On the Round Complexity of Randomized Byzantine Agreement

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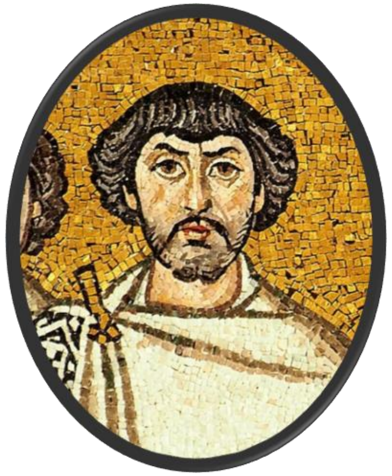

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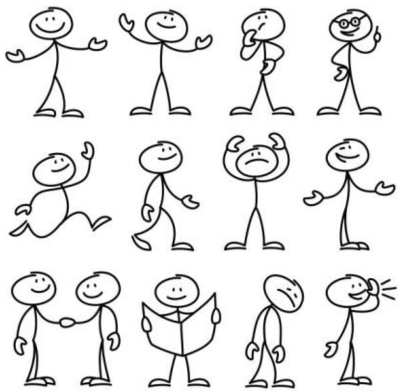
Definition of Byzantine Agreement (BA)

[Pease-Lamport-Shostak'80, Lamport-Shostak-Pease'82]

- Each P_i holds input $v_i \in \{0,1\}$.
- Agreement: All honest parties output the same bit.
- Validity: $\exists\ i$ s.t. (honest) P_i outputs v_i.



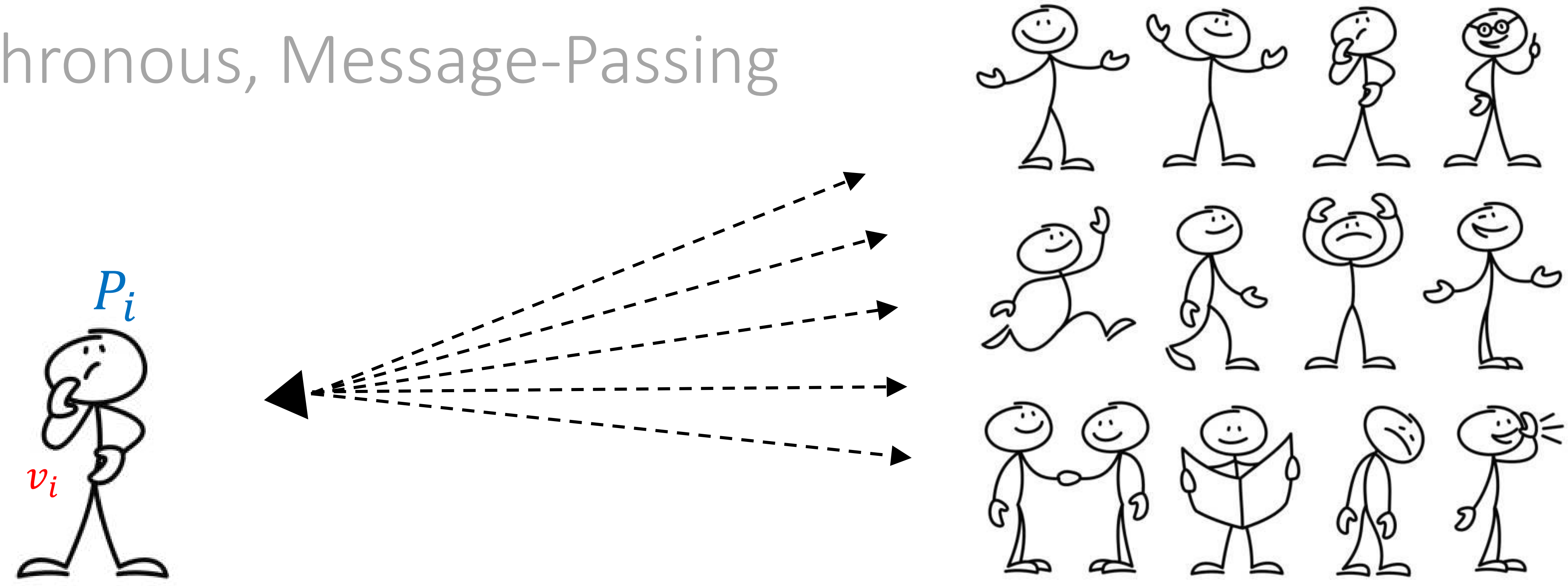
BA is very closely related to **Broadcast**

Sender sends a message to many receivers s.t. all receivers agree on the message

- ➢ Fault-tolerant distributed systems
- ➢ Cryptography (Multi-Party Computation)
- ➢ Blockchain (Cryptocurrencies)

Model

Synchronous, Message-Passing



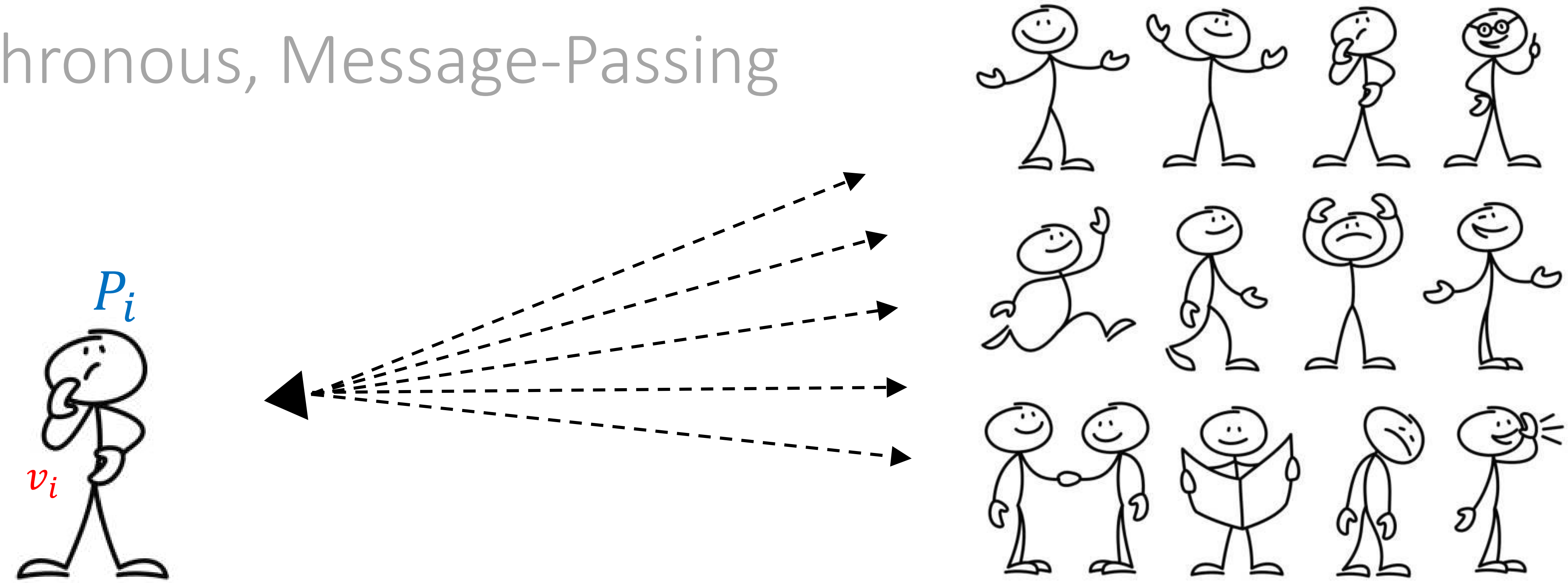
- Parties are connected in a point-to-point network with *synchronous rounds*.

  Round := every party sending a message to all other parties.

- Allow Setup (e.g. Digital signatures, PKI).

Model

Synchronous, Message-Passing



Problem Statement:

What is the minimal number of (expected) rounds needed to reach Byzantine Agreement?

When facing t-out-of-n corrupted players.

Previous Results

Synchronous, Message-Passing

t = security threshold

Deterministic protocols:

- $\#\text{Rounds} = t + 1$ [Lamport-Shostak-Pease’82 , Dolev-Strong’83, Garay-Moses’93]
- $\#\text{Rounds} \geq t + 1$ [Fischer-Lynch’82, Dolev-Strong’83]

Randomized protocols:

- Constant-round impossibility
  [Chor-Merritt-Shmoys’85, Karlin-Yao’84]
- Expected constant-round BA
  [BenOr’83, Rabin’83, Feldman-Micali’88, Katz-Koo’06, Micali’17]
  [Micali-Vaikuntanathan’17, Abraham-Devadas-Dolev-Nayak-Ren’18]
  [Abraham-Chan-Dolev-Nayak-Pass-Ren-Shi’19]

Our Work

We prove bounds on the halting probability after 1 and 2 rounds.

Micali's BA (ITCS'17) halts after 3 rounds with constant probability.

We Show

For every BA resilient against $t = n/3$ corruptions

Halting Probability in round 1	Halting Probability in round 2
$o(1) \approx 0$	$1 - \Theta(1) \ll 1$

Under plausible combinatorial assumption:

Halting Probability in round 2
$o(1) \approx 0$

Outline

1. Adversarial Model
   Local Consistent Adversaries
2. Our Attack(s)
   i. 1^{st} round halting
   ii. 2^{nd} round halting

Adversarial Model

Adversarial Model

Locally Consistent Adversaries

- Efficient (PPTM) limited to the following adversarial behavior
  i. Adversary corrupts a subset of parties
  ii. Corrupted parties may send conflicting inputs to honest parties
  iii. Adversary may abort (some corrupted parties) at any given round

Adversary may **not**
- Manipulate randomness
- Lie about (honest) incoming messages

Adversarial Model

Locally Consistent Adversaries

- Efficient (PPTM) limited to the following adversarial behavior
  i. Adversary corrupts a subset of parties
  ii. Corrupted parties may send conflicting inputs to honest parties
  iii. Adversary may abort (some corrupted parties) at any given round
- We show lower bounds via locally consistent attacks

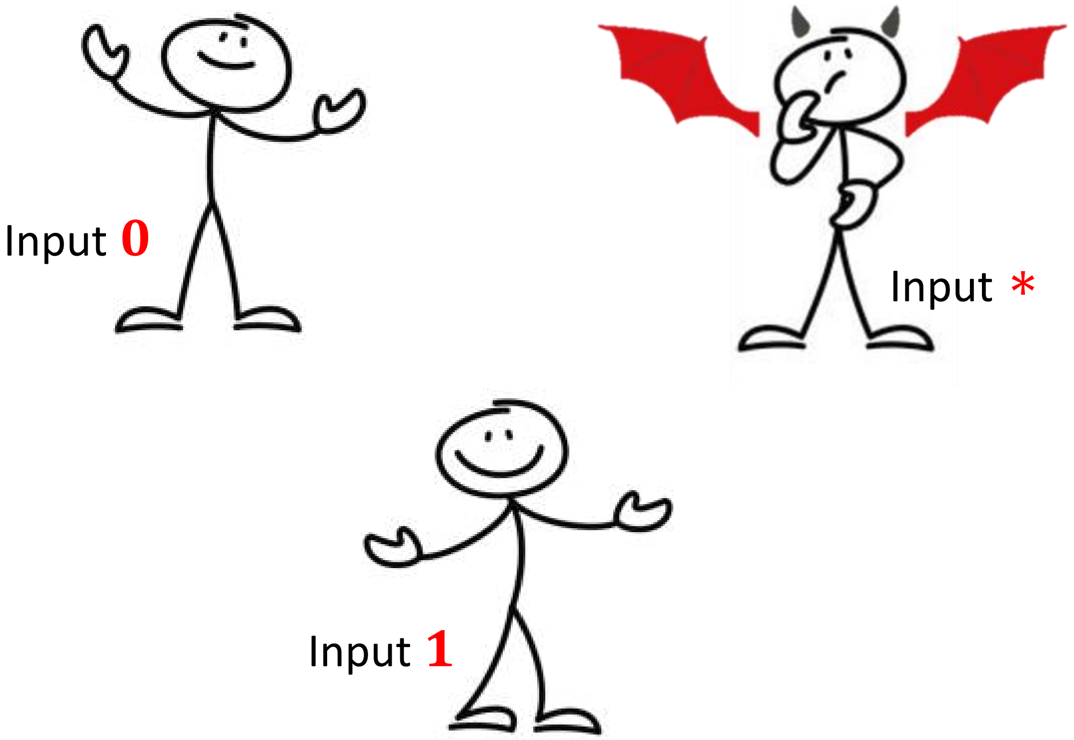
- On the positive (protocols) side

Additional Contribution (See Full Version of the Paper)

Locally consistent security $\Longrightarrow$ Malicious security

Our Attack
1st Round Halting

Lower bound for 1st Round Halting

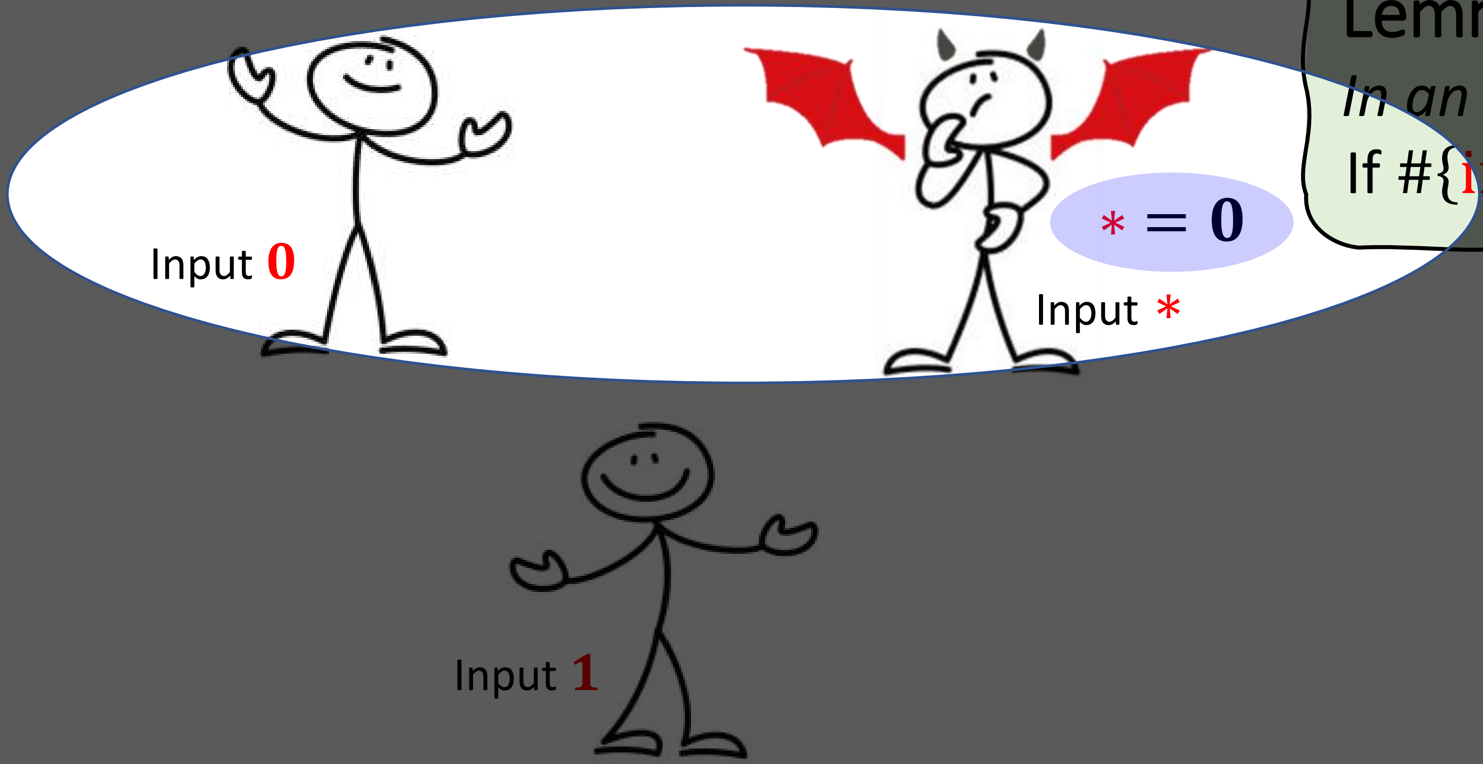


Lemma (Folklore)

In an honest *execution*:

If $\#\{\text{inputs} = z\} \geq 2n/3$ then $\text{output} = z$

Lower bound for 1st Round Halting



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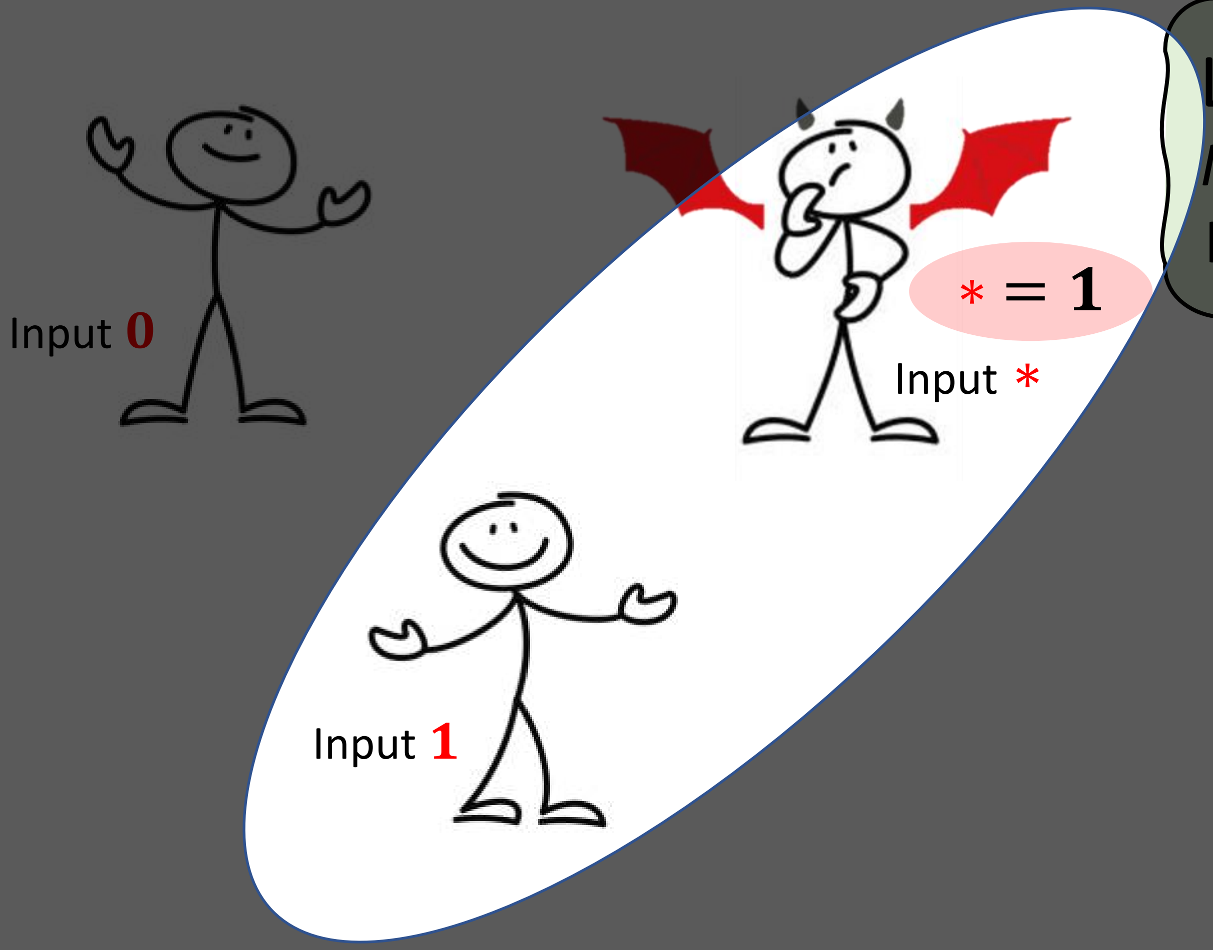

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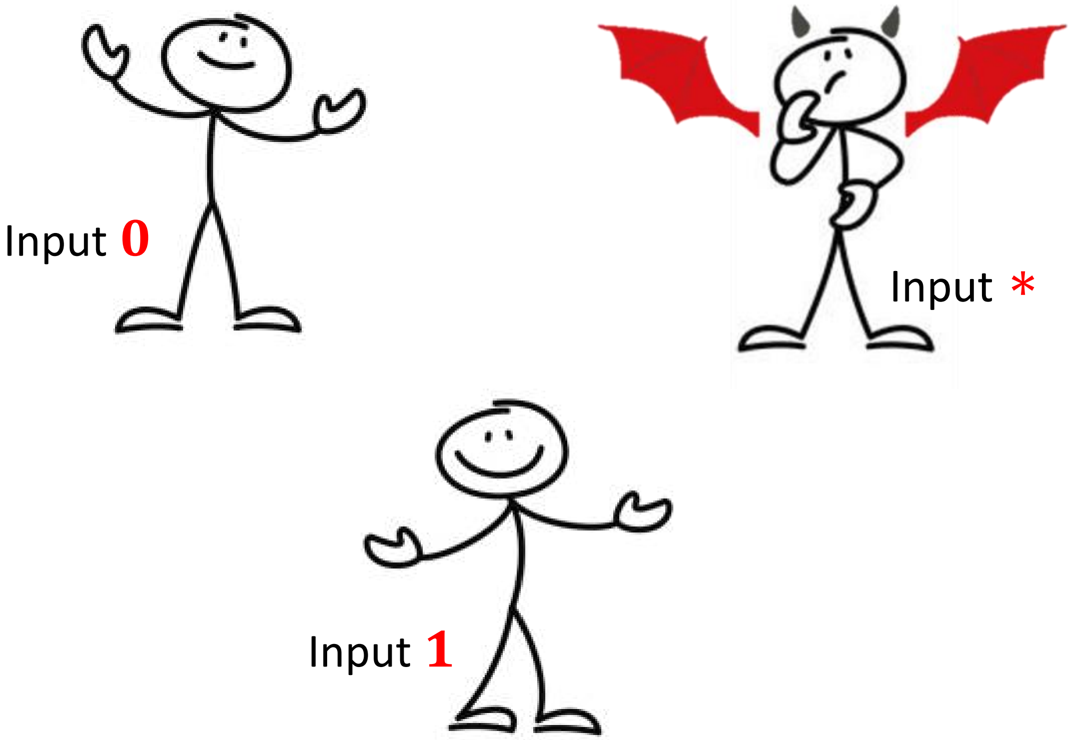

Lemma (Folklore)

In an honest *execution*:

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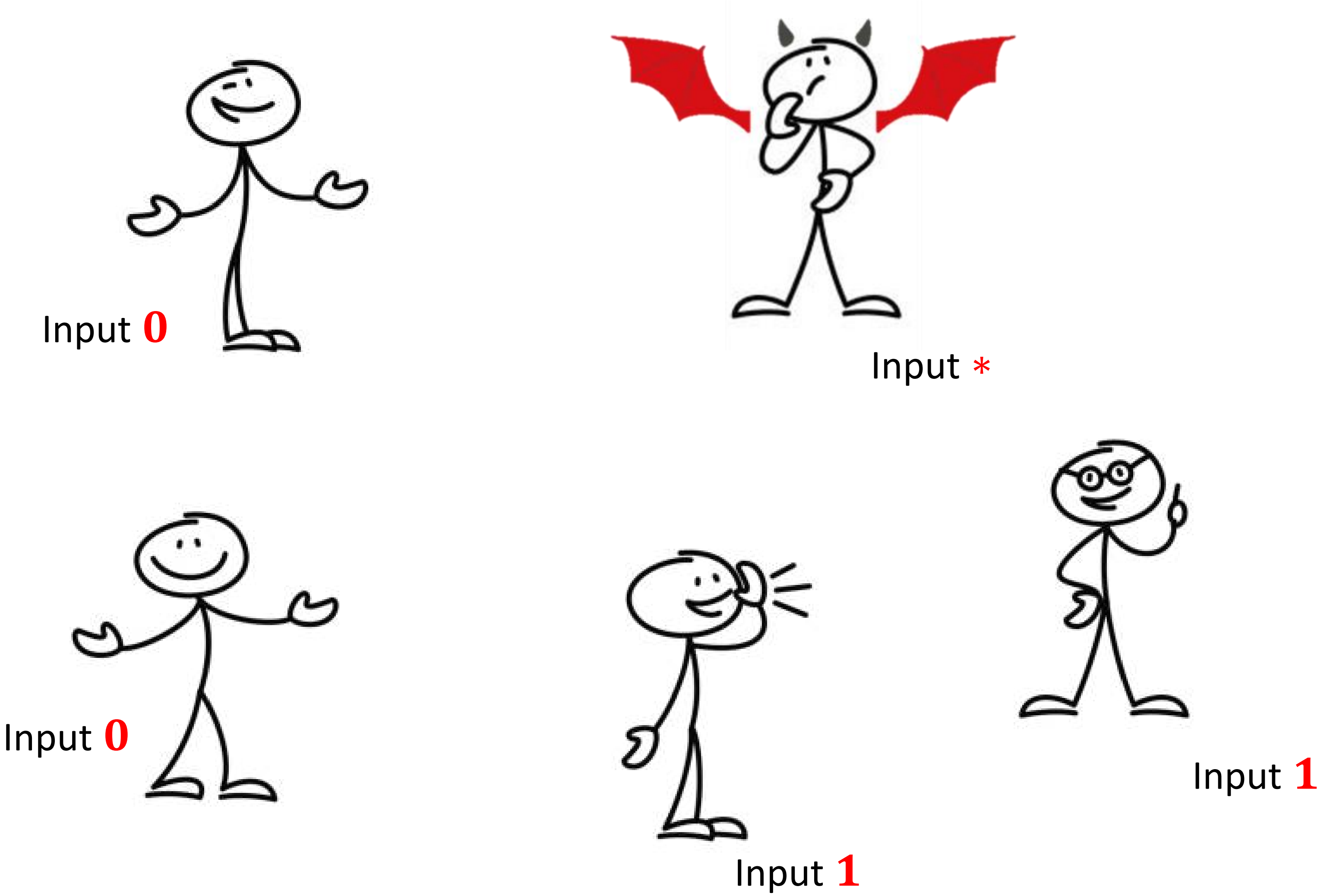
Theorem

BA resilient against $n/3$ corruptions never halts at the 1st round.

Our Attack
2nd Round Halting

Lower bound for 2nd Round Halting



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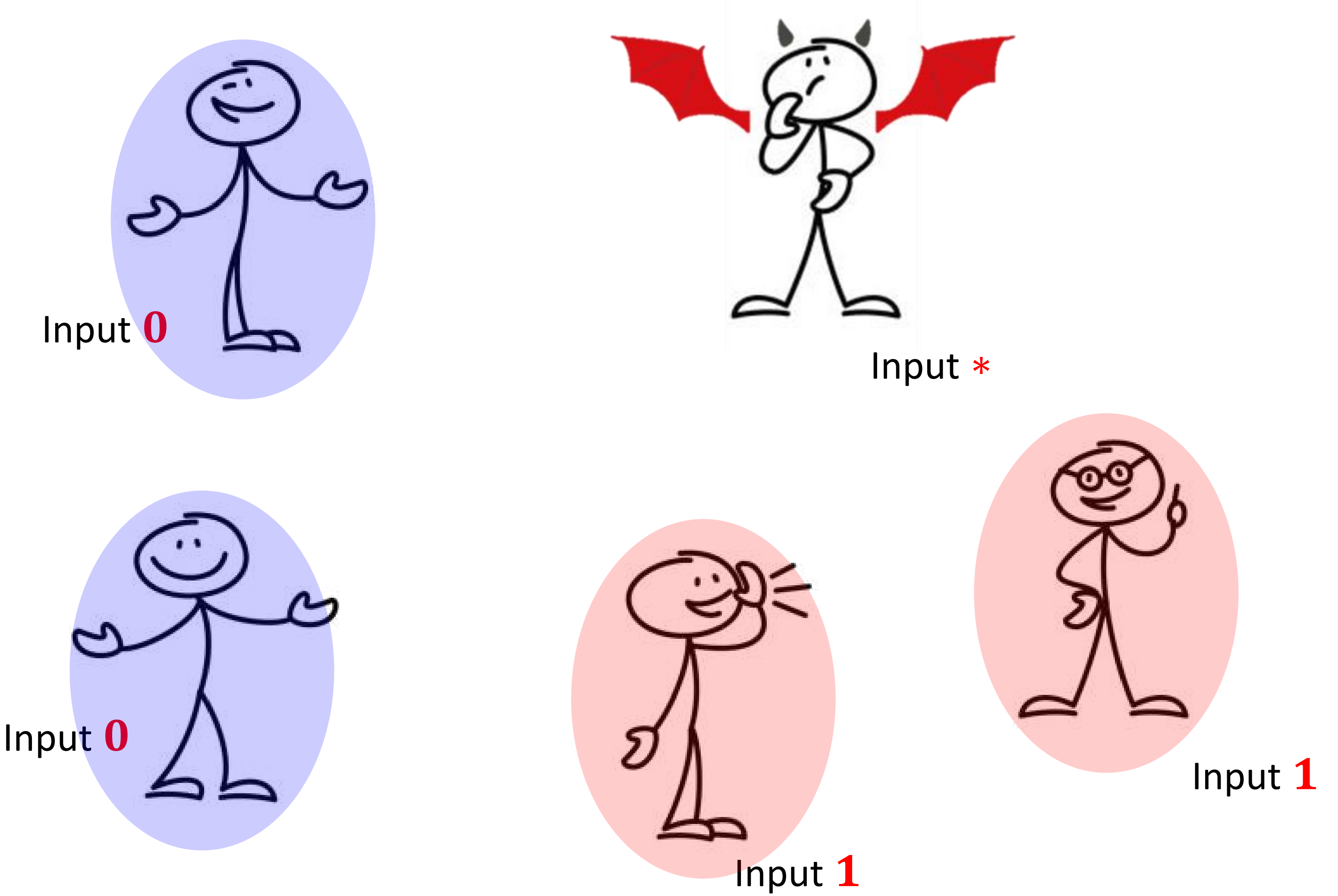


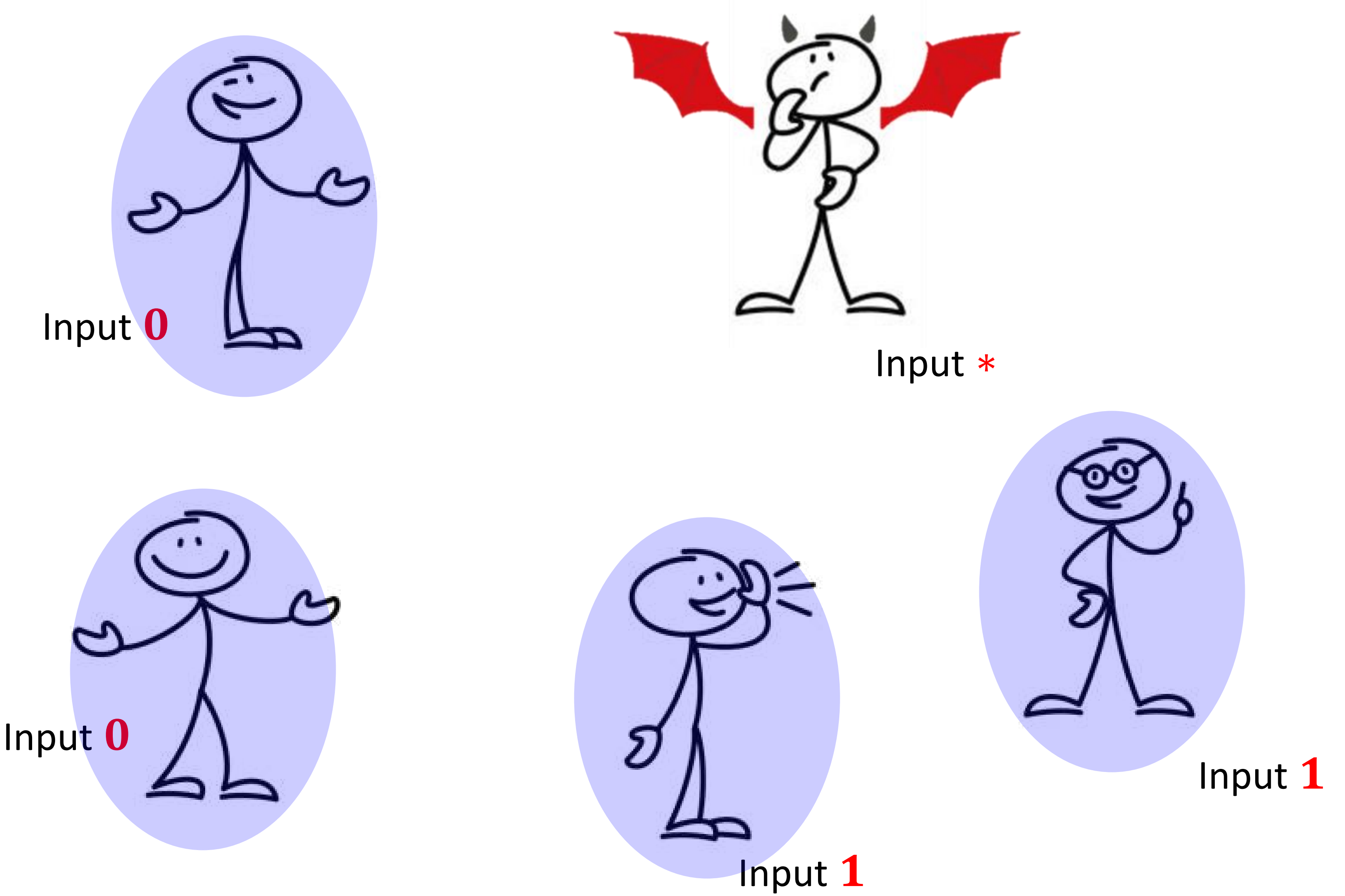
$* = 0$

$* = 1$

Parties can use the second round to ***spot liars!***

Lower bound for 2nd Round Halting

(Assume parties always halt at 2nd round)



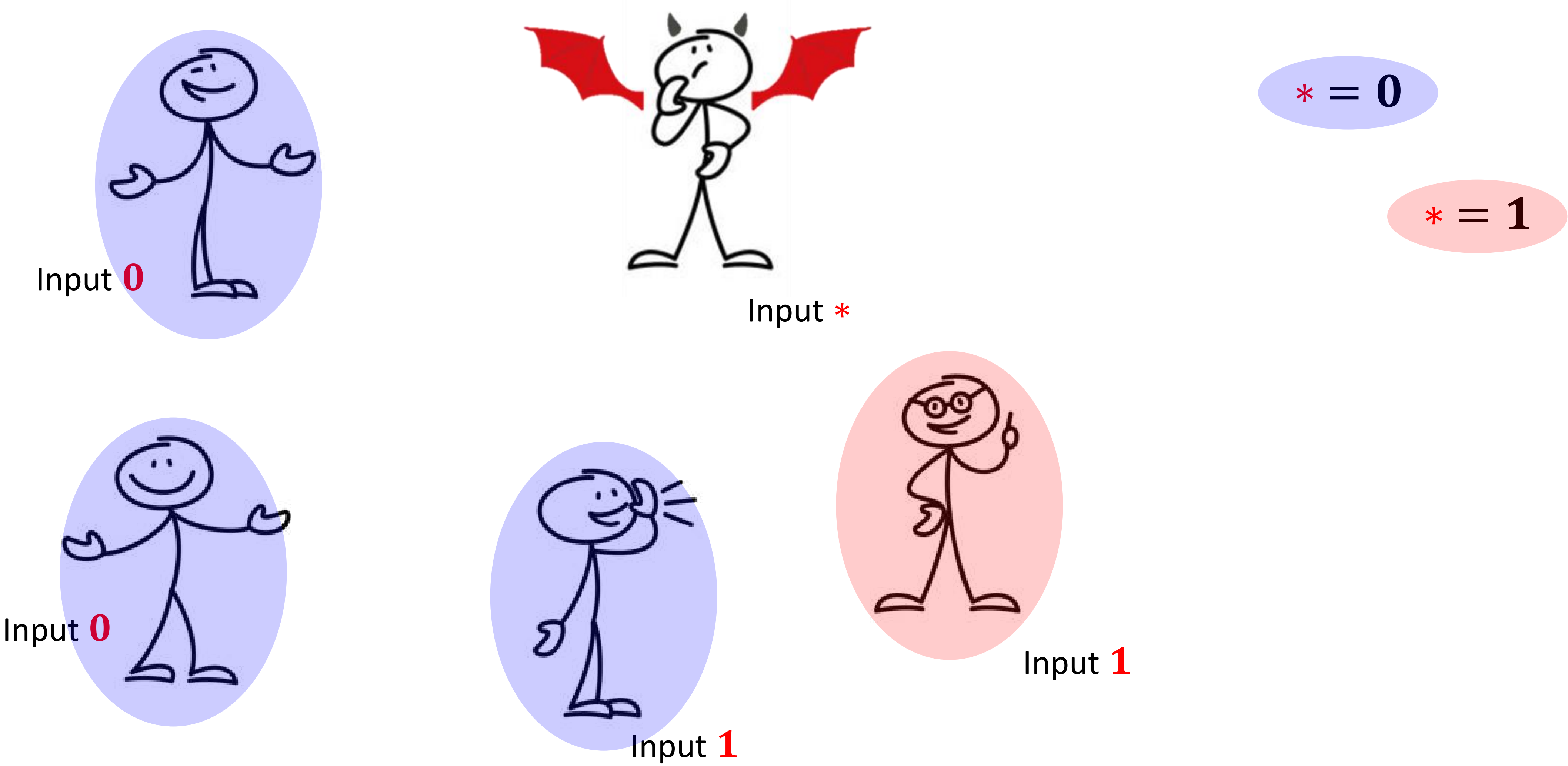
$* = 0$

$* = 1$

Honest execution with 2/3 inputs = 0

Lower bound for 2^{nd} Round Halting

(Assume parties always halt at 2^{nd} round)



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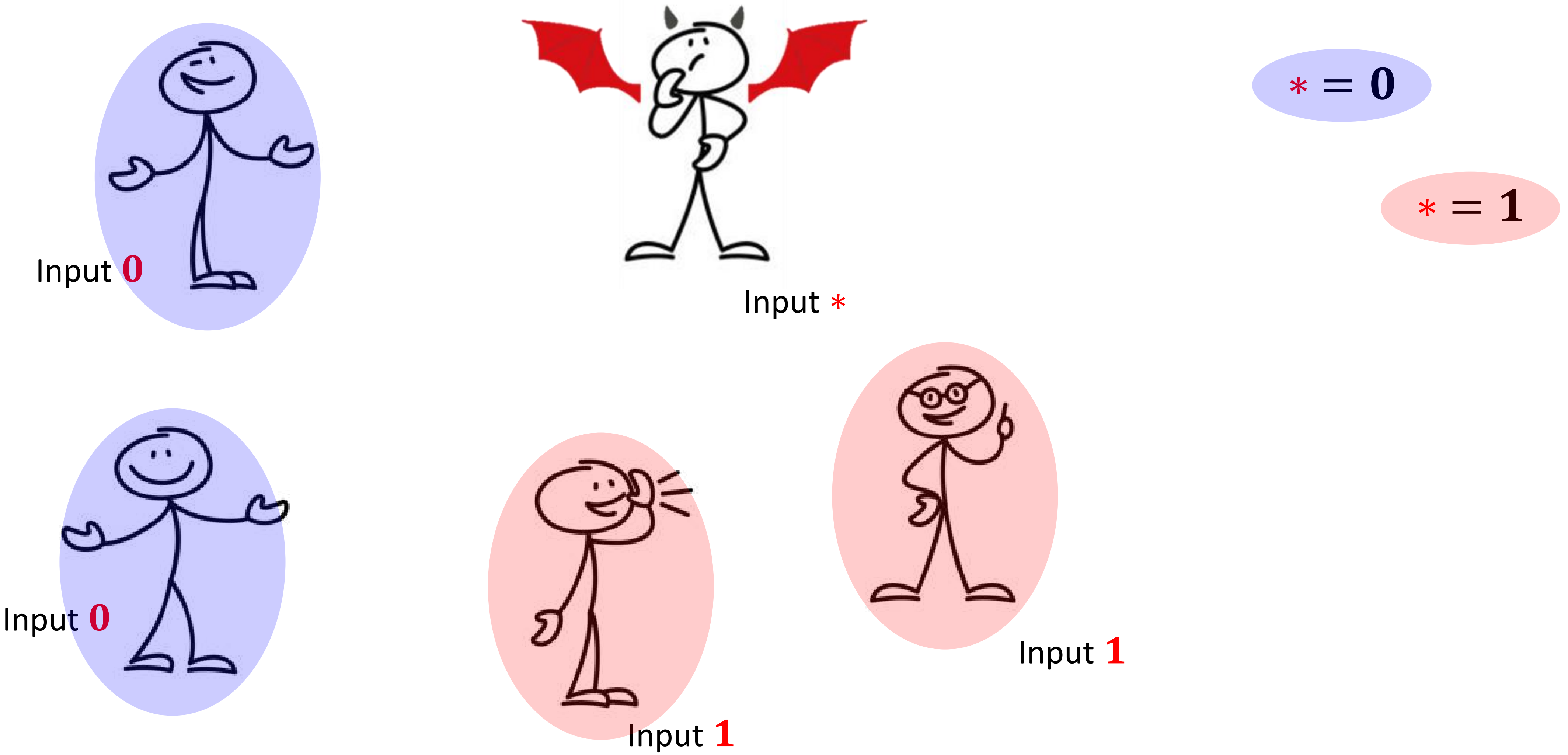

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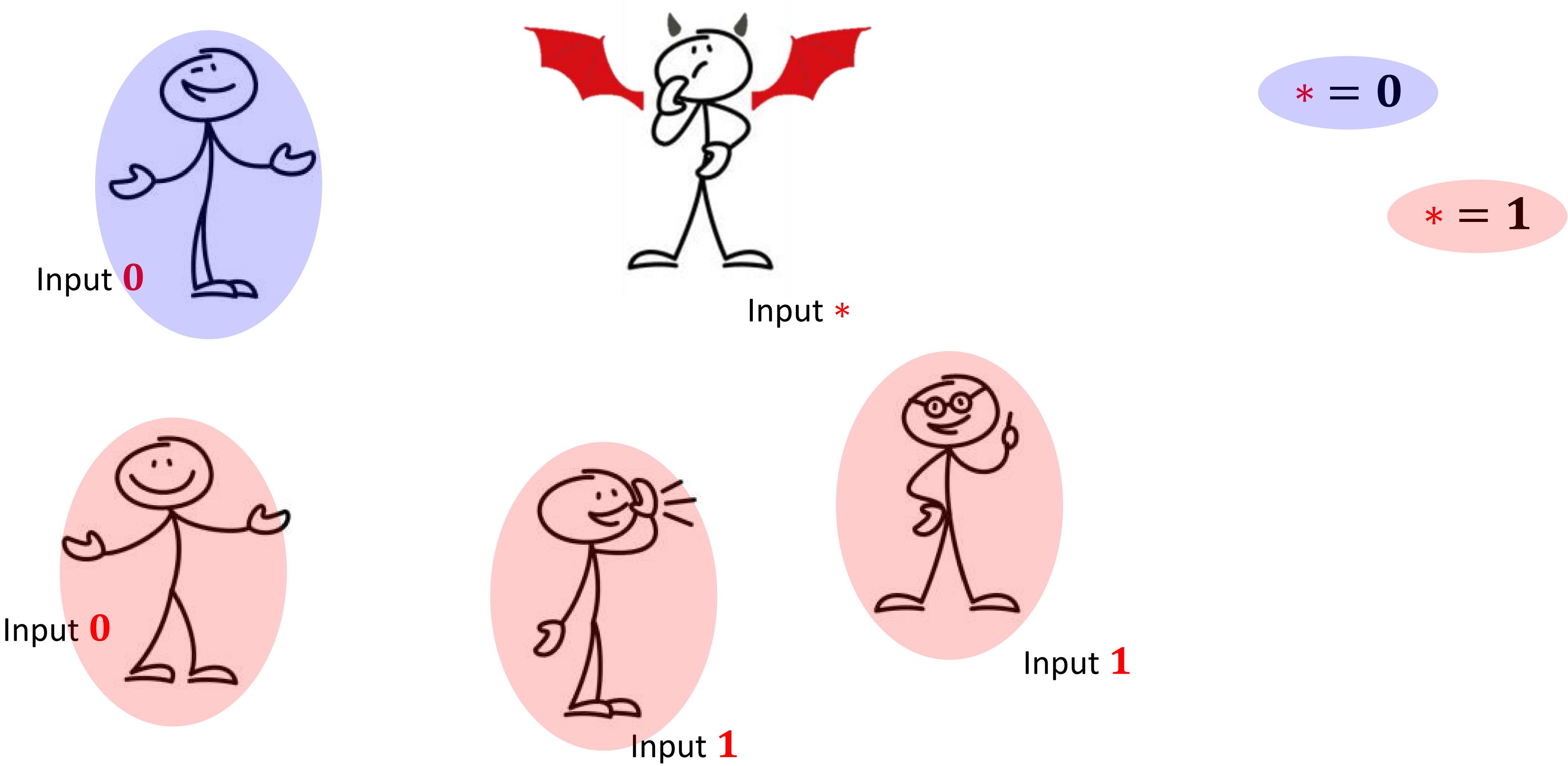

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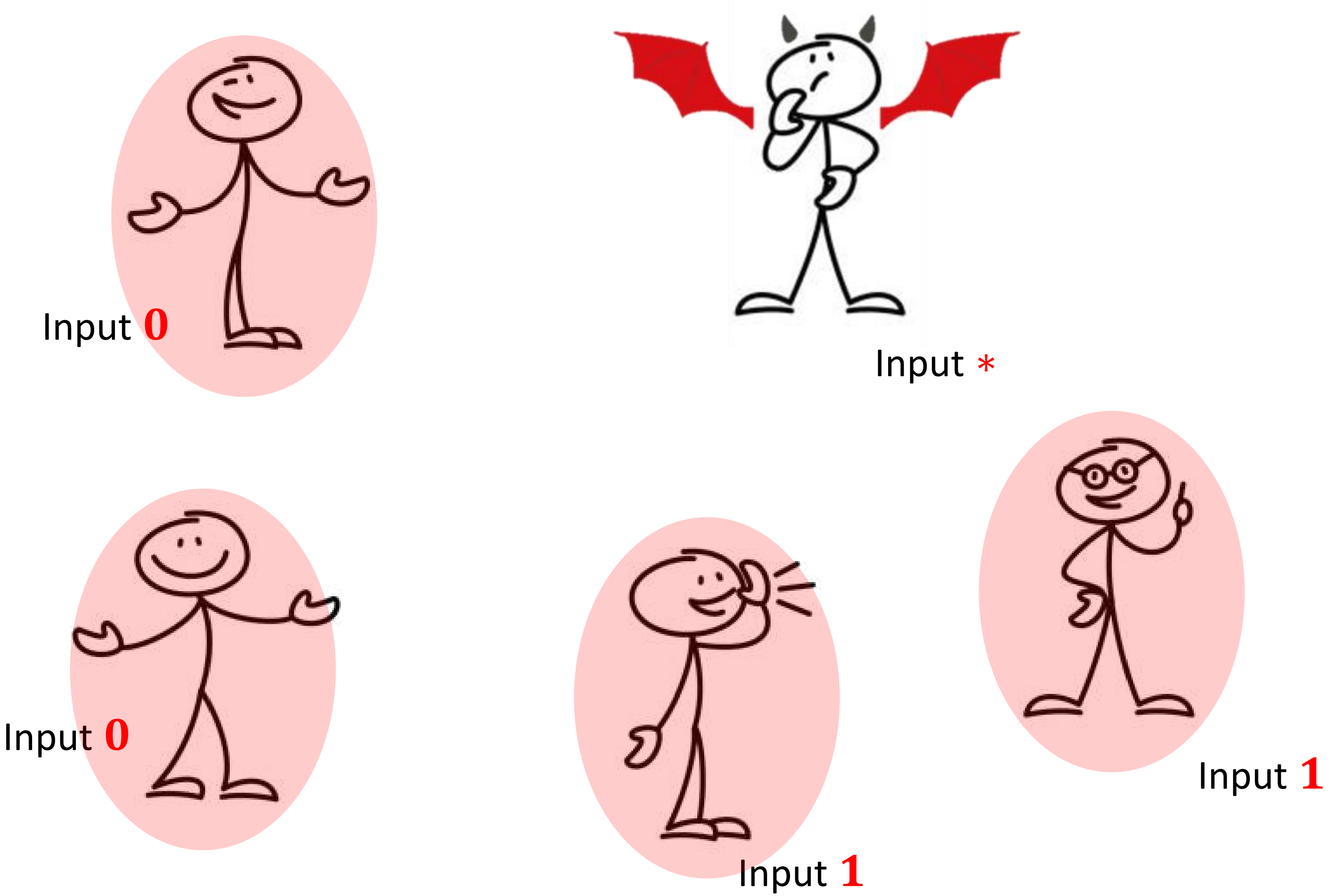


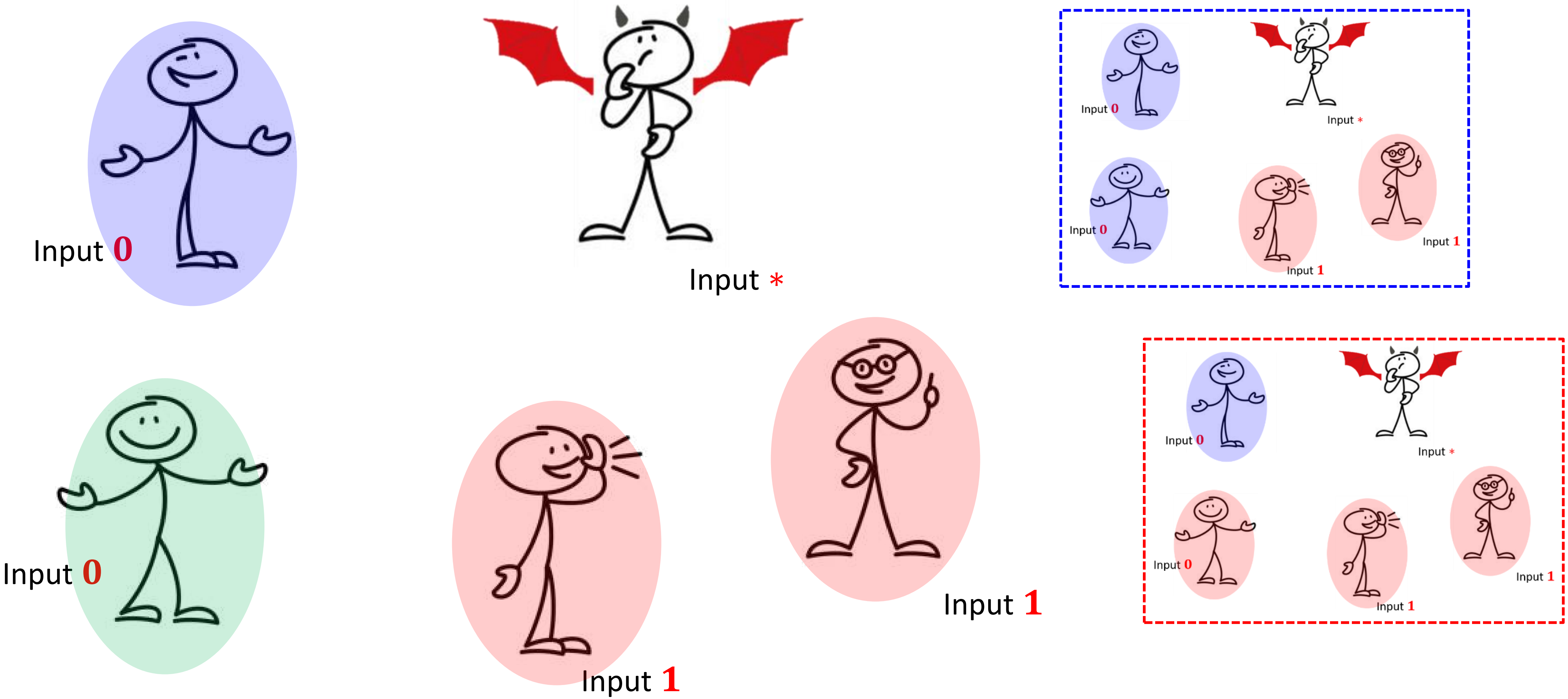
$* = 0$

$* = 1$

Honest execution with 2/3 inputs = 1

Lower bound for 2nd Round Halting

(Assume parties always halt at 2nd round)



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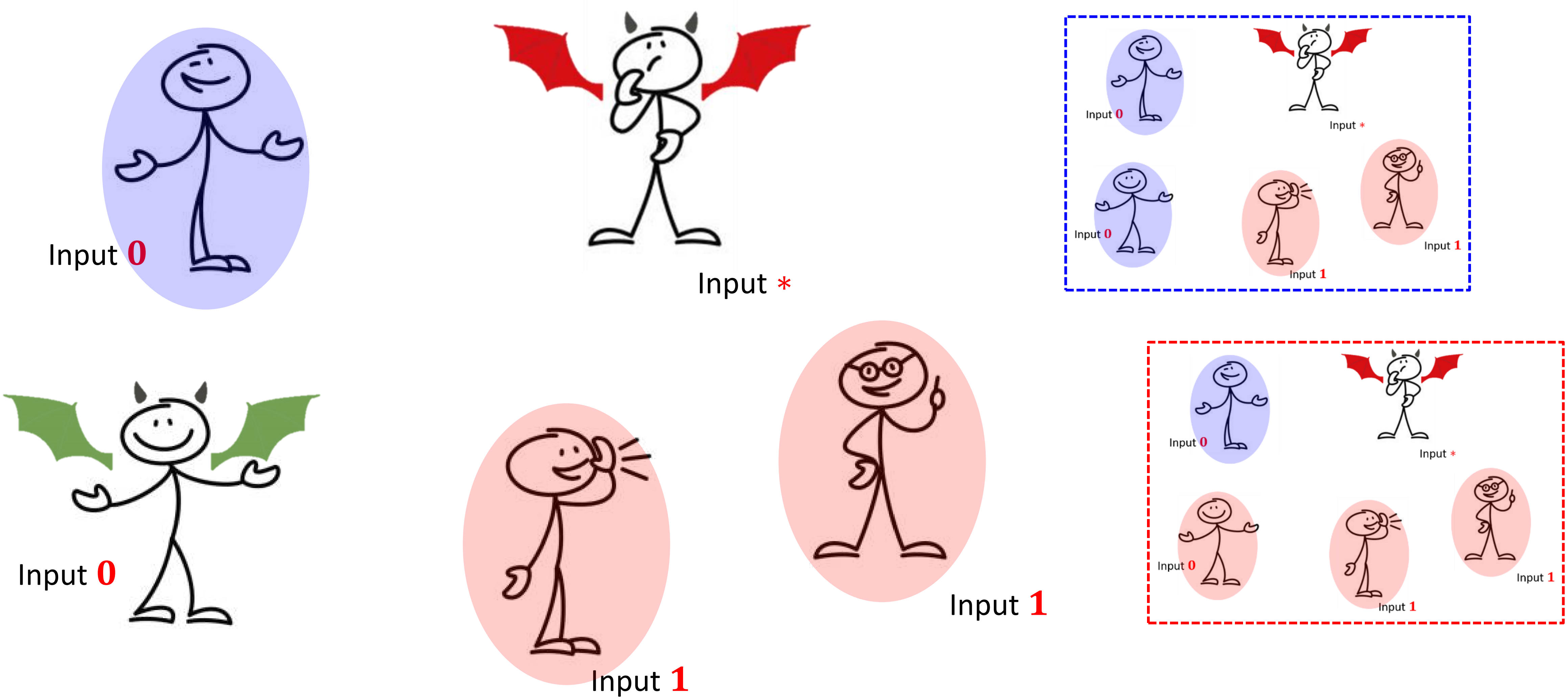

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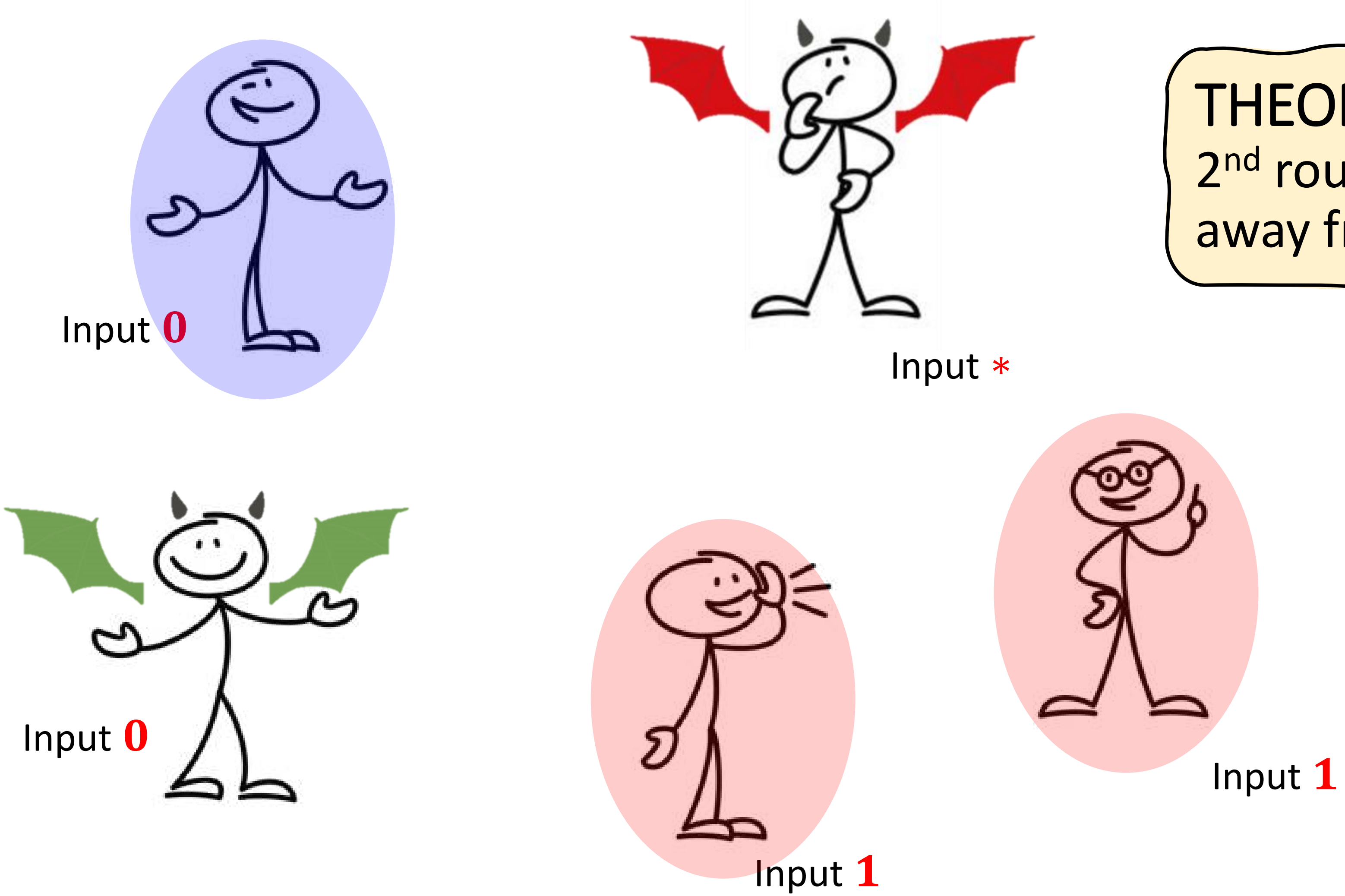

THEOREM
2^{nd} round halting is bounded away from 1.

Lower bound for 2^{nd} Round Halting

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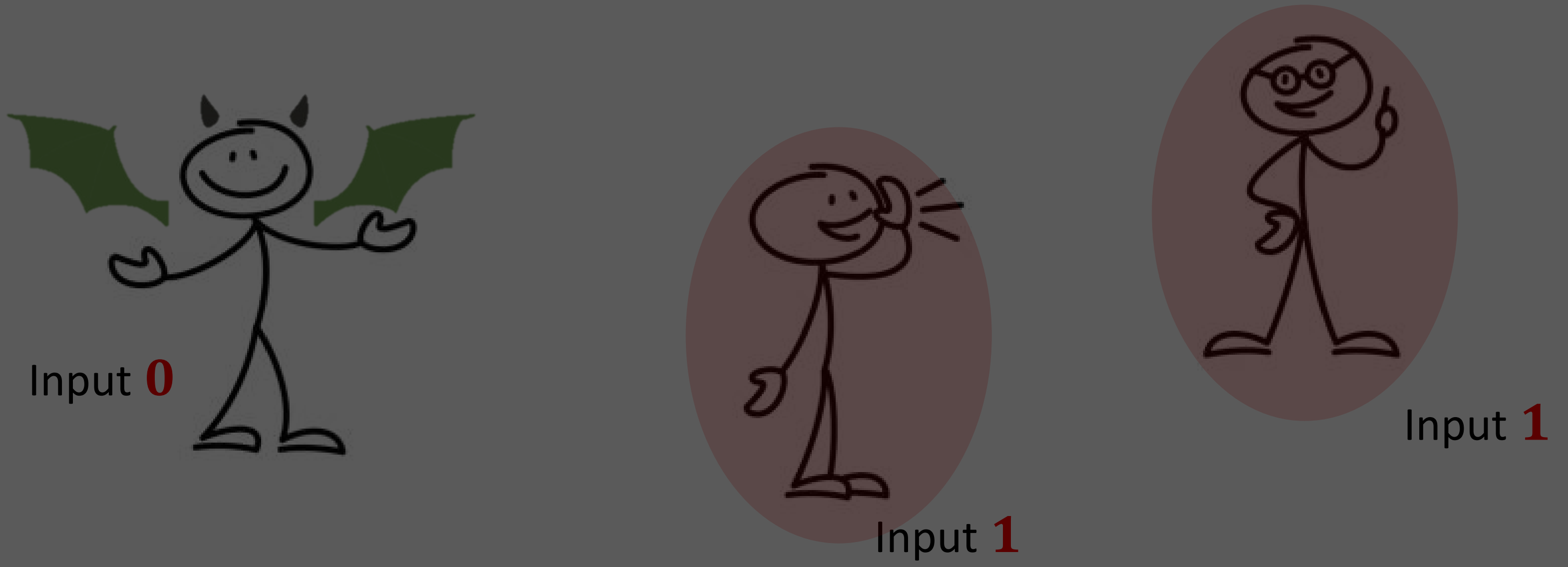
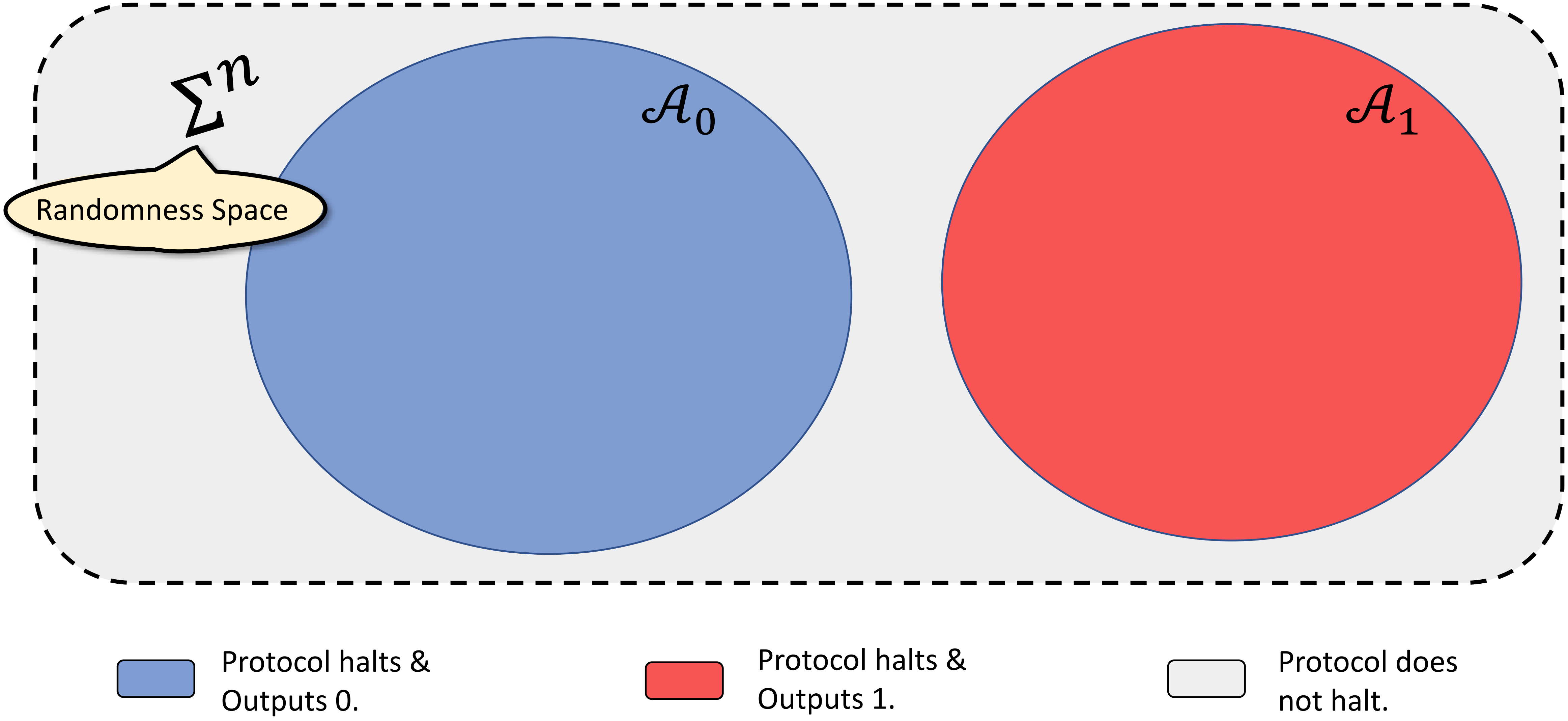
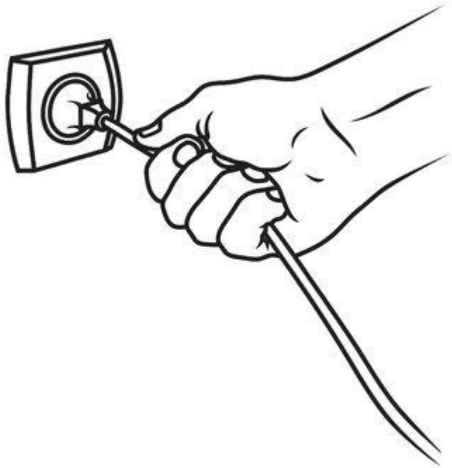


Limits of Attack



Our Attack
2nd Round Halting
with Abort

Attack with Aborting Parties

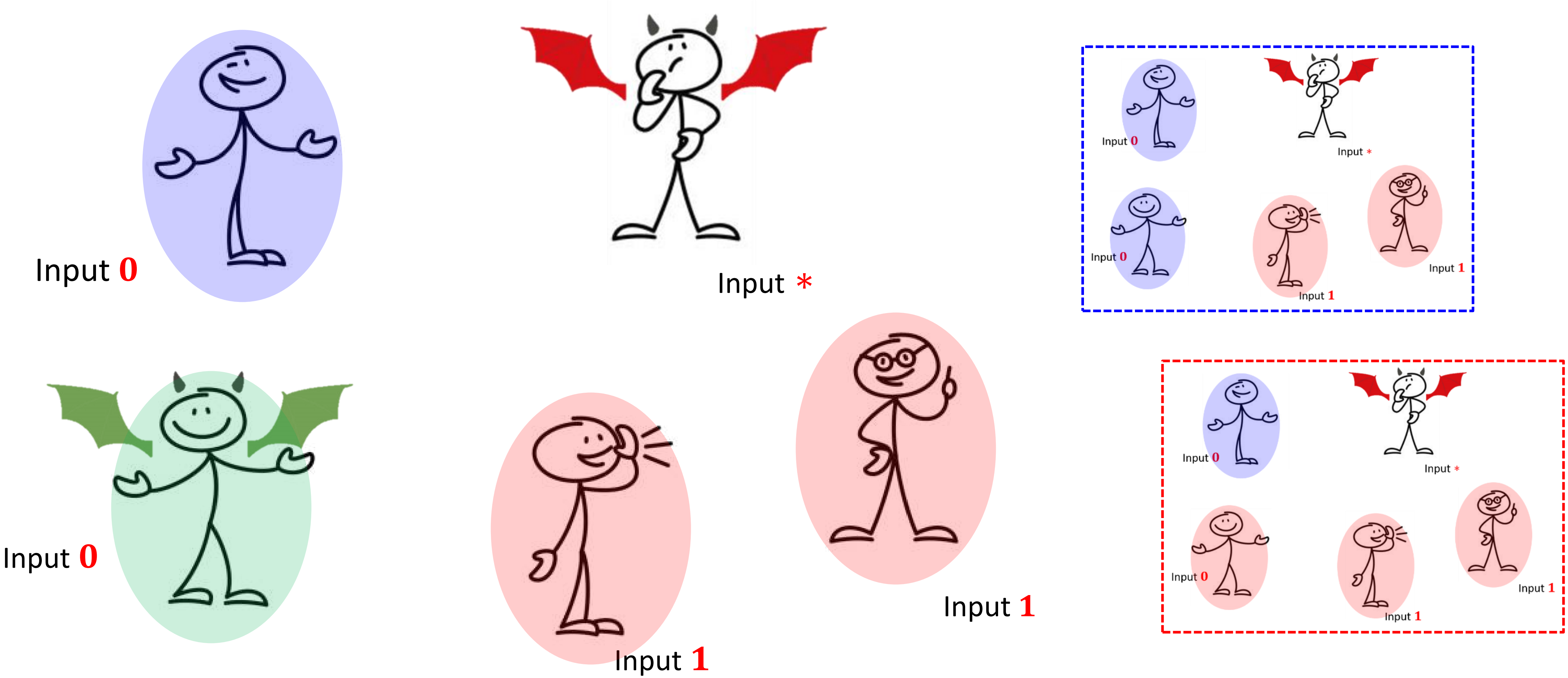


We add another dimension to our attack by instructing (certain) corrupted parties to **abort prematurely**

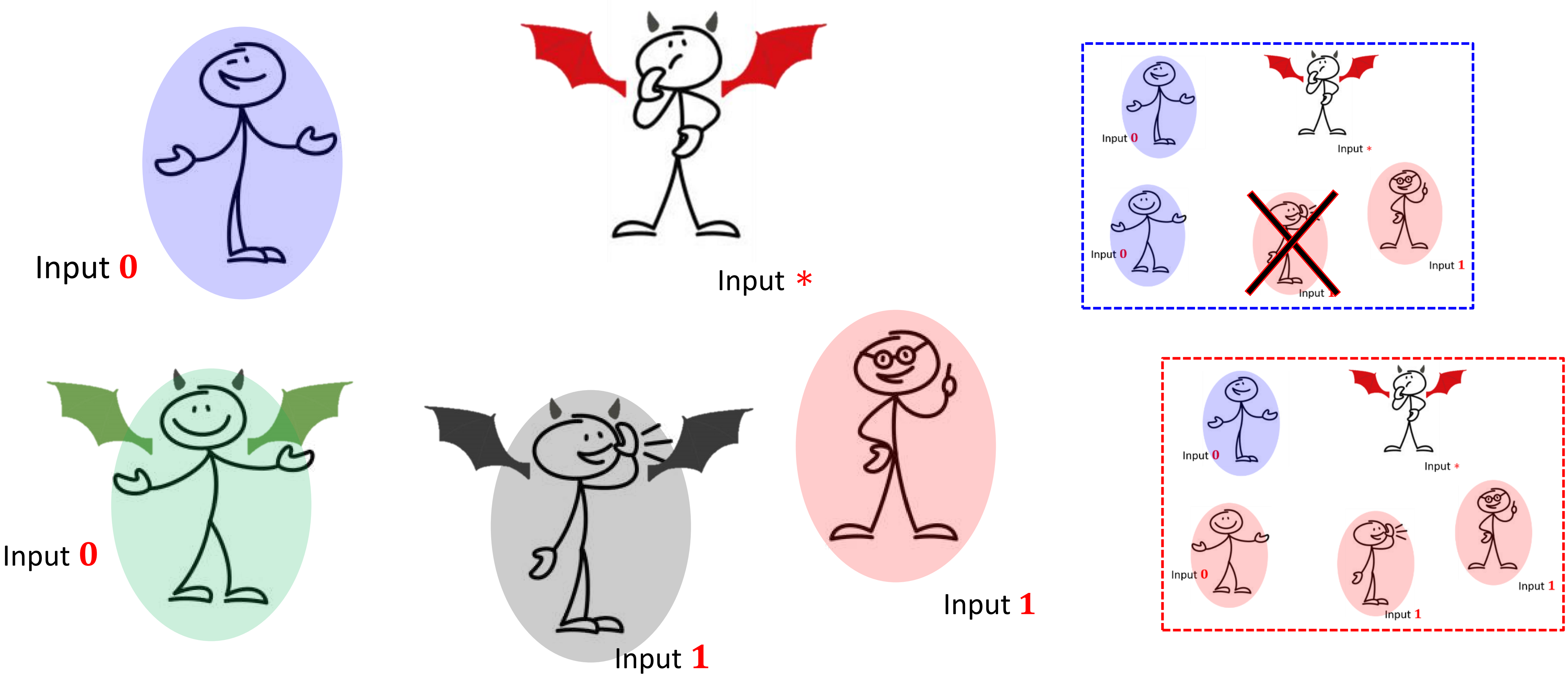
ATTACK w/ Aborting Parties

- Follow previous attack.
- At round **2**:
  Choose a random set $\mathcal{S}$ and **abort** it for a **subset** of honest parties.

Attack with Aborting Parties



Attack with Aborting Parties



Theorem Statement

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Conjecture 1.5. *For any $\sigma, \lambda > 0$ there exists $\delta > 0$ such that the following holds for large enough $n \in \mathbb{N}$: let Σ be a finite alphabet, and let $\mathcal{A}_0, \mathcal{A}_1 \subseteq \{\Sigma \cup \perp\}^n$ be two sets such that for both $b \in \{0,1\}$:*

$$\Pr_{\mathcal{S} \leftarrow \mathrm{D}_{n,\sigma}} \left[\Pr_{\boldsymbol{r} \leftarrow \Sigma^n} [\boldsymbol{r}, \perp_{\mathcal{S}}(\boldsymbol{r}) \in \mathcal{A}_b] \geq \lambda \right] \geq 1 - \delta.$$

Then,

$$\Pr_{\substack{\mathcal{S} \leftarrow \mathrm{D}_{n,\sigma} \\ \boldsymbol{r} \leftarrow \Sigma^n}} [\forall b \in \{0,1\} \colon \{\boldsymbol{r}, \perp_{\mathcal{S}}(\boldsymbol{r})\} \cap \mathcal{A}_b \neq \emptyset] \geq \delta.$$

We know how to handle limited (and unrealistic) cases without the conjecture.

THEOREM

Conj. 1.5. $\Longrightarrow$ *BA protocols* halt after two rounds with probability 0.

Public Randomness (PR) Protocols

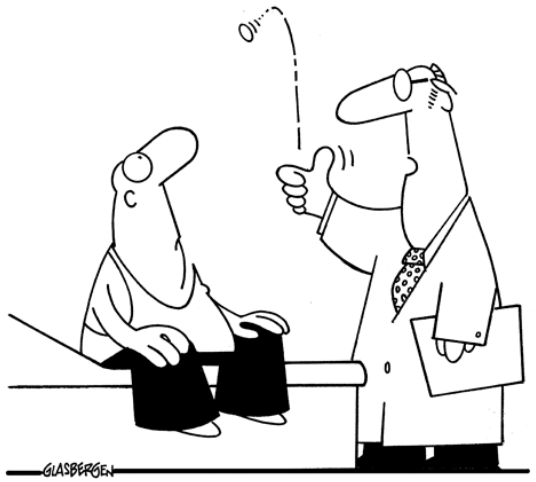
Analogues of (inputless) public coin protocols

Randomness is sent in the clear

Public Randomness Protocols:

The ℓ-th round message from P_i to P_k is a pair $(m_{i,k}^{(\ell)}, r_i^{(\ell)})$ s.t.
$m_{i,k}^{(\ell)}$ is a deterministic function of P_i's view.

- Such protocols are typically
  - ✓ Conceptually Simple(r)
  - ✓ Highly Efficient (inputless regime).
- All known BA protocols can be cast as PR protocols.



Summary

For every BA resilient against $t = n/3$ corruptions

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FIN