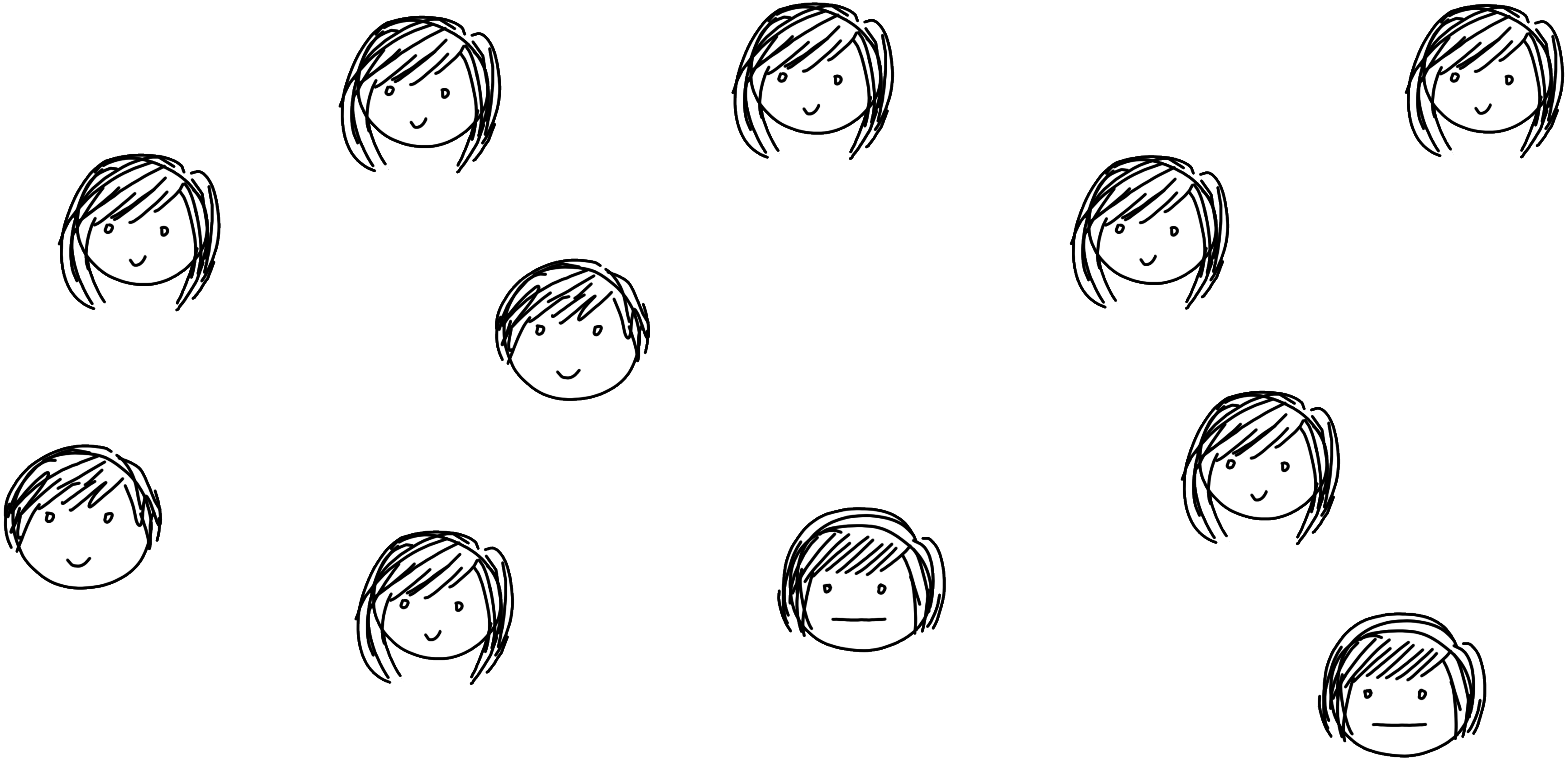


Adaptive corruptions

Adv picks corrupted parties at any time.

Adv



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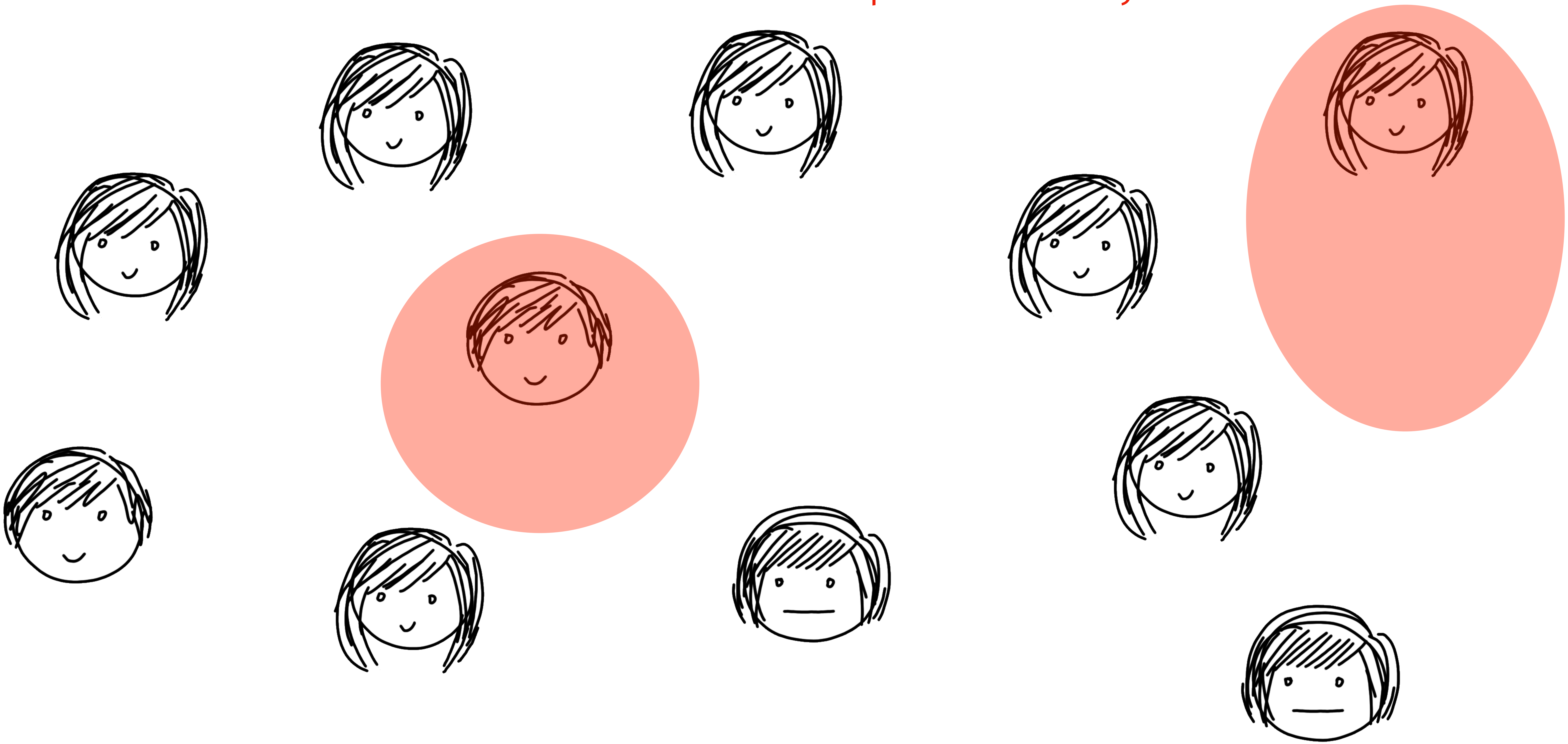

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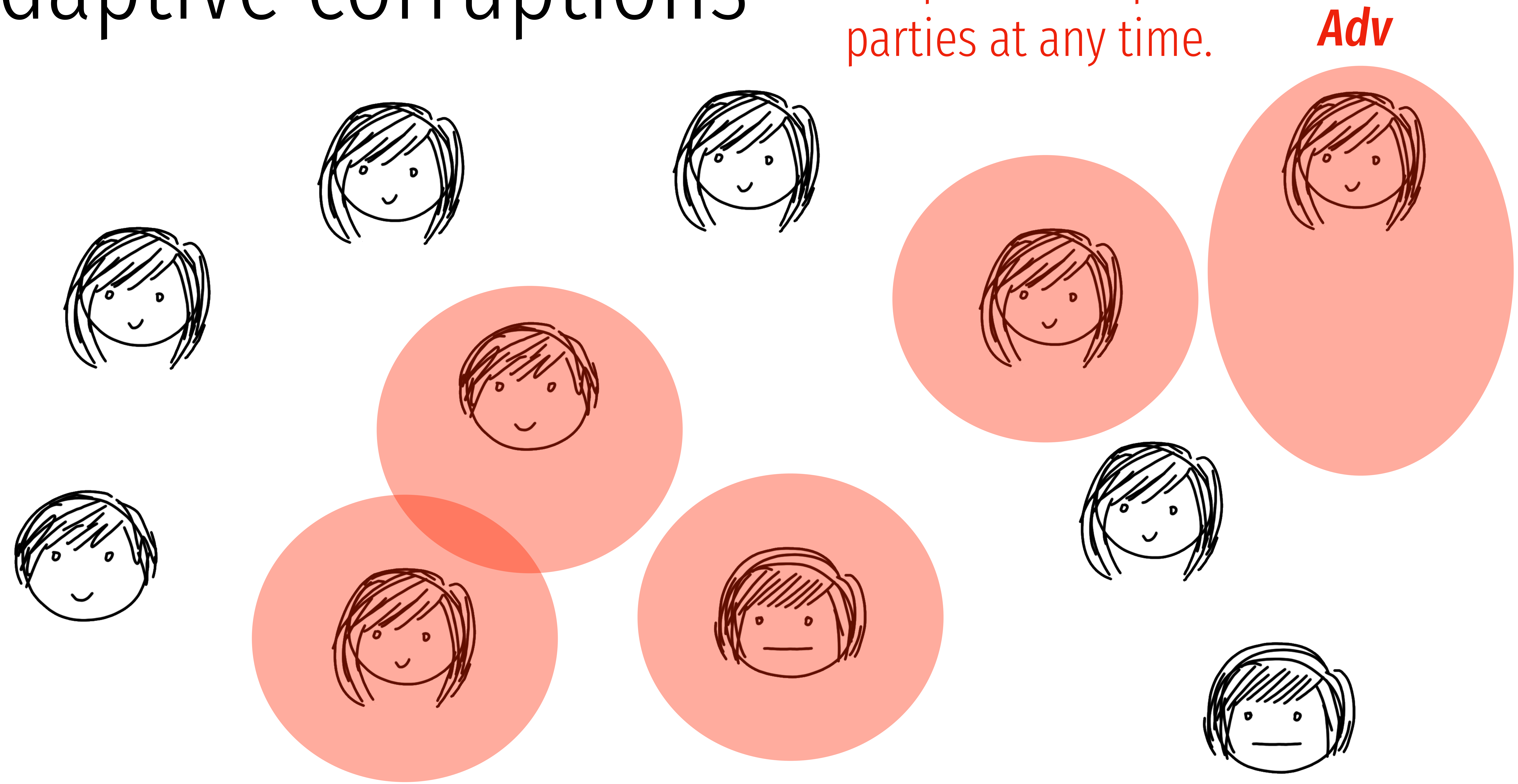
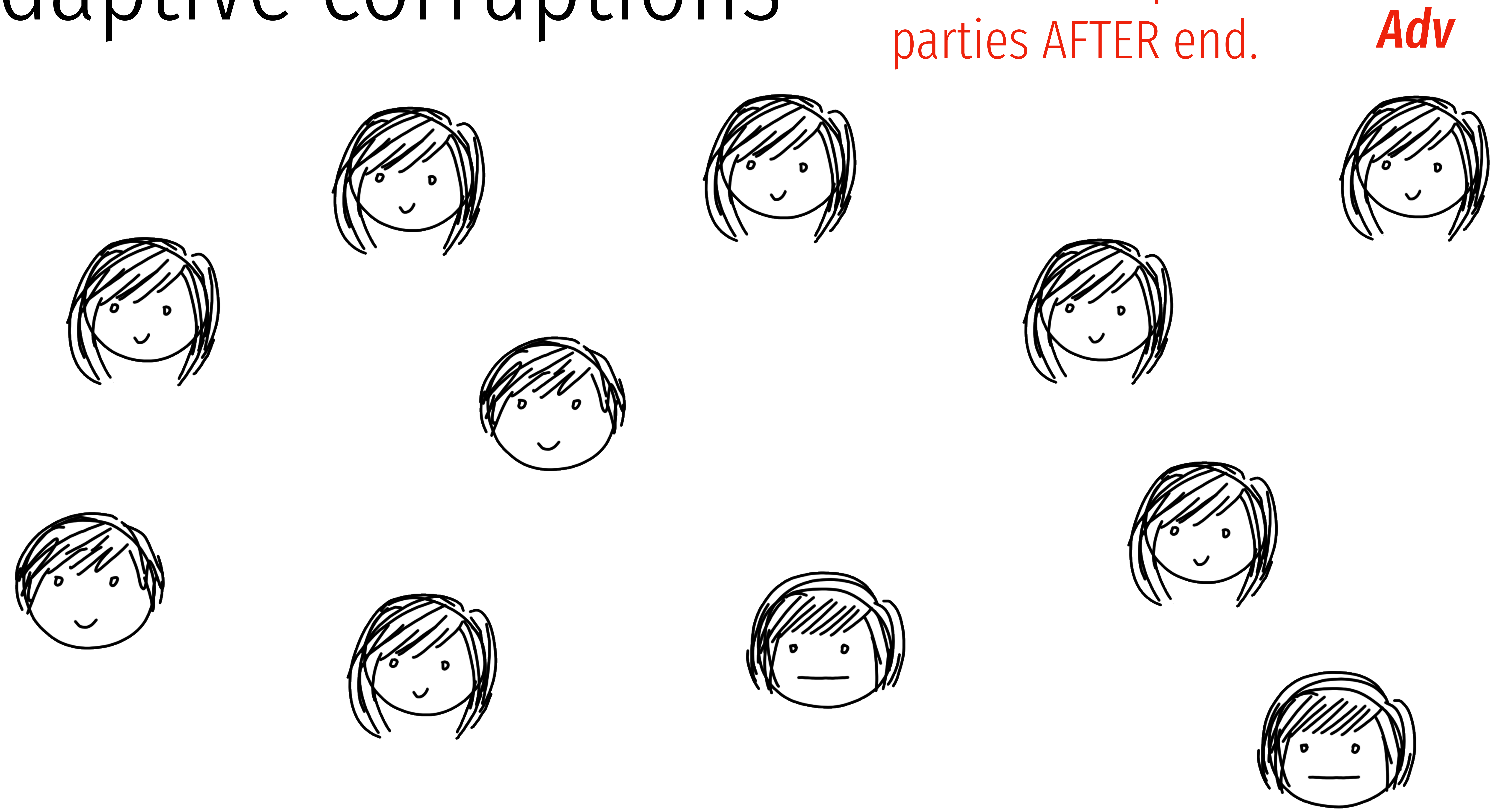


Adaptive corruptions

Adv can corrupt ALL parties AFTER end.



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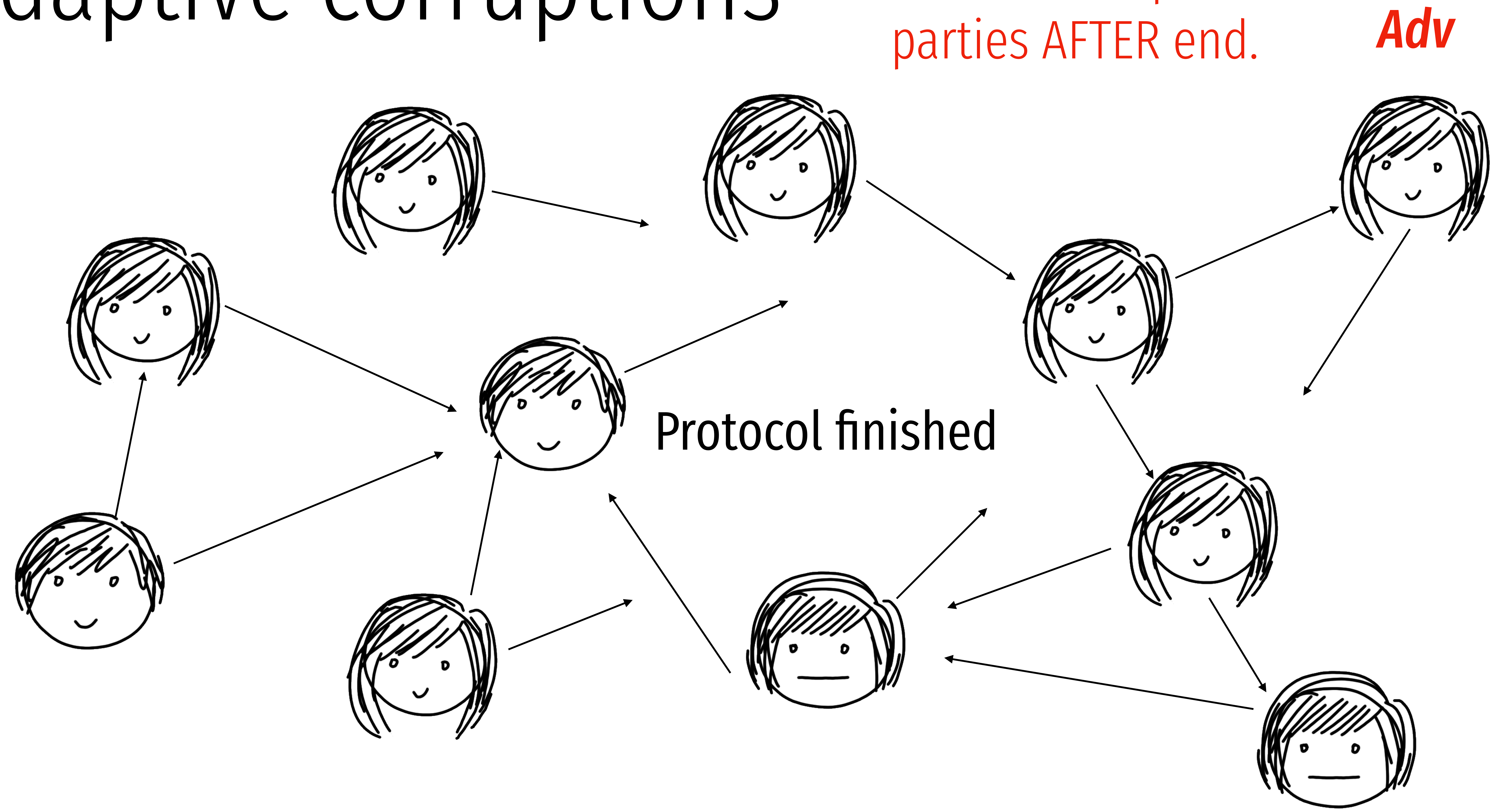

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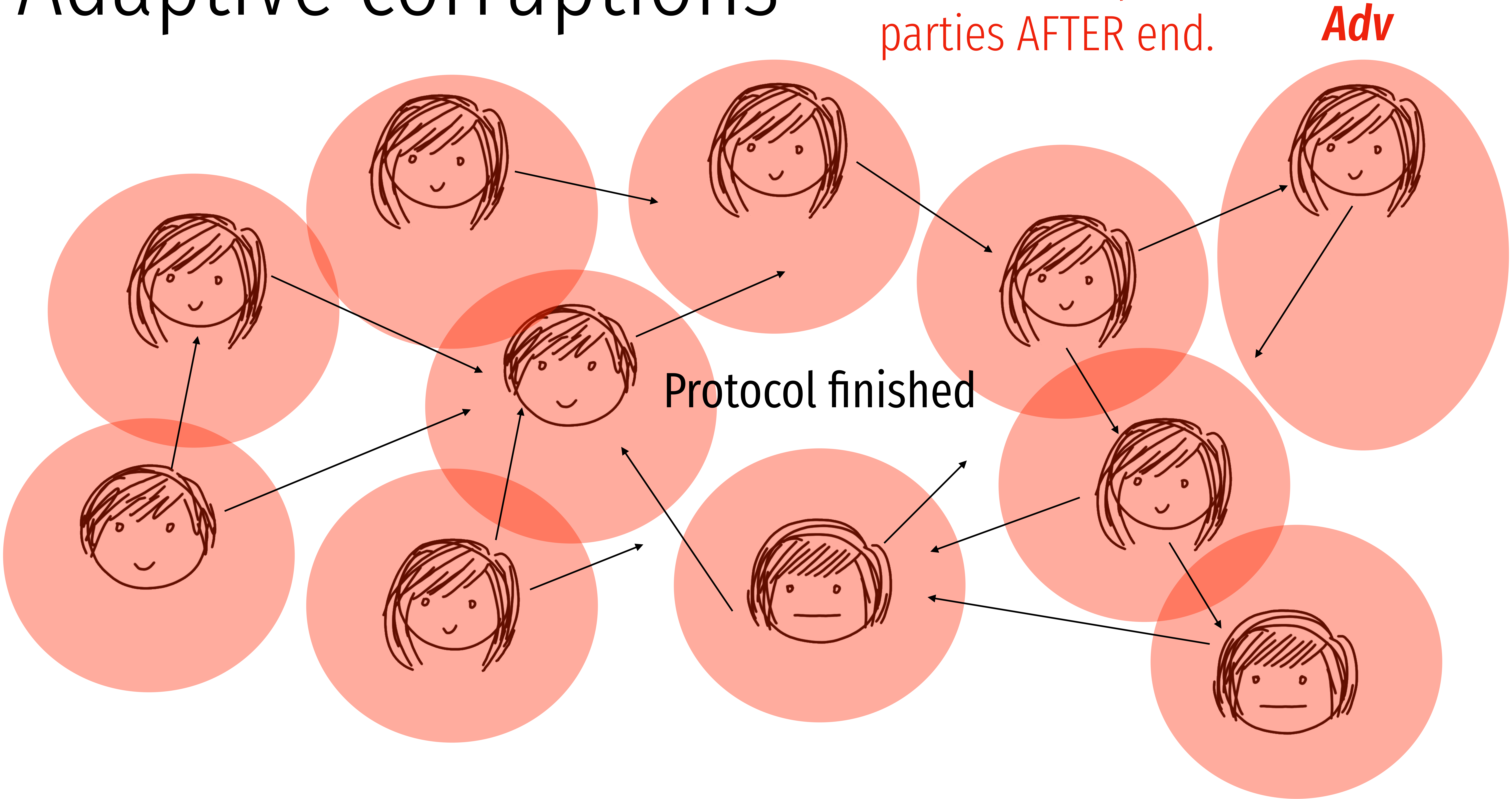

Adaptive corruptions

(without erasures)

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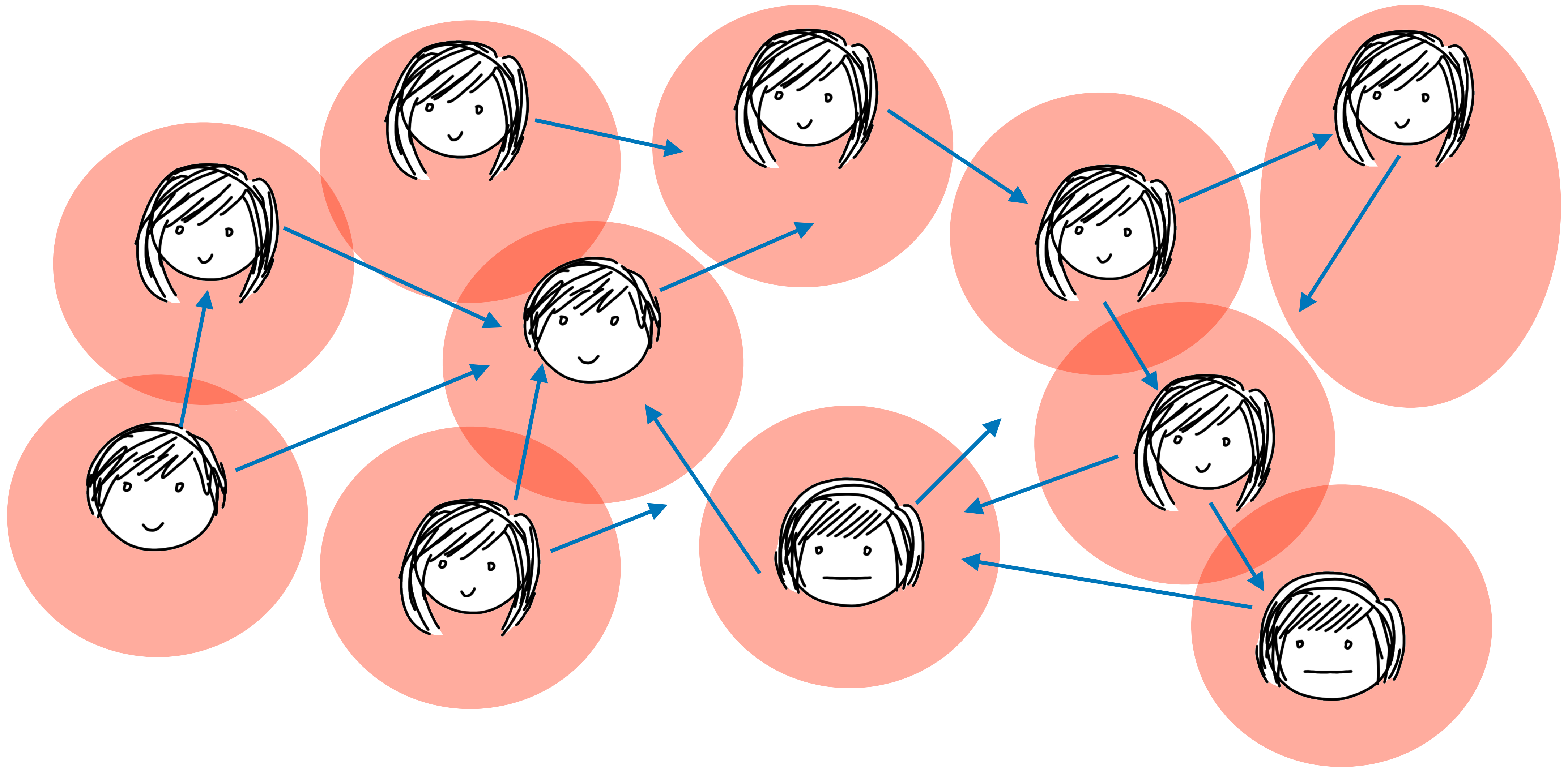

Simulator S must produce transcript T without knowing inputs or outputs.

After corruption, S learns inputs and outputs.

S must explain transcript T by producing random tapes for each party!

At what cost adaptive security?

Partial history (static)

GMW'87
O(d) rounds
OT

BMR'90
O(1) rounds
OT

AJLTVW'12
2 rounds, comm
LWE, NIZK,
Threshold-PKI

GGHR'14
2 rounds
iO, NIZK, CRS

MW'16
2 rounds, comm
LWE, NIZK, CRS

GS'18 / BL'18
2 rounds
OT, CRS

QWW'18
2 rounds
comm & online work
Adaptive-LWE, NIZK, CRS

Partial history (static)

CLOS'02
O(d) rounds
OT, CRS

DKR'15 / CGP'15
O(1) rounds
iO, OT, RefStr

GP'15
2 rounds, o(C) comm
iO,DenEnc,O(C) Ref

CPV'17
O(1) rounds
OT, CRS

BLPV'18
2 rounds, |C|
OT, CRS

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comm & online work
Adaptive-LWE, NIZK, CRS

RB'89
O(d)
rounds

BMR'90
O(1) rounds
OWF

AJLTVW'12
4 rounds, comm
LWE, NIZK,
Threshold-PKI

GLS'15
2 rounds, comm
LWE, NIZK,
Threshold-PKI

ACGJ'18
3 rounds
PKE, Zaps

Partial history (static)

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O(d) rounds
OT, CRS

DKR'15 / CGP'15
O(1) rounds
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2 rounds, o(C) comm
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O(1) rounds
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O(1) rounds
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2 rounds, comm
LWE, NIZK,
Threshold-PKI

GGHR'14
2 rounds
iO, NIZK, CRS

MW'16
2 rounds, comm
LWE, NIZK, CRS

GS'18 / BL'18
2 rounds
OT, CRS

QWW'18
2 rounds
comm & online work
Adaptive-LWE, NIZK, CRS

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O(d)
rounds

BMR'90
O(1) rounds
OWF

AJLTVW'12
4 rounds, comm
LWE, NIZK,
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GLS'15
2 rounds, comm
LWE, NIZK,
Threshold-PKI

ACGJ'18
3 rounds
PKE, Zaps

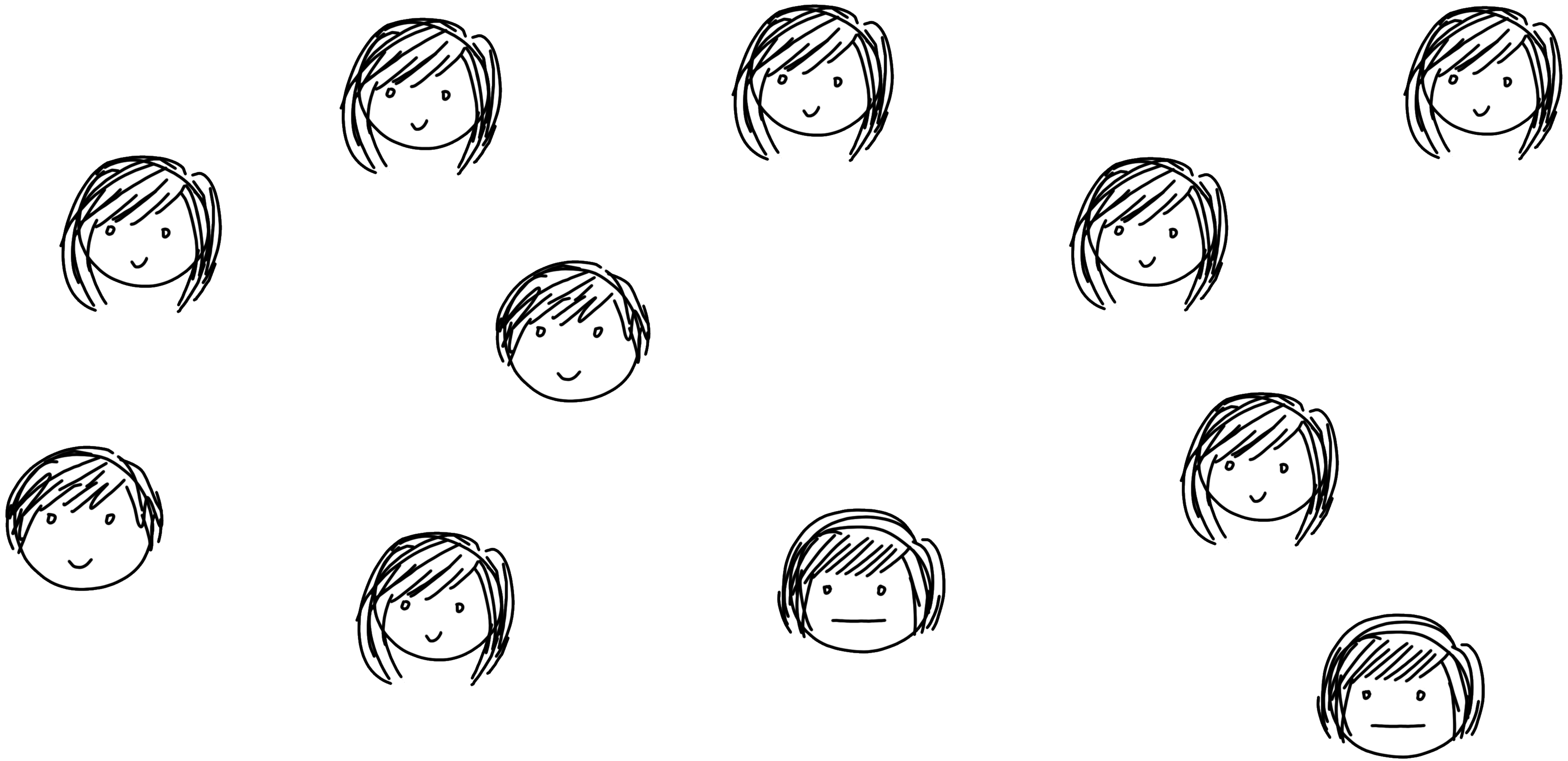
IPS'08
O(1) rounds
OT, CRS

DPR'16
3 rounds, o(C) comm
LWE, NIZK, Threshold-
PKI

CDDHR'99
O(d) rounds

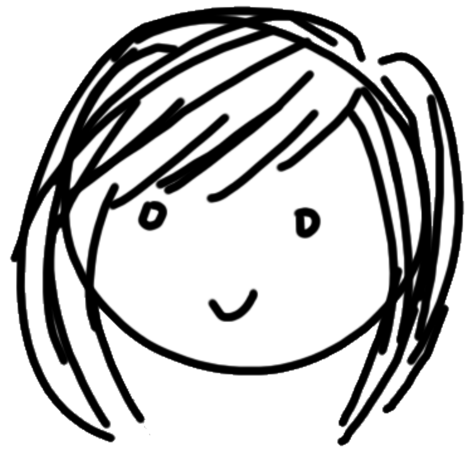
DI'05
O(1) rounds
OWF

Framework for 2-round sub-|C| MPC



Framework for 2-round sub-|C| MPC

pk, sk_i



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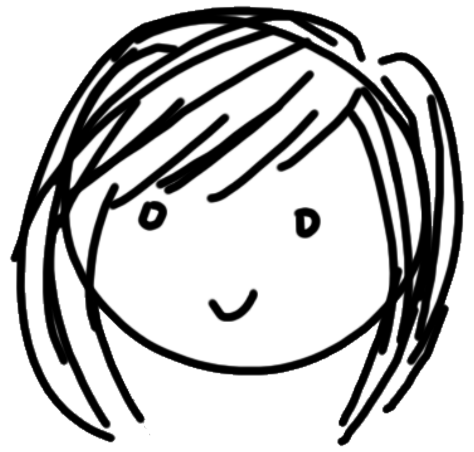

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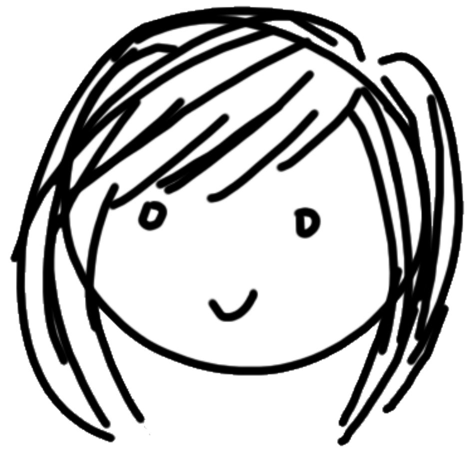

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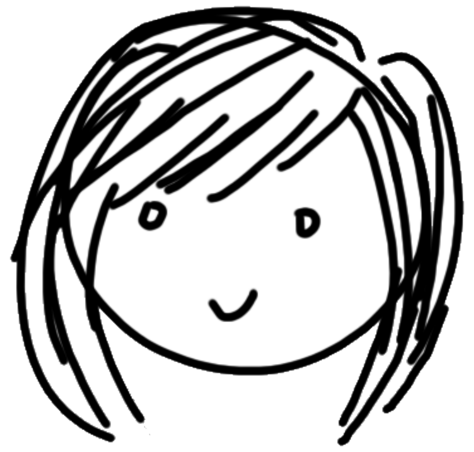

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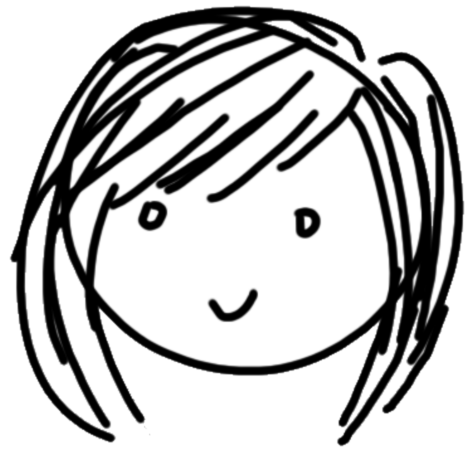

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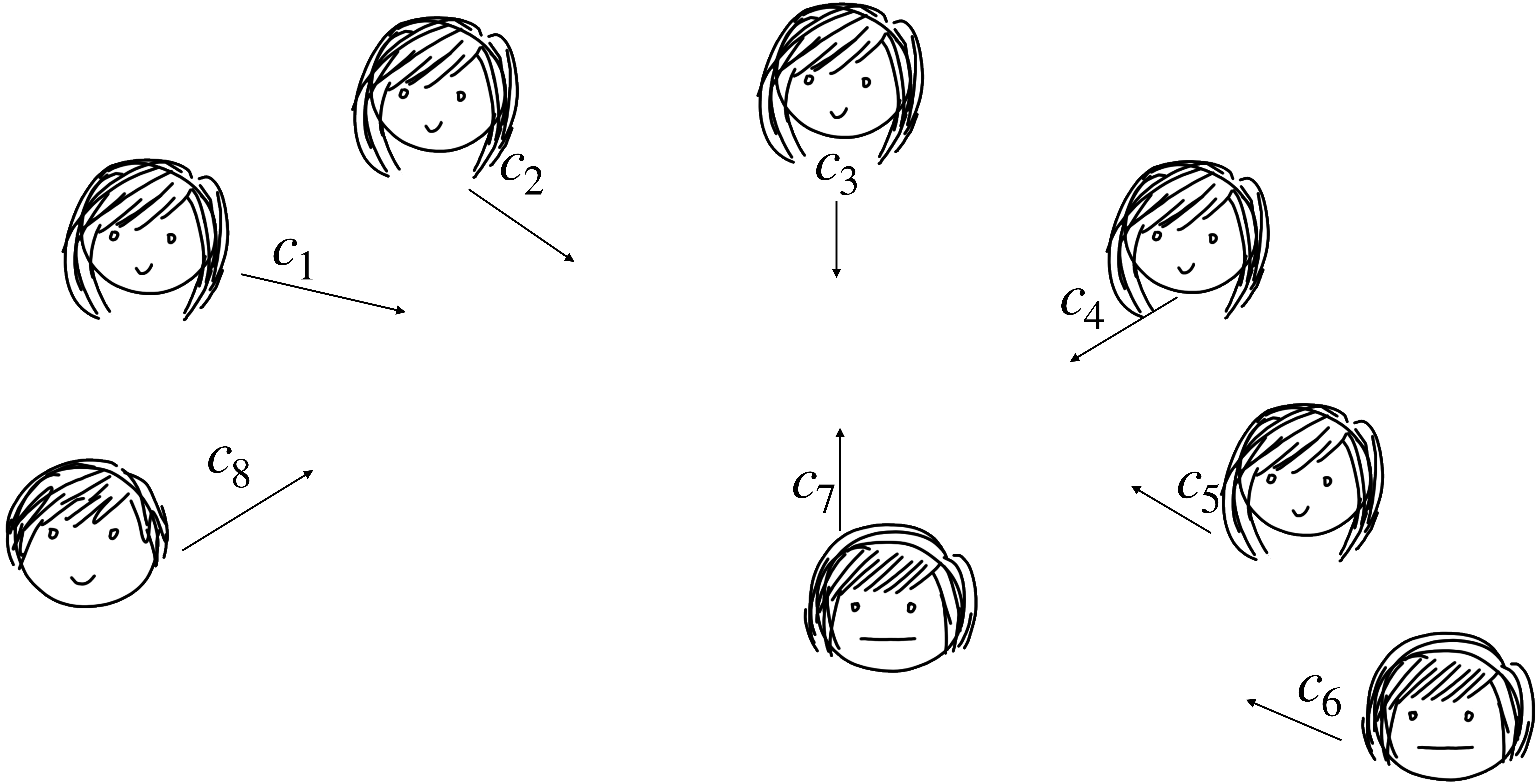

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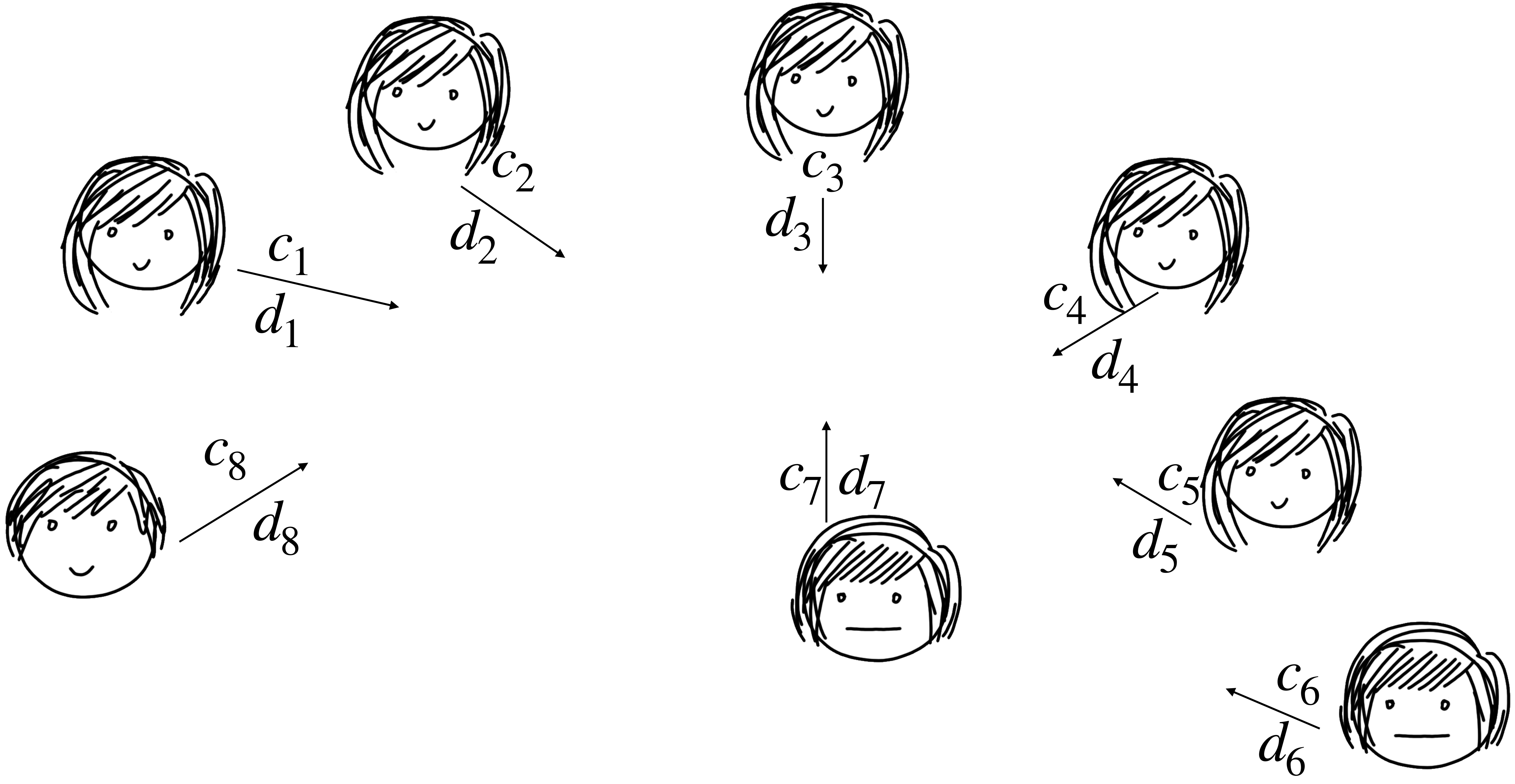

Adaptive Secure FHE

(sk, pk) <—- Gen(1^k)
Enc, Dec, Eval as usual

$\underline{\mathsf{Ideal}_{\mathcal{A},\mathcal{S}}(k)}$

$$
\begin{aligned}
&(pk, c_1, \ldots, c_\ell, s) \leftarrow \mathcal{S}_1(1^k);\\
&(m_1, \ldots, m_\ell, \tau) \leftarrow \mathcal{A}_1(1^k);\\
&sk \leftarrow \mathcal{S}_2(s, m_1, \ldots, m_\ell);\\
&b \leftarrow \mathcal{A}_2(\tau, pk, c_1, \ldots, c_\ell, sk);\\
&\mathit{Return}\ b.
\end{aligned}
$$

Adaptive Secure FHE Impossible Katz-Thiruvengadam-Zhou

$$(pk, c_1, \ldots, c_\ell, s) \leftarrow \mathcal{S}_1(1^k)$$

$$c' \leftarrow \text{Eval}_{\mathsf{pk}}(C_f, c_1, \ldots, c_\ell)$$

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Size of circuit computing f is:

Impossibility of adaptive FHE

Erasures don't help

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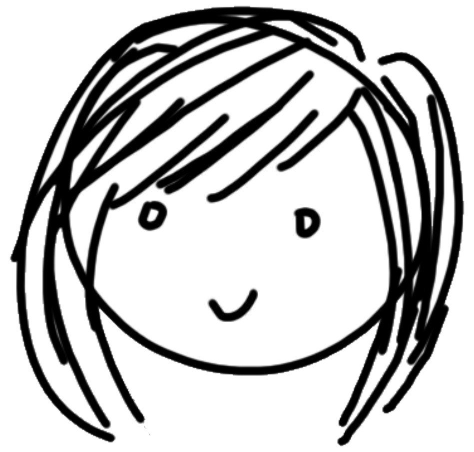

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Erase sk_i.

(receive d1,...,dn from everyone)

$f(x_1, \ldots, x_n) \leftarrow \text{Combine}(d_1, \ldots, d_n)$

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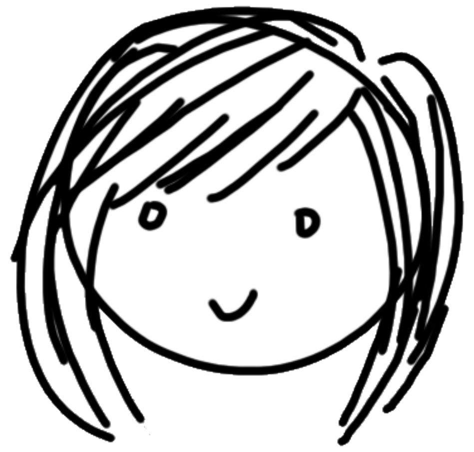

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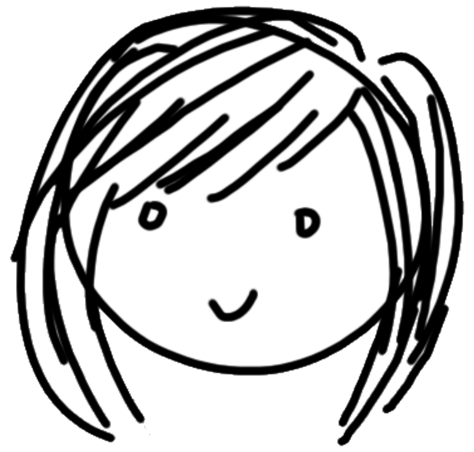

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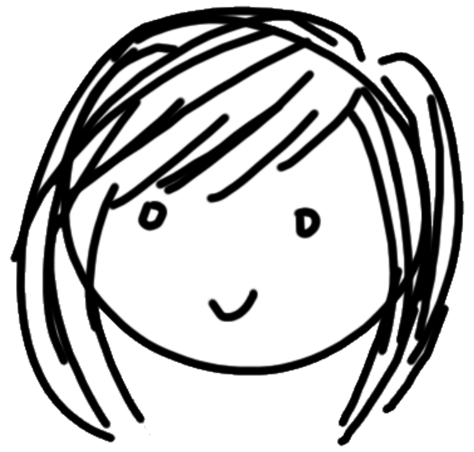

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Need new ideas for adaptive+succinct

Succinct
But not
Adaptive

Adaptive
but not
Succinct

Succinct
and
Adaptive

Laconic Function Evaluation (LFE)

Quach-Wee-Wichs'18

$$\mathsf{crs} \leftarrow \mathsf{LFE.crsGen}(1^\kappa, \mathsf{params})$$

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$$y = \mathsf{LFE.Dec}(\mathsf{crs}, C, r, \mathsf{ct})$$

LFE Avoids Impossibility

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Fully Adaptive Succinct MPC

$$\mathsf{crs} \leftarrow \mathsf{LFE.crsGen}(1^\kappa, f.\mathsf{params})$$

Succinct

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$$\mathcal{F}^{\mathsf{LFE.Enc}}_{\mathsf{sfe\text{-}abort}}(\mathsf{input}, \mathsf{sid}, (\mathsf{crs}, \mathsf{digest}_f, x_i, r_i)).$$

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Benhamouda-Lin-Polychroniado-Muthu

C_f

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C_f

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LFE is all-but-one adaptive secure.

Removing erasures

Explainability Compiler

Dachman-Soled—Katz-Rao'15

$$\mathsf{EC}(\mathrm{Alg}) \to (\widetilde{\mathrm{Alg}}, \mathsf{Explain})$$

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Poly-time overhead

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Correctness: $\mathrm{Alg}(x) \approx \widetilde{\mathrm{Alg}}(x) \quad \forall x$

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For any input/output (x,y), **Explain** produces coins r s.t. ~Alg(x,r) = y

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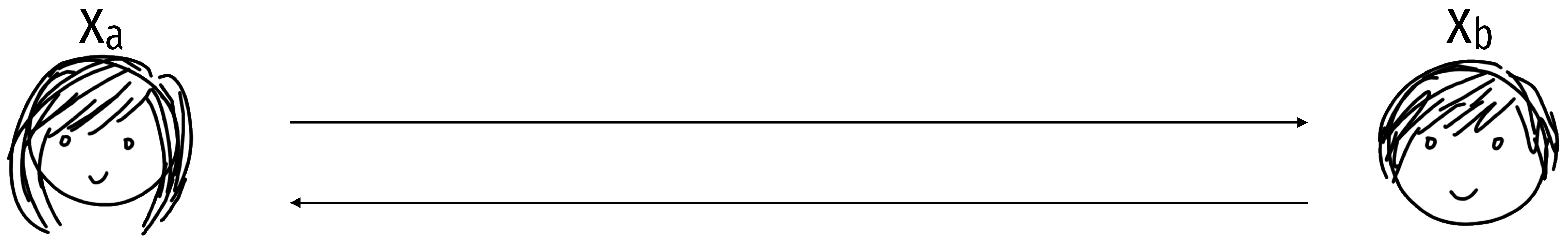
Corollary A.7. *Assuming the existence of an indistinguishable obfuscator for* P/poly *and of one-way functions, both with sub-exponential security, there exists an explainability compiler with adaptive security for* P/poly.

Fully-adaptive summary

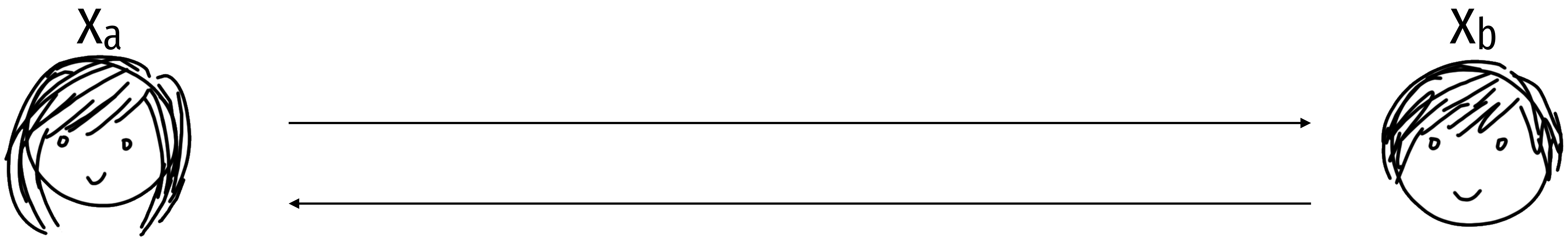
Protocol	Security (erasures)	Rounds	Communication	Online Computation	Setup size	Setup type	Assumption
MW [79]	static	2	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	$\text{poly}(\|C\|, \kappa)$	$\text{poly}(\kappa, d)$	CRS	LWE, NIZK
QWW [85] ABJMS [3]	static	2	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	$\text{poly}(\kappa, d)$	CRS	ALWE LWE
CLOS [24]	adaptive(no)	$O(d)$	$\|C\| \cdot \text{poly}(\kappa, n)$	$\text{poly}(\|C\|, \kappa)$	$\text{poly}(\kappa)$	CRS	TDP, NCE dense-crypto
GS [50]*	adaptive(no)	$O(d)$	$\|C\| \cdot \text{poly}(\kappa, n)$	$\text{poly}(\|C\|, \kappa)$	-	-	CRH TDP, NCE dense-crypto
DKR [40] CGP [27]	adaptive(no)	$O(1)$	$\|C\| \cdot \text{poly}(\kappa, n)$	$\text{poly}(\|C\|, \kappa)$	$\text{poly}(\|C\|, \kappa)$	Ref	OWF, iO
GP [49]	adaptive(no)	2	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, \kappa, n)$	$\text{poly}(\|C\|, \kappa)$	$\text{poly}(\|C\|, \kappa)$	Ref	OWF, iO
CPV [30]	adaptive(no)	$O(1)$	$\|C\| \cdot \text{poly}(\kappa, n)$	$\text{poly}(\|C\|, \kappa)$	$\text{poly}(\kappa)$	CRS	NCE dense-crypto
BLPV [13]	adaptive(no)	2	$\|C\| \cdot \text{poly}(\kappa, n)$	$\text{poly}(\|C\|, \kappa)$	$\text{poly}(\kappa)$	Ref	adaptive 2-round OT
This work	adaptive(yes) adaptive(no)	2	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	$\text{poly}(\kappa, d)$ $\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	CRS Ref	ALWE ALWE, iO

Alice-optimal



Alice learns $y = f(x_a, x_b)$

Alice-optimal



Alice learns $y = f(x_a, x_b)$

Comm: |xa| + |y|

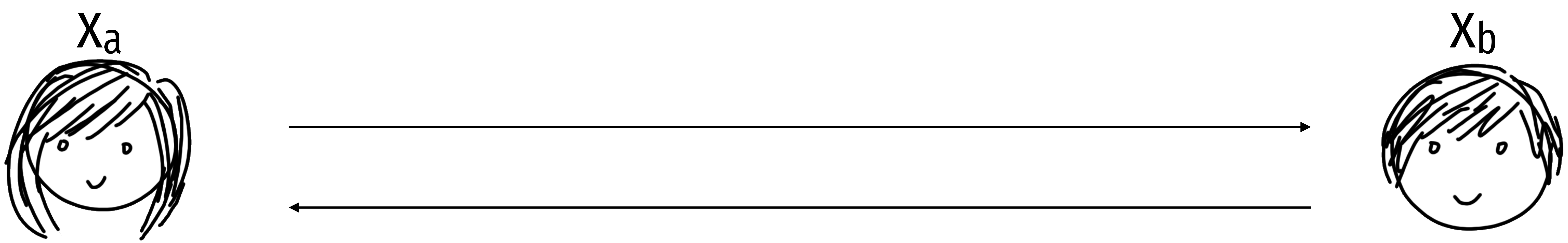
Comp: |f|

Comp: |xa| + |y|

Bob-optimal

Alice learns $y = f(x_a, x_b)$

Bob-optimal



Alice learns $y = f(x_a, x_b)$

Comp: $|f|$

Comm: $|x_b| + |y|$

Comp: $|x_b| + |y|$

Approach	**Security** (erasures)	**CRS**	**Communication** Alice	Bob	**Computation** Alice	Bob	**Assumptions**
GC [92]	static	-	ℓ_A	$\|f\|$	$\|f\|$	$\|f\|$	static OT
LOT [32]	static	$O(1)$	$O(1)$	$\|f\|$	$\|f\|$	$\|f\|$	DDH, etc.
FHE [52]	static	-	ℓ_A	ℓ_{out}	$\ell_A + \ell_{\mathsf{out}}$	$\|f\|$	LWE
LFE [85]	static	$O(1)$	$O(1)$	$\ell_B + \ell_{\mathsf{out}}$	$\|f\|$	$\ell_B + \ell_{\mathsf{out}}$	ALWE
equivocal GC [30]	adaptive (no)	-	ℓ_A	$\|f\|$	$\|f\|$	$\|f\|$	adaptive OT
This work	adaptive (yes)	$O(1)$	$O(1)$	$\ell_B + \ell_{\mathsf{out}}$	$\|f\|$	$\ell_B + \ell_{\mathsf{out}}$	ALWE
	adaptive (no)	$\ell_B + \ell_{\mathsf{out}}$	$O(1)$	$\ell_B + \ell_{\mathsf{out}}$	$\|f\|$	$\ell_B + \ell_{\mathsf{out}}$	ALWE and iO
	adaptive (yes)	$\|f\|$	$\|f\|$	$\ell_{\mathsf{out}} + o(\ell_B)$	$\|f\|$	$\|f\|$	impossible

Table 2: Comparison of two-message semi-honest protocols for $f : \{0,1\}^{\ell_A} \times \{0,1\}^{\ell_B} \to \{0,1\}^{\ell_{\mathsf{out}}}$. Alice talks first, Bob the second, and only Alice learns the output. For simplicity, multiplicative factors that are polynomial in the security parameter κ or the circuit depth d are suppressed.

At what cost lesser adaptive security?

Adaptive UC-NIZK

Groth-Ostrovsky-Sahai

Using bilinear pairings, Adaptive NIZK of size |C|*poly(k).

Succinct NIZK

Gentry-Groth-Ishai-Peikert-Sahai-Smith

NIZK crs

Prover(x,w)

sk,pk = FHE.Gen(r)

v_i = FHE.Enc_{pk}(w_i)

{v_i}, pi →

u* = FHE.$Eval_{pk}$(R,x,w_i,…w_i)

pi = Nizk{ FHE.Dec(sk,u*) = 1 }

Succinct + Adaptive NIZK ?

Homomorphic Trapdoor Function

Gorbunov-Vinod-Wichs

$$(pk, sk) \leftarrow \mathsf{HTDF.Gen}(1^k, 1^d)$$

$$f_{\mathsf{pk},x} : \mathcal{U} \to \mathcal{V}$$

$$\mathsf{HTDF.Inv}_{\mathsf{sk},x} : \check{\mathcal{V}} \to \mathcal{U}.$$

Homomorphic Trapdoor Function

Gorbunov-Vinod-Wichs

$$(pk, sk) \leftarrow \mathsf{HTDF.Gen}(1^k, 1^d)$$

$$f_{\mathsf{pk},x} : \mathcal{U} \to \mathcal{V},$$

$$\mathsf{HTDF.Inv}_{\mathsf{sk},x} : \mathcal{V} \to \mathcal{U}.$$

$$\mathsf{HTDF.Eval}^{\mathsf{in}}(g, (x_1, u_1), \ldots, (x_\ell, u_\ell))$$

$$v^* = \mathsf{HTDF.Eval}^{\mathsf{out}}(g, v_1, \ldots, v_\ell).$$

Impossibility doesn't apply to HTDF

$$(pk, c_1, \ldots, c_\ell, s) \leftarrow \mathcal{S}_1(1^k)$$

$$c' \leftarrow \mathrm{Eval}_{\mathsf{pk}}(C_f, c_1, \ldots, c_\ell)$$

Given input $m = (m_1, \ldots, m_\ell)$ compute f(m) as:

$$sk \leftarrow \mathcal{S}_2(s, m_1, \ldots, m_\ell);$$

$$f(m) \leftarrow \mathrm{Dec}_{sk}(c')$$

Size of circuit computing f is:

Succinct Adaptive NIZK

crs = HTDF.pk

Prover(x,w)

$v_i = \mathrm{HTDF}_{pk}(w_i)$

Succinct Adaptive NIZK

crs = HTDF.pk

Prover(x,w)

$v_i = \text{HTDF}_{pk}(w_i)$

$u^* = \text{HTDF.Eval}_{pk}(R,x,w_i,\ldots w_i)$

Succinct Adaptive NIZK

crs = HTDF.pk

Prover(x,w)

$v_i = HTDF_{pk}(w_i)$

$u^* = HTDF.Eval_{pk}(R,x,w_i,\ldots w_i)$

$v^* = HTDF.Eval_{pk}(R,x,v_i,..,v_i)$

Succinct Adaptive NIZK

crs = HTDF.pk

Prover(x,w)

v_i = $HTDF_{pk}(w_i)$

u* = $HTDF.Eval_{pk}(R,x,w_i,...w_i)$

v* = $HTDF.Eval_{pk}(R,x,v_i,..,v_i)$

pi = Adp-Nizk{$f_{pk}(u^*) = v^*$}

Succinct Adaptive NIZK

crs = HTDF.pk

Prover(x,w)

v_i = $HTDF_{pk}(w_i)$

$\{v_i\}$, pi →

u^* = $HTDF.Eval_{pk}(R,x,w_i,...w_i)$

v^* = $HTDF.Eval_{pk}(R,x,v_i,..,v_i)$

pi = Adp-Nizk$\{f_{pk}(u^*) = v^*\}$

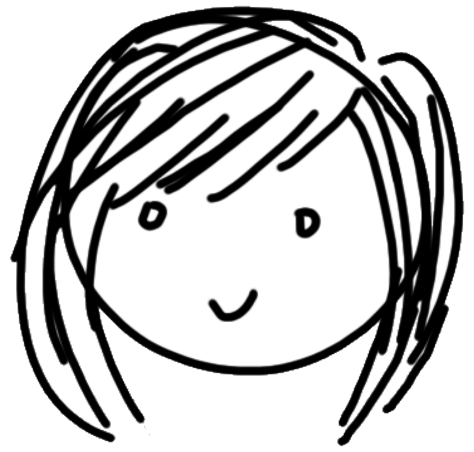
Adaptive NIZK

Protocol	Security (erasures)	CRS size	Proof size	Assumptions
Groth [60]	static	$\|C\| \cdot \text{poly}(\kappa)$	$\|C\| \cdot \text{poly}(\kappa)$	TDP
Groth [60]	static	$\|C\| \cdot \text{polylog}(\kappa) + \text{poly}(\kappa)$	$\|C\| \cdot \text{poly}(\kappa)$	Naccache-Stern
GOS [61]	adaptive (no)	$\text{poly}(\kappa)$	$\|C\| \cdot \text{poly}(\kappa)$	pairing based
Gentry [52]	adaptive (yes)	$\text{poly}(\kappa)$	$\|w\| \cdot \text{poly}(\kappa, d)$	LWE, NIZK
GGIPSS [56]	adaptive (yes)	$\text{poly}(\kappa)$	$\|w\| + \text{poly}(\kappa, d)$	LWE, NIZK
This work	adaptive (no)	$\text{poly}(\kappa)$	$\|w\| \cdot \text{poly}(\kappa, d)$	LWE, NIZK

Table 3: NIZK arguments with security parameter κ, for circuit size $|C|$, depth d, and witness size $|w|$.

All-but-one in 2 rounds

pk, sk_i



$c_i \leftarrow \mathsf{TEFHE.Enc}_{pk}(x_i), s = [0]$

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$

$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

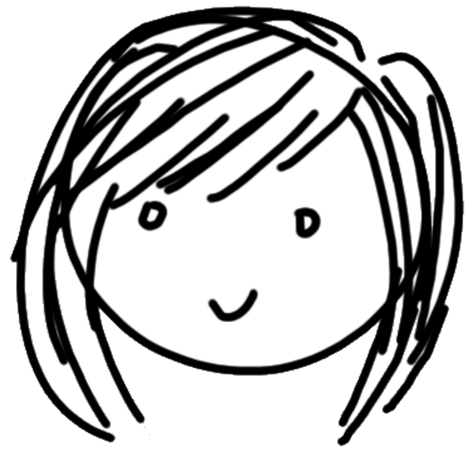
(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

+ Adaptive NIZK for malicious security

All-but-one in 2 rounds

pk, sk_i



$c_i \leftarrow \mathsf{TEFHE.Enc}_{pk}(x_i), s = [0]$

(receive c1,...,cn from everyone)

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$
$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

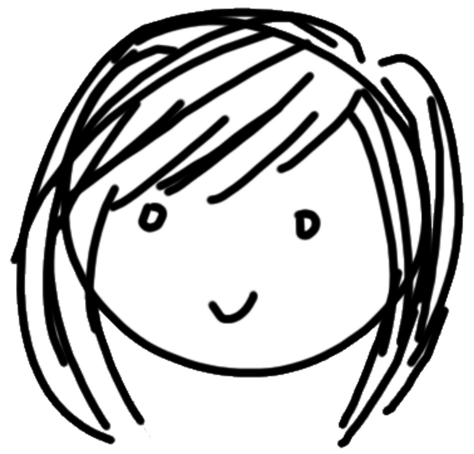
(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

+ Adaptive NIZK for malicious security

All-but-one in 2 rounds

pk, sk_i



$c_i \leftarrow \mathsf{TEFHE.Enc}_{pk}(x_i), s = [0]$

(receive c1,...,cn from everyone)

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$

$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

$d_i + s_i$

(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

+ Adaptive NIZK for malicious security

All-but-one corruptions

Protocol	Security	Rounds	Communication	Assumptions	Setup
AJLTVW [5]	static	2 3	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI CRS
MW [79]	static	2	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	LWE, NIZK	CRS
IPS [70]	adaptive	$O(1)$	$\|C\| + \mathrm{poly}(d, \log \|C\|, \kappa, n)$	OT-hybrid	-
GS [50]	adaptive	$O(1)$	$\|C\| + \mathrm{poly}(d, \log \|C\|, \kappa, n)$	CRH, TDP, NCE dense crypto	-
DPR [45]	adaptive	3	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI
This work	adaptive	2 4	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI CRS

Table 4: Comparison of maliciously secure MPC for $f : (\{0,1\}^{\ell_{\mathsf{in}}})^n \to \{0,1\}^{\ell_{\mathsf{out}}}$ represented by a circuit C of depth d, tolerating $n-1$ corruptions. $(*)$ The results in [50] only hold in the stand-alone model.

Honest majority results

Protocol	Security	Rounds	Communication	Assumptions	Setup
AJLTVW [5]	static	4 5	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI CRS
GLS [59]	static	2 3	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI CRS
ACGJ [4]	static	3	$\|C\| \cdot \text{poly}(\kappa, n)$	PKE and zaps	-
BJMS [6]	static	2 3	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, zaps, dense crypto	threshold PKI -
DI [41]	adaptive	$O(1)$	$\|C\| \cdot \text{poly}(\kappa, n)$	OWF	-
This work	adaptive	2 $O(1)$	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI -

Table 5: Comparison of maliciously secure MPC for $f : (\{0,1\}^{\ell_{\text{in}}})^n \to \{0,1\}^{\ell_{\text{out}}}$ represented by circuit C of depth d, in the honest-majority setting.

Open questions

Are erasures/io necessary for adaptive succinct MPC?

Protocol	Security (erasures)	Rounds	Communication	Online Computation	Setup size	Setup type	Assumption
MW [79]	static	2	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	$\mathrm{poly}(\kappa, d)$	CRS	LWE, NIZK
QWW [85] ABJMS [3]	static	2	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathrm{poly}(\kappa, d)$	CRS	ALWE LWE
CLOS [24]	adaptive(no)	$O(d)$	$\lvert C\rvert \cdot \mathrm{poly}(\kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	$\mathrm{poly}(\kappa)$	CRS	TDP, NCE dense-crypto
GS [50]*	adaptive(no)	$O(d)$	$\lvert C\rvert \cdot \mathrm{poly}(\kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	-	-	CRH TDP, NCE dense-crypto
DKR [40] CGP [27]	adaptive(no)	$O(1)$	$\lvert C\rvert \cdot \mathrm{poly}(\kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	Ref	OWF, iO
GP [49]	adaptive(no)	2	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, \kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	Ref	OWF, iO
CPV [30]	adaptive(no)	$O(1)$	$\lvert C\rvert \cdot \mathrm{poly}(\kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	$\mathrm{poly}(\kappa)$	CRS	NCE dense-crypto
BLPV [13]	adaptive(no)	2	$\lvert C\rvert \cdot \mathrm{poly}(\kappa, n)$	$\mathrm{poly}(\lvert C\rvert, \kappa)$	$\mathrm{poly}(\kappa)$	Ref	adaptive 2-round OT
This work	adaptive(yes) adaptive(no)	2	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathrm{poly}(\kappa, d)$ $\mathrm{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	CRS Ref	ALWE ALWE, iO

Open questions

Are erasures/io necessary for adaptive succinct MPC?

Are Ref strings/erasures necessary for fully adaptive succinct MPC?

Protocol	Security (erasures)	Rounds	Communication	Online Computation	Setup size	Setup type	Assumption
MW [79]	static	2	$\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	$\mathsf{poly}(\kappa, d)$	CRS	LWE, NIZK
QWW [85] ABJMS [3]	static	2	$\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathsf{poly}(\kappa, d)$	CRS	ALWE LWE
CLOS [24]	adaptive(no)	$O(d)$	$\lvert C \rvert \cdot \mathsf{poly}(\kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	$\mathsf{poly}(\kappa)$	CRS	TDP, NCE dense-crypto
GS [50]*	adaptive(no)	$O(d)$	$\lvert C \rvert \cdot \mathsf{poly}(\kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	-	-	CRH TDP, NCE dense-crypto
DKR [40] CGP [27]	adaptive(no)	$O(1)$	$\lvert C \rvert \cdot \mathsf{poly}(\kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	Ref	OWF, iO
GP [49]	adaptive(no)	2	$\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, \kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	Ref	OWF, iO
CPV [30]	adaptive(no)	$O(1)$	$\lvert C \rvert \cdot \mathsf{poly}(\kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	$\mathsf{poly}(\kappa)$	CRS	NCE dense-crypto
BLPV [13]	adaptive(no)	2	$\lvert C \rvert \cdot \mathsf{poly}(\kappa, n)$	$\mathsf{poly}(\lvert C \rvert, \kappa)$	$\mathsf{poly}(\kappa)$	Ref	adaptive 2-round OT
This work	adaptive(yes) adaptive(no)	2	$\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	$\mathsf{poly}(\kappa, d)$ $\mathsf{poly}(\ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, \kappa, n)$	CRS Ref	ALWE ALWE, iO

Open questions

Are erasures/io necessary for adaptive succinct MPC?

Are Ref strings/erasures necessary for fully adaptive succinct MPC?

Are setup relaxations possible for all-but-one adaptive succinct MPC?

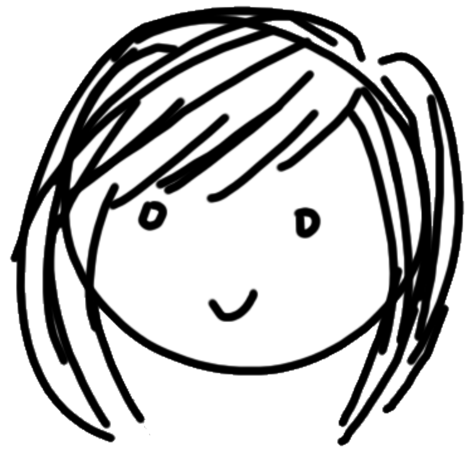
Protocol	Security	Rounds	Communication	Assumptions	Setup
AJLTVW [5]	static	2 3	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI CRS
MW [79]	static	2	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	CRS
IPS [70]	adaptive	$O(1)$	$\lvert C\rvert + \text{poly}(d, \log \lvert C\rvert, \kappa, n)$	OT-hybrid	-
GS [50]	adaptive	$O(1)$	$\lvert C\rvert + \text{poly}(d, \log \lvert C\rvert, \kappa, n)$	CRH, TDP, NCE dense crypto	-
DPR [45]	adaptive	3	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI
This work	adaptive	2 4	$\text{poly}(\ell_{\text{in}}, \ell_{\text{out}}, d, \kappa, n)$	LWE, NIZK	threshold PKI CRS

Table 4: Comparison of maliciously secure MPC for $f : (\{0,1\}^{\ell_{\text{in}}})^n \to \{0,1\}^{\ell_{\text{out}}}$ represented by a circuit C of depth d, tolerating $n-1$ corruptions. $(*)$ The results in [50] only hold in the stand-alone model.

All-but-one corruptions prior work

Damgard-Polychroniadou-Rao

pk, sk_i



$c_i \leftarrow \mathsf{EquivFHE.Enc}_{pk}(x_i)$

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$

$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

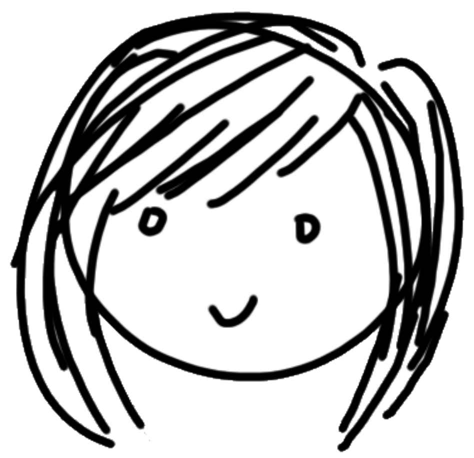
(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

All-but-one corruptions prior work

Damgard-Polychroniadou-Rao

pk, sk_i



$c_i \leftarrow \mathsf{EquivFHE.Enc}_{pk}(x_i)$

(receive c1,...,cn from everyone)

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$

$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

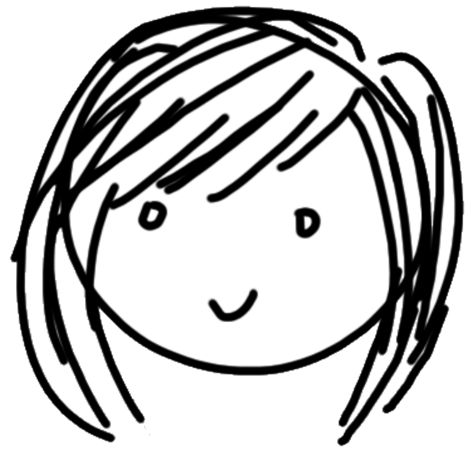
(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

All-but-one corruptions prior work

Damgard-Polychroniadou-Rao

pk, sk_i



$c_i \leftarrow \mathsf{EquivFHE.Enc}_{pk}(x_i)$

(receive c1,...,cn from everyone)

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$

$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

d_i

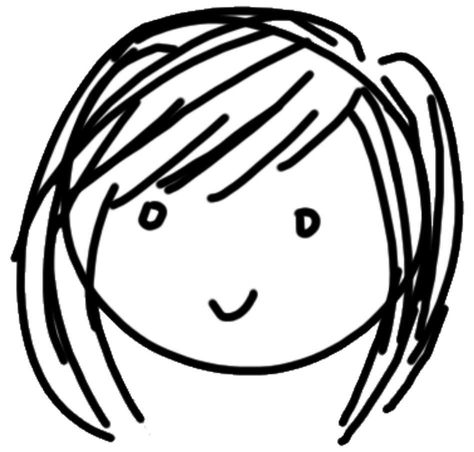
(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

All-but-one corruptions prior work

Damgard-Polychroniadou-Rao

pk, sk_i



$c_i \leftarrow \mathsf{EquivFHE.Enc}_{pk}(x_i)$ →

(receive c1,...,cn from everyone)

$y \leftarrow \mathsf{Eval}_{pk}(f, c_1, c_2, \ldots, c_n)$

$r_i \leftarrow \mathsf{EquivFhe.Enc}(0)$ →

$d_i \leftarrow \mathrm{Dec}_{sk_i}(y + r_i)$

d_i →

(receive from everyone)

$f(x_1, \ldots, x_n) \leftarrow \mathsf{Combine}(d_1, \ldots, d_n)$

Adaptive LWE

- *The Challenger picks k random matrices $A_i \leftarrow \mathbb{Z}_q^{n\times m}$ for $i \in [k]$, and sends them to $\mathcal{A}$.*
- *$\mathcal{A}$ adaptively picks $x_1, \ldots, x_k \in \{0,1\}$, and sends it to the Challenger.*
- *The Challenger samples $\boldsymbol{s} \leftarrow \mathbb{Z}_q^n$ and computes for all $i \in [k]$*

$$\begin{cases} \boldsymbol{b}_i = \boldsymbol{s}^T(\boldsymbol{A}_i - x_i \cdot \boldsymbol{G}) + \boldsymbol{e}_i \text{ where } \boldsymbol{e}_i \leftarrow \chi^m, & \text{if } \beta = 0. \\ \boldsymbol{b}_i \leftarrow \mathbb{Z}_q^m, & \text{if } \beta = 1. \end{cases}$$

The Challenger also picks $\boldsymbol{A}_{k+1} \leftarrow \mathbb{Z}_q^{n\times m'}$ and computes

$$\begin{cases} \boldsymbol{b}_{k+1} = \boldsymbol{s}^T \boldsymbol{A}_{k+1} + \boldsymbol{e}_{k+1} \text{ where } \boldsymbol{e}_{k+1} \leftarrow \chi^{m'}, & \text{if } \beta = 0. \\ \boldsymbol{b}_{k+1} \leftarrow \mathbb{Z}_q^{m'}, & \text{if } \beta = 1. \end{cases}$$

The challenger sends $\boldsymbol{A}_{k+1}$ and $\{\boldsymbol{b}_i\}_{i\in[k+1]}$ to the adversary.

HTDF

- **Correctness.** Let $x_1, \ldots, x_\ell \in \{0,1\}$ and $v_i = f_{pk,x_i}(u_i)$ for $i \in [\ell]$. Then, for $u^* = \mathsf{HTDF.Eval}^{\mathsf{in}}(g, (x_1, u_1), \ldots, (x_\ell, u_\ell))$ and $v^* = \mathsf{HTDF.Eval}^{\mathsf{out}}(g, v_1, \ldots, v_\ell)$ it holds that $f_{pk,y}(u^*) = v^*$, where $y = g(x_1, \ldots, x_\ell)$.

- **Distributional equivalence of inversion.** For a bit $x \in \{0,1\}$, the tuple (pk, x, u, v) computed as $v = f_{\mathsf{pk},x}(u)$ for a random $u \leftarrow \mathcal{U}$ is statistically close to sampling $v \leftarrow \mathcal{V}$ at random and computing $u = \mathsf{HTDF.Inv}_{\mathsf{sk},x}(v)$.

- **Claw-free security.** Given the public key, no efficient adversary can come up with u and u' such that $f_{\mathsf{pk},0}(u) = f_{\mathsf{pk},1}(u')$ with more than a negligible probability.

Full adaptive case

Theorem 4.1 (Theorem 1.1, secure-erasures version, restated)**.** *Assume the existence of LFE schemes for* $\mathsf{P/poly}$*, of 2-round adaptively and maliciously secure OT, and of secure erasures, and let* $f : (\{0,1\}^{\ell_{\mathsf{in}}})^n \to \{0,1\}^{\ell_{\mathsf{out}}}$ *be an* n*-party function of depth* d*.*

Then, $\mathcal{F}^f_{\mathsf{sfe\text{-}abort}}$ *can be UC-realized tolerating a malicious, adaptive PPT adversary by a 2-round protocol in the common random string model. The size of the common random string is* $\mathrm{poly}(\kappa, d)$*, whereas the communication and online-computational complexity of the protocol are* $\mathrm{poly}(\kappa, \ell_{\mathsf{in}}, \ell_{\mathsf{out}}, d, n)$*.*