Bounded Budget Connection Games

or how to make friends and
influence people, on a budget

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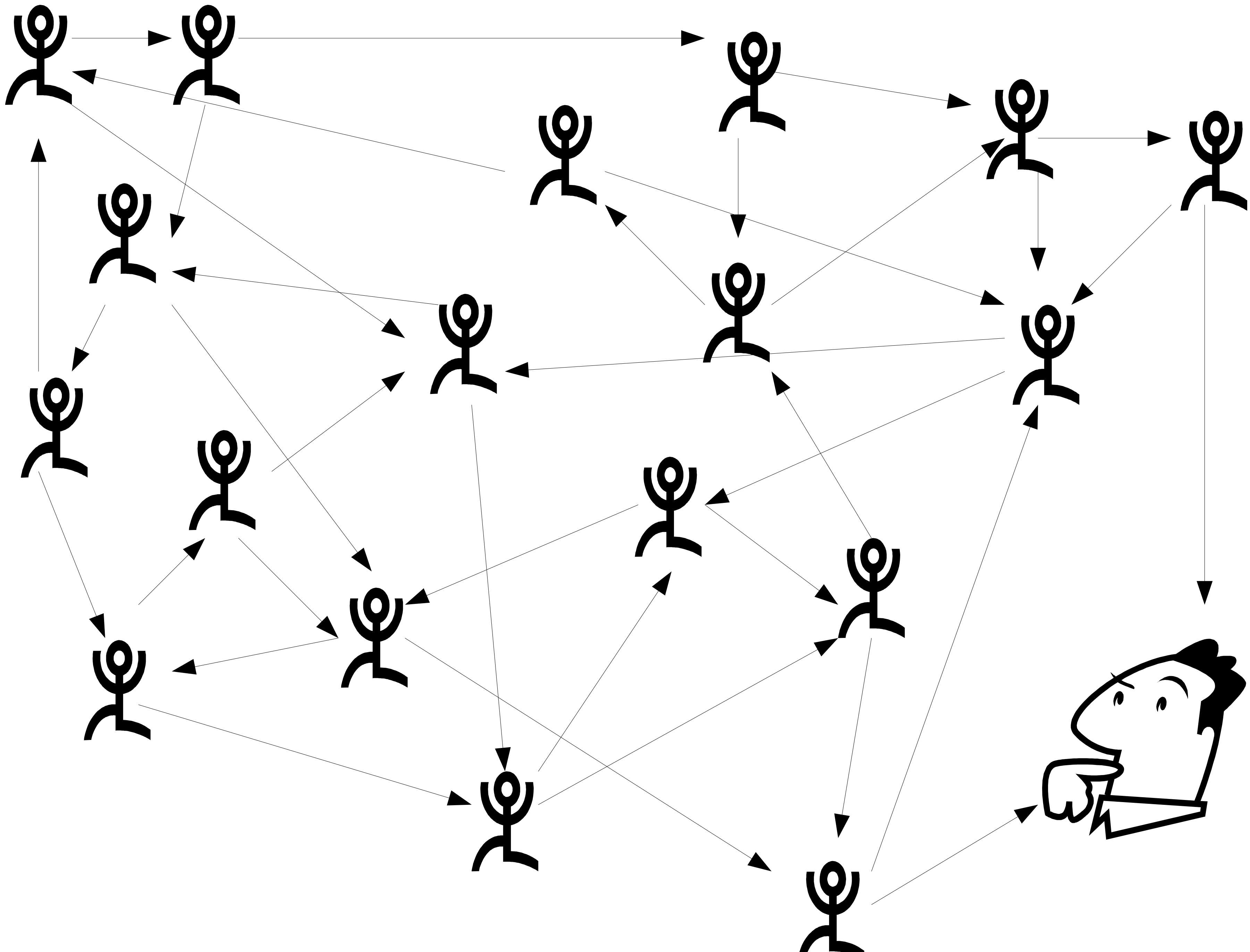
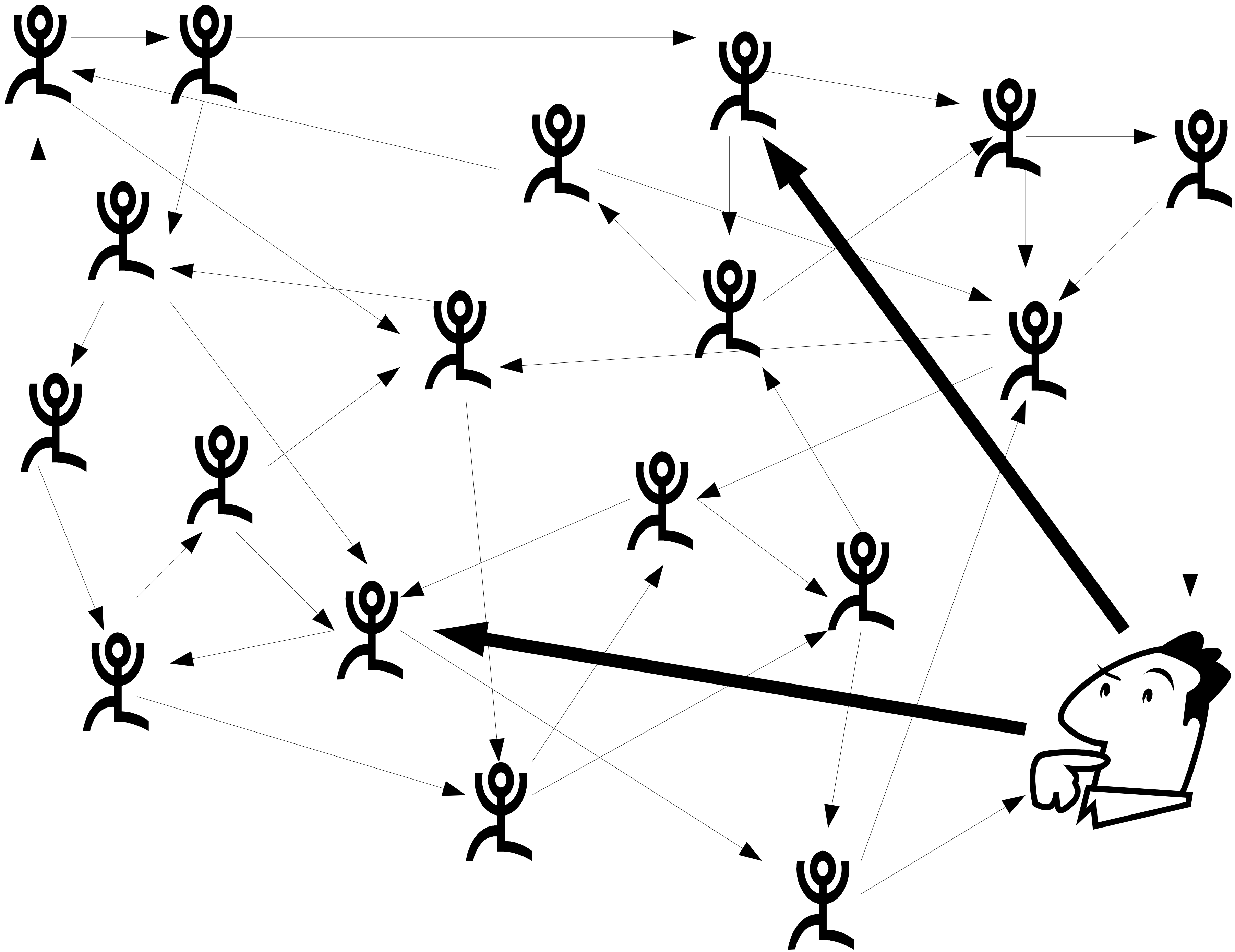
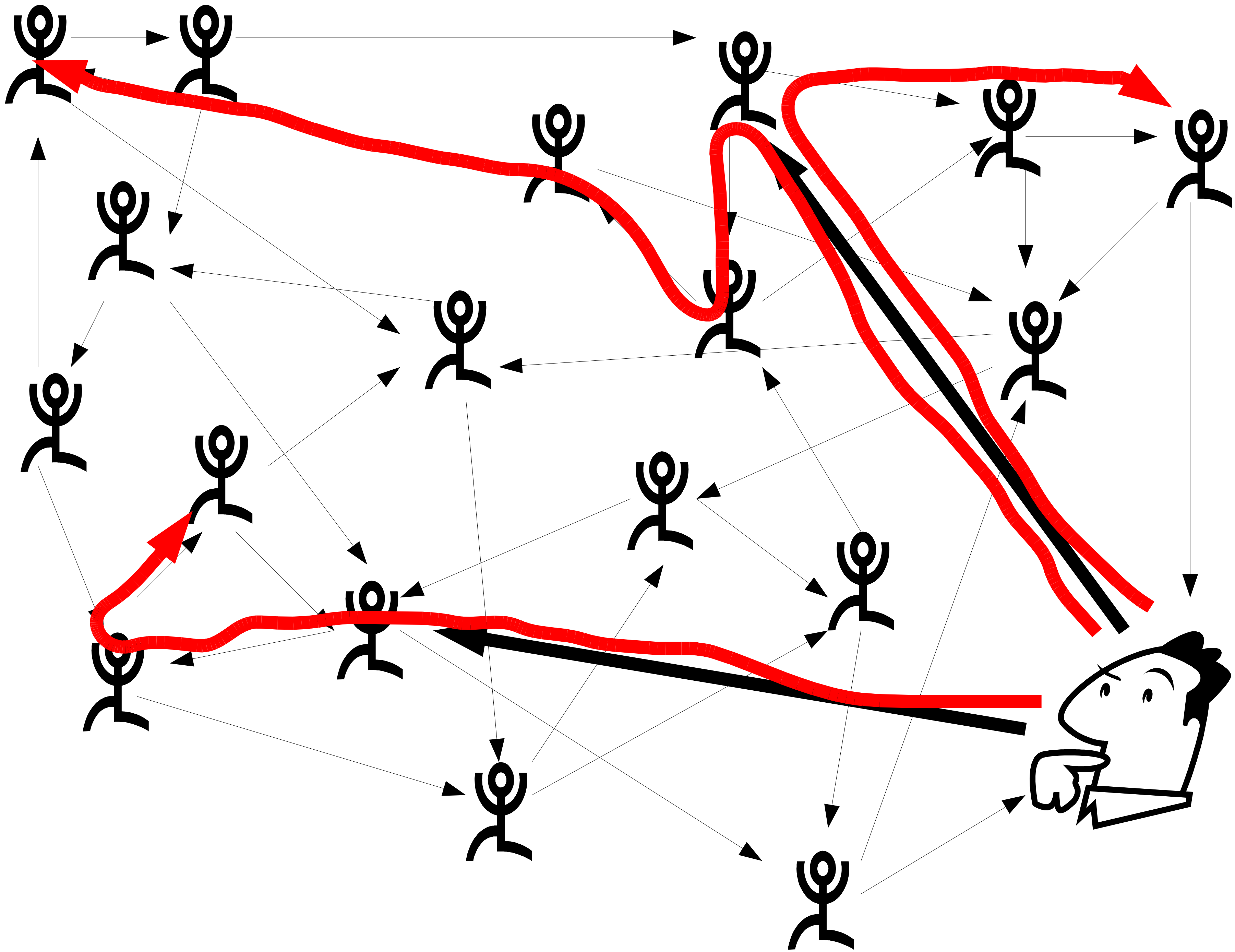
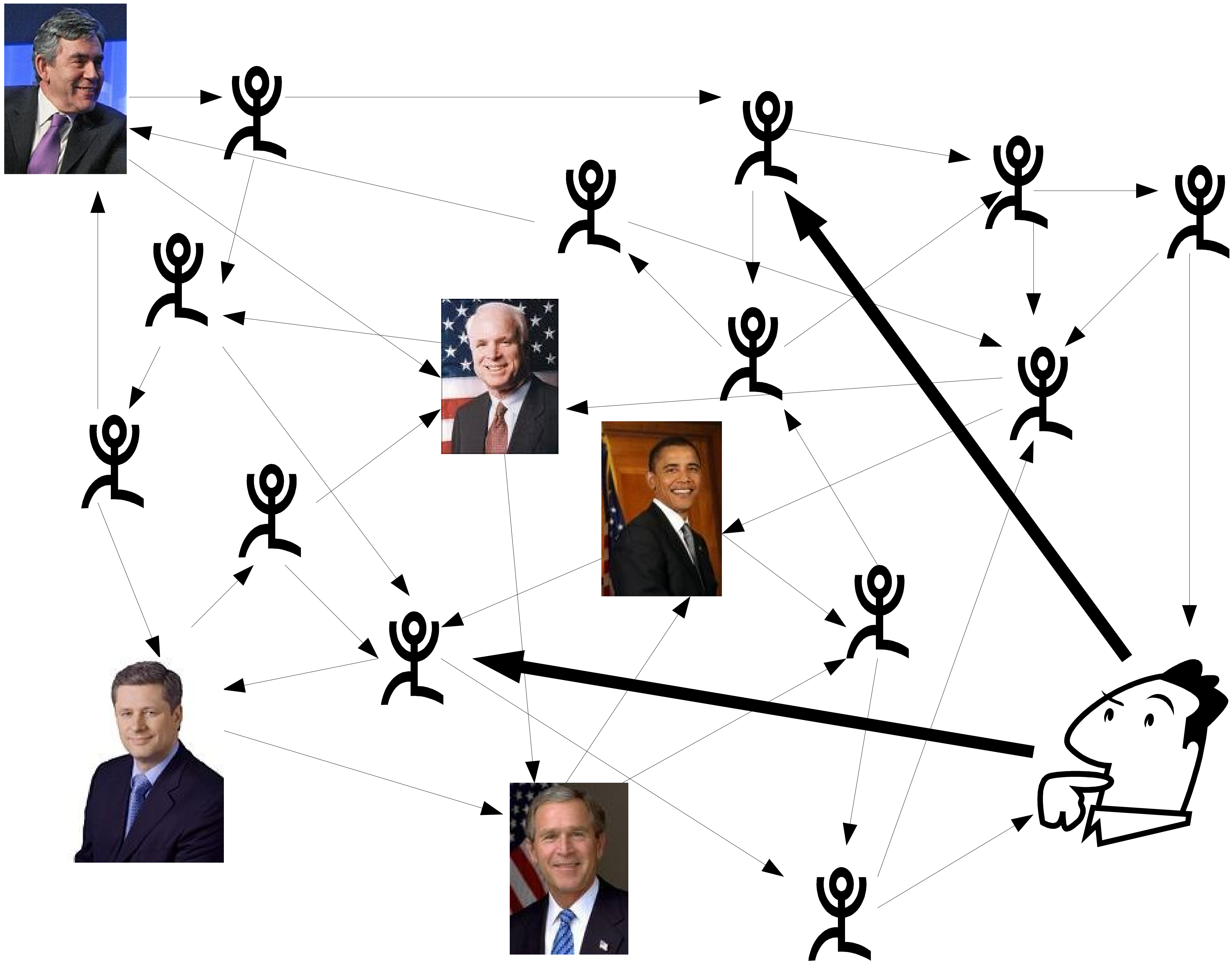
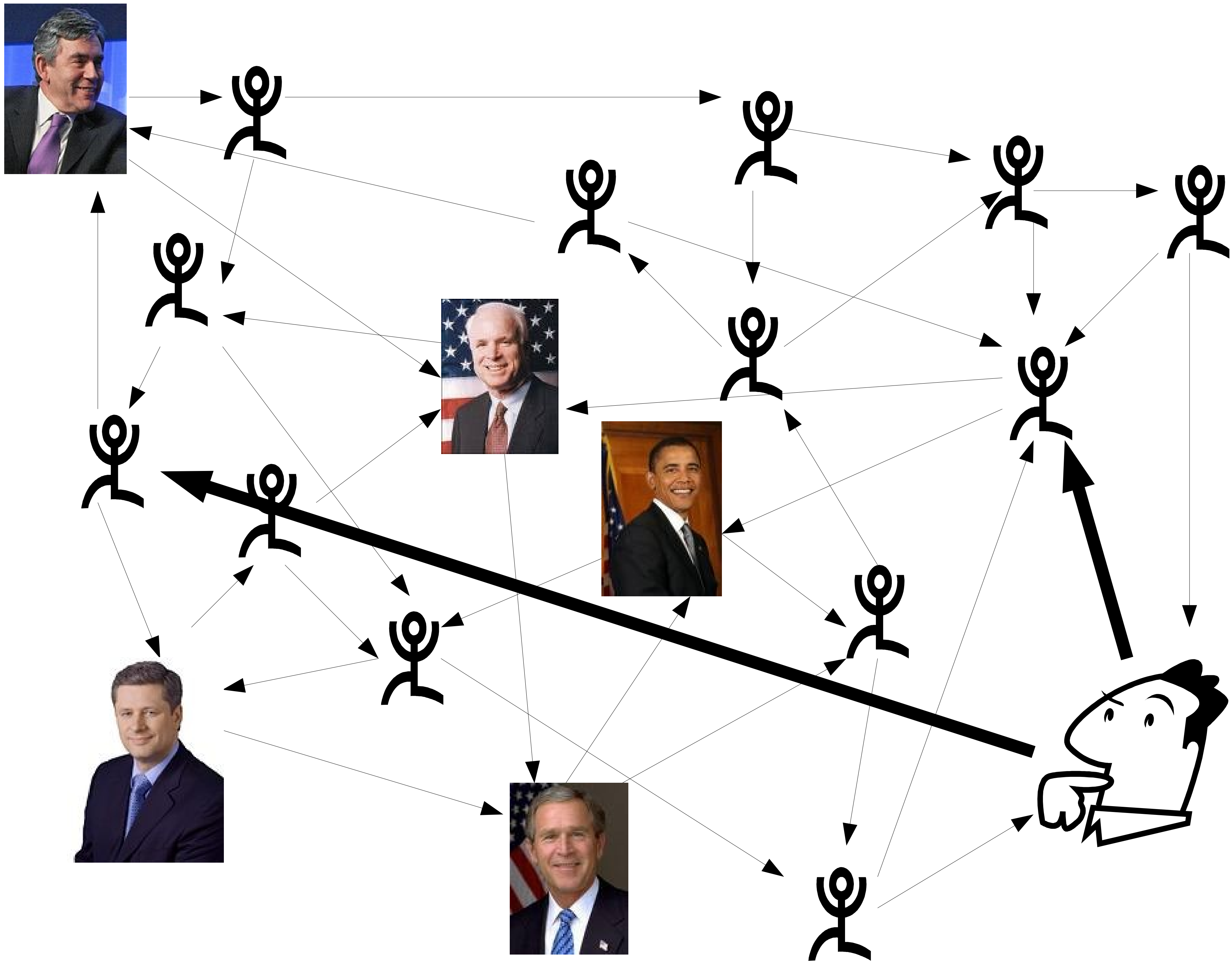
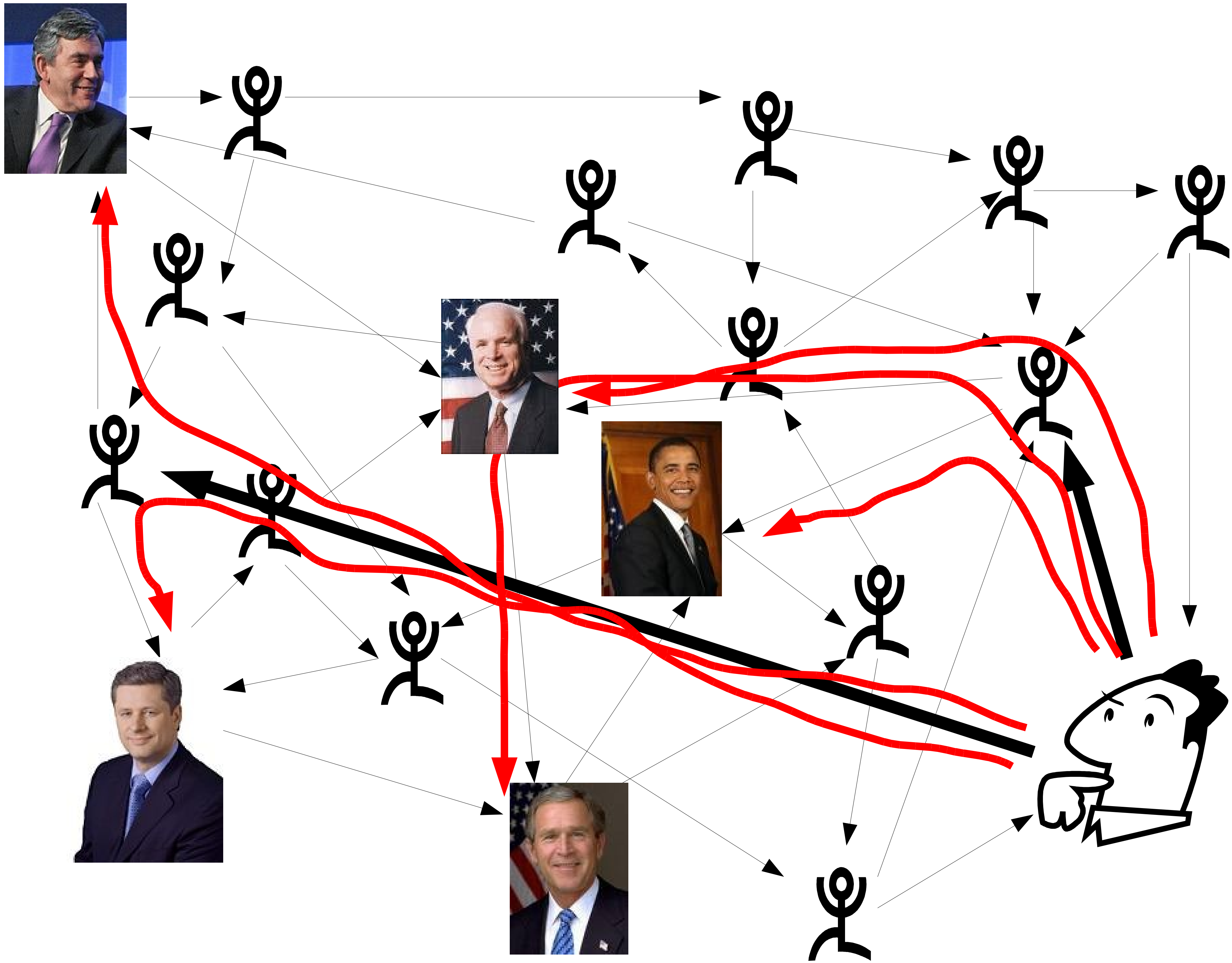


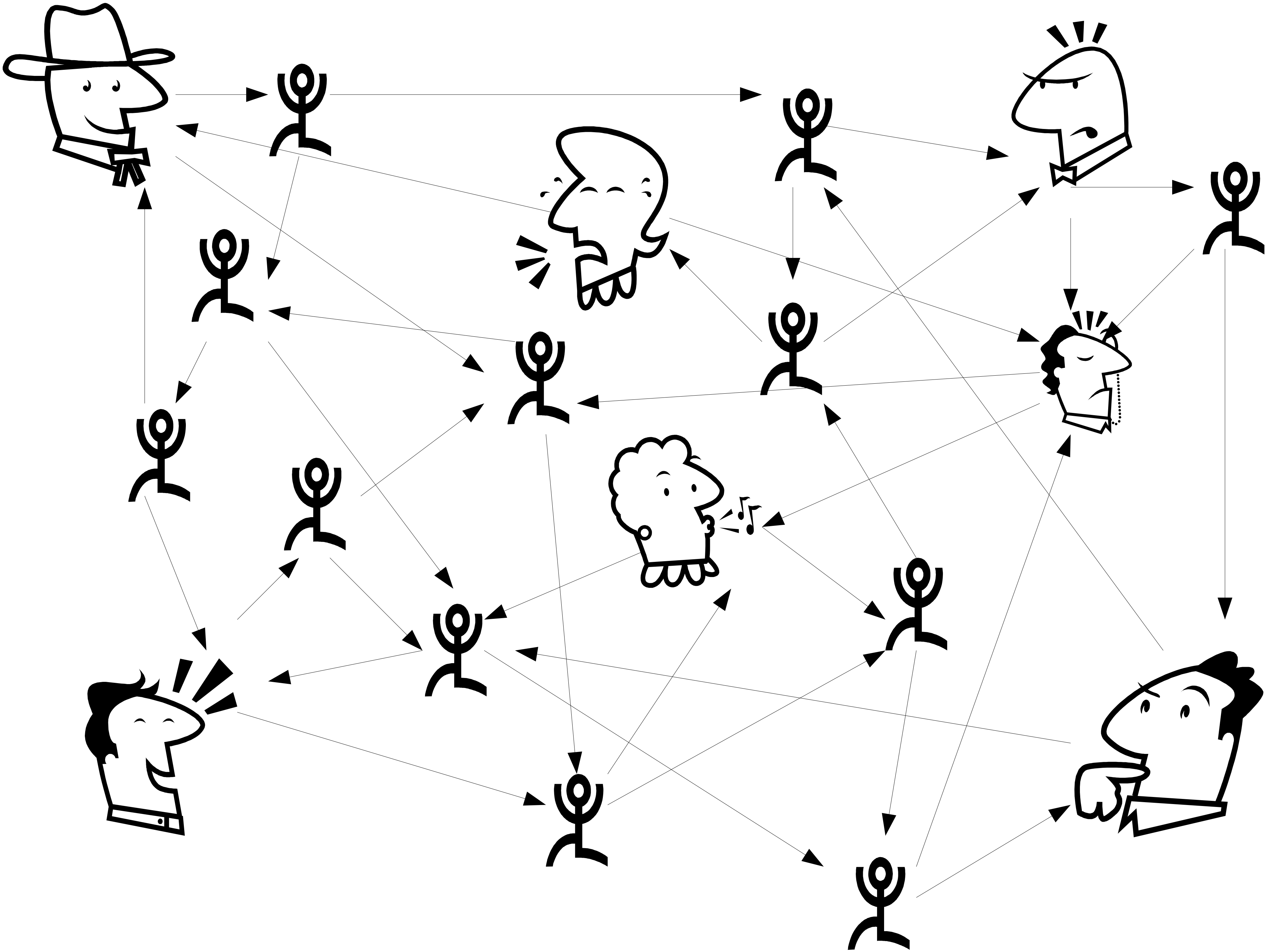


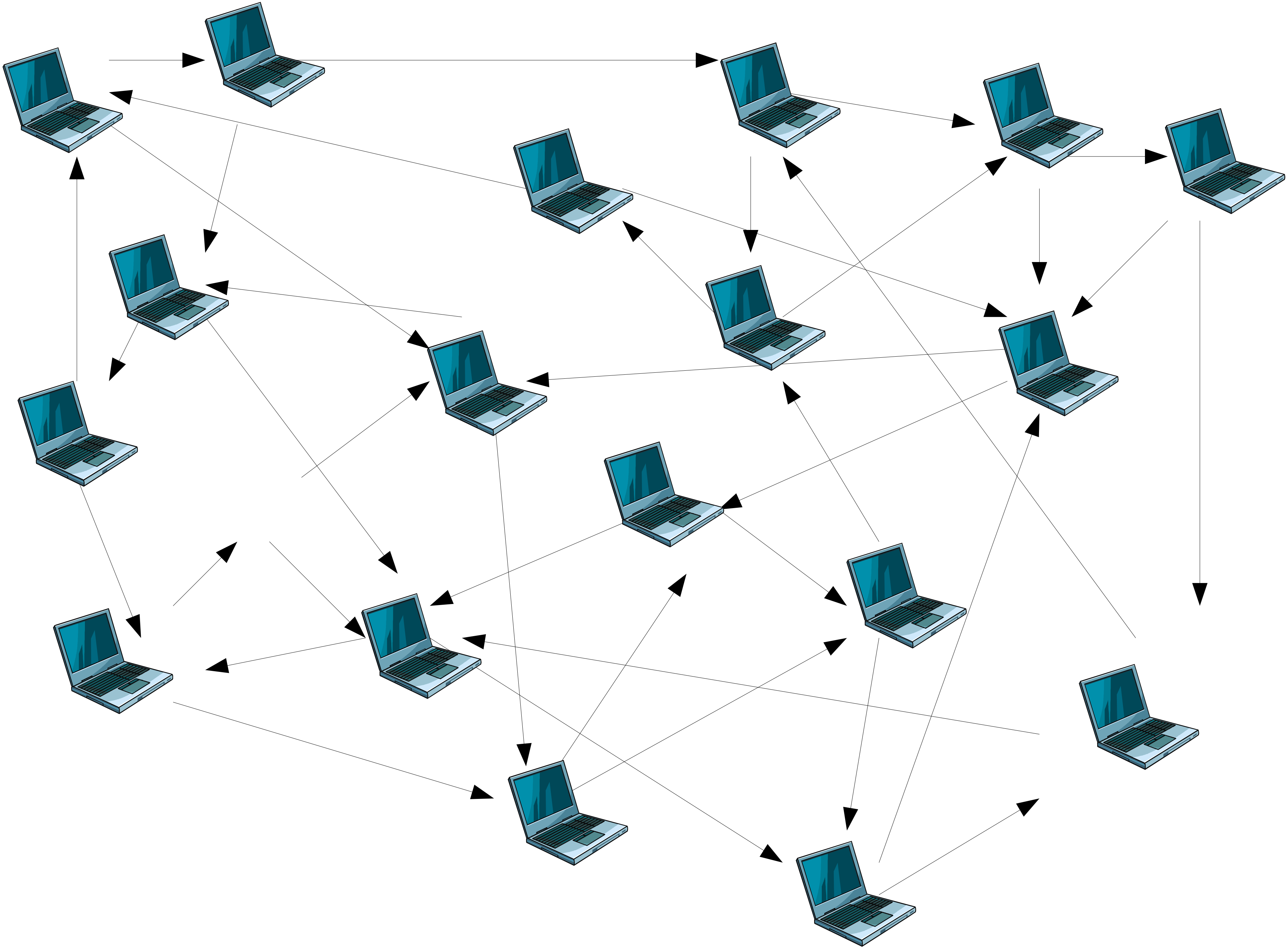












The Model

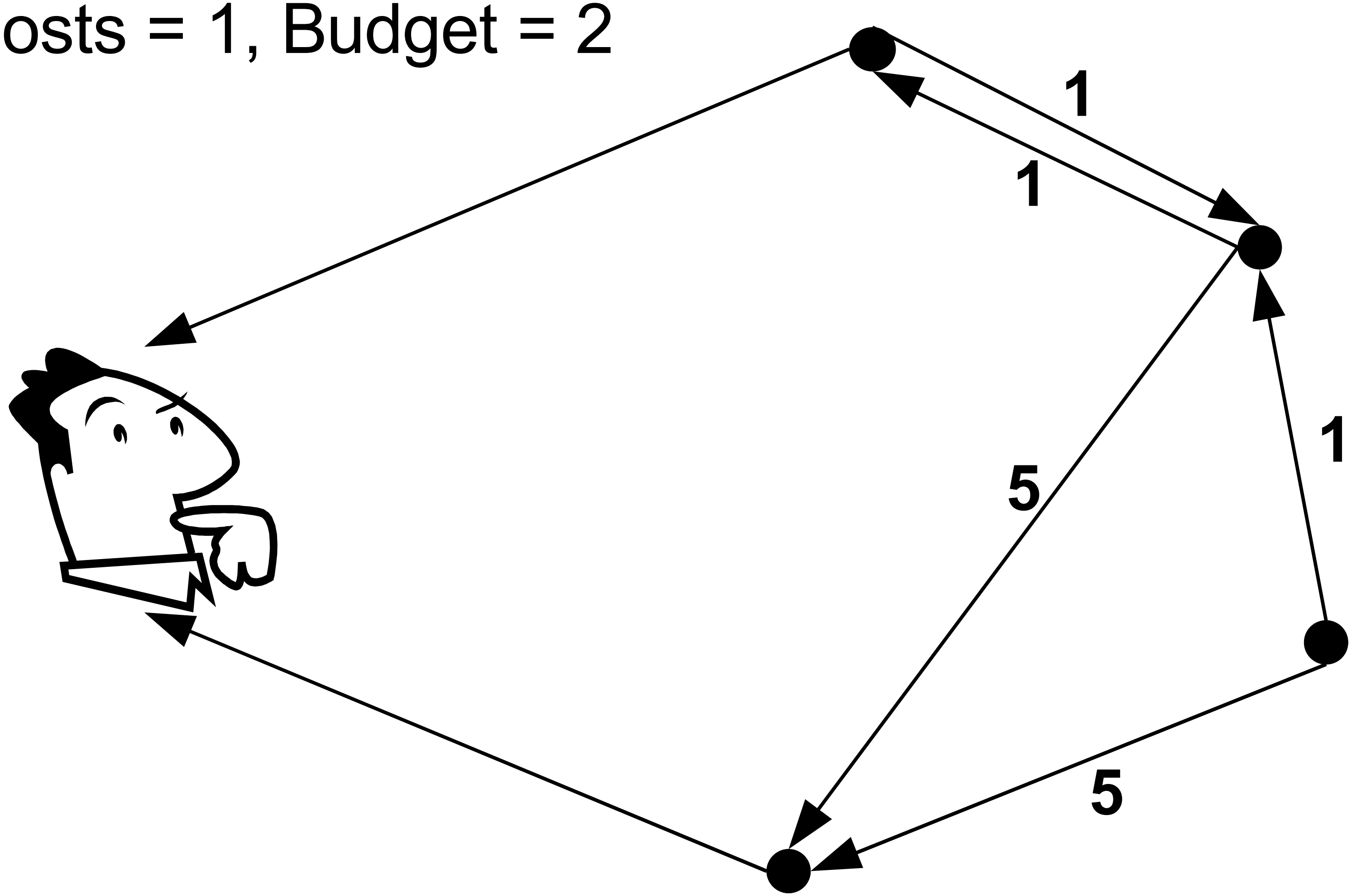
- Number of nodes
- Preference for each directed pair of nodes
- Link cost for each pair of nodes
- Budget of allowed link cost per node, *k(v)*
- Link length between each pair of nodes
- Each node *v* spends ≤ *k(v)* on links to minimize

$$\sum_{\text{other nodes}} (\text{preference} * \text{shortest path distance})$$

↑ or disconnection penalty if no path exists.

Example

Costs = 1, Budget = 2



Example

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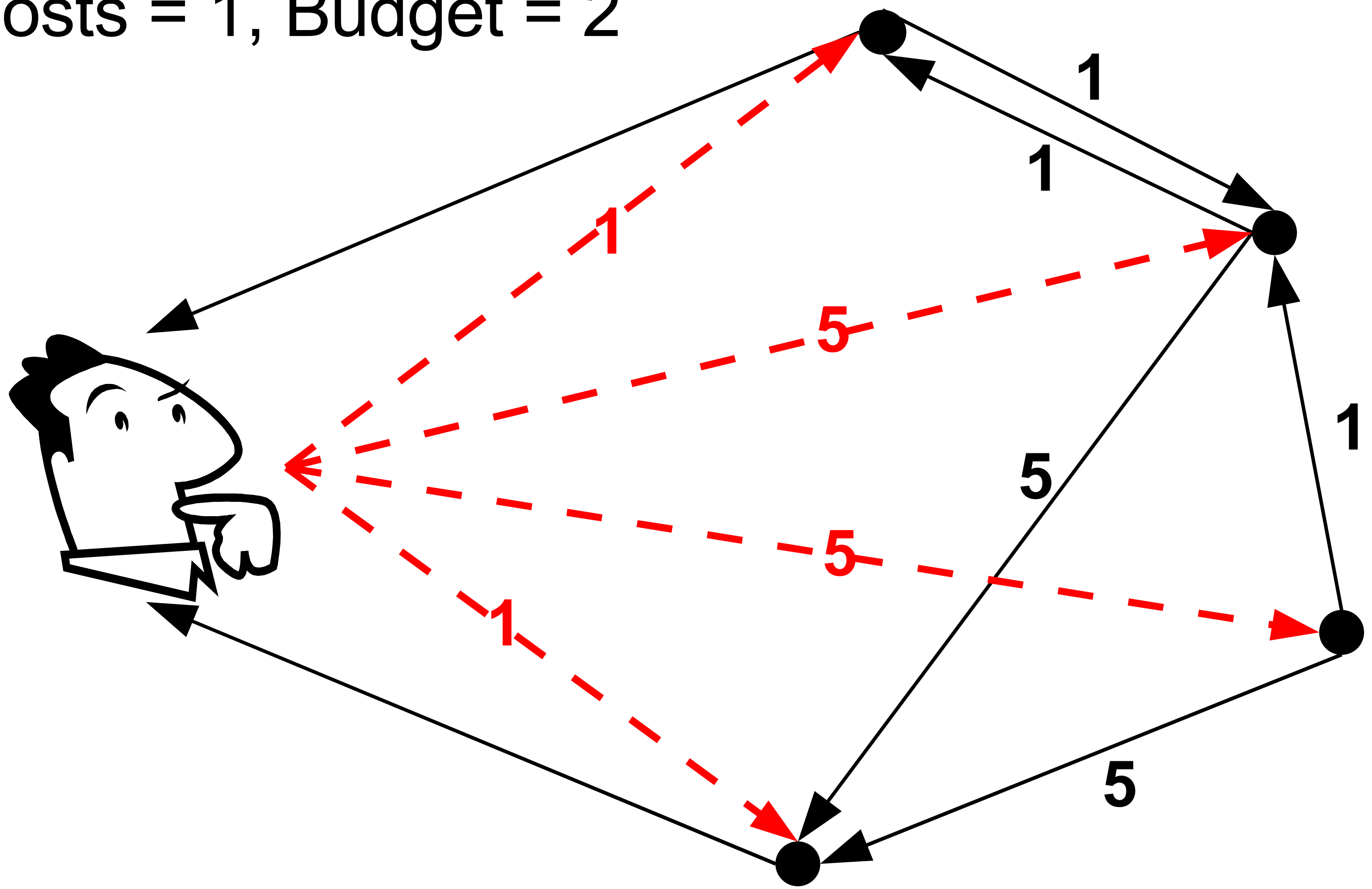

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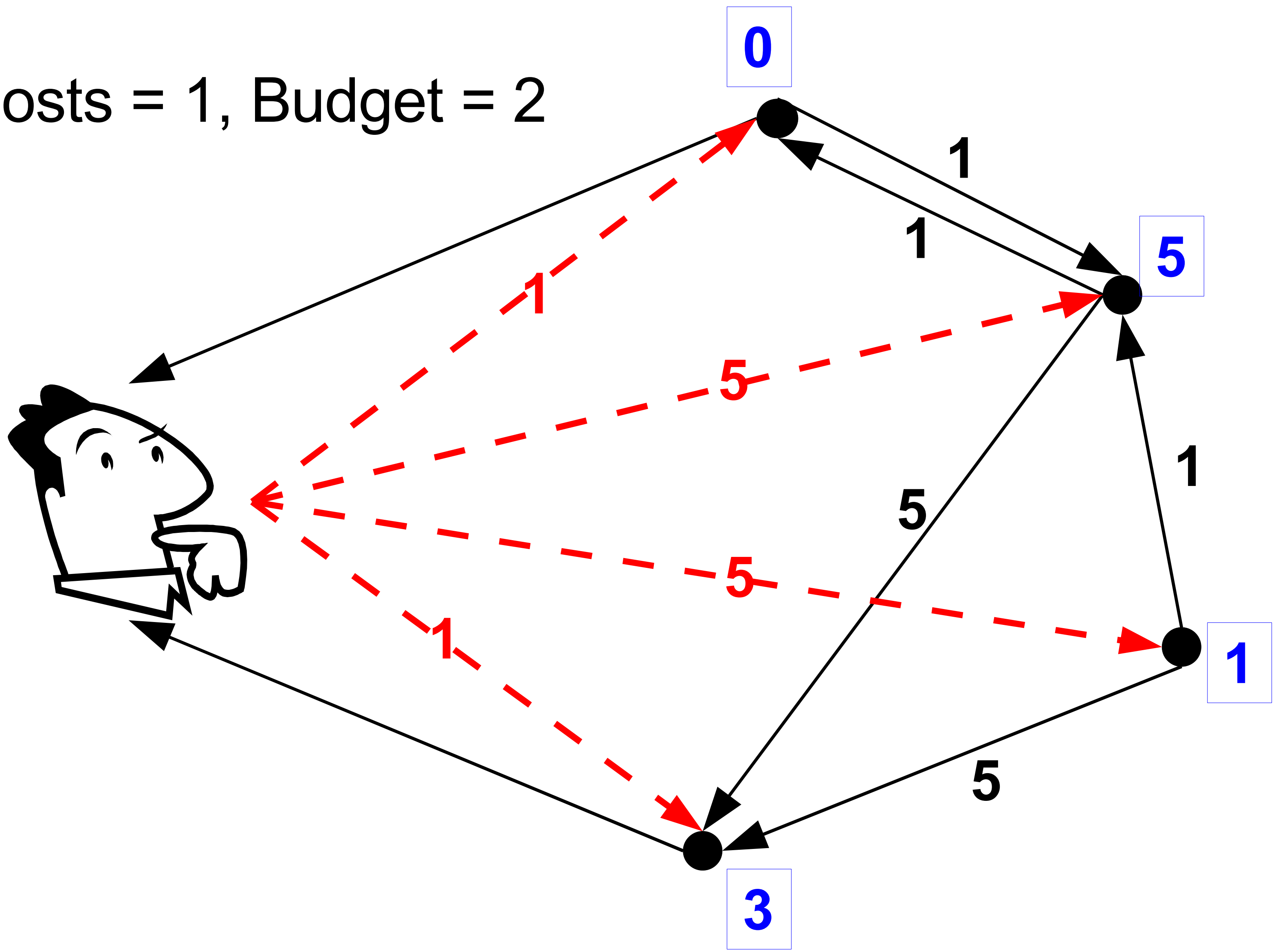

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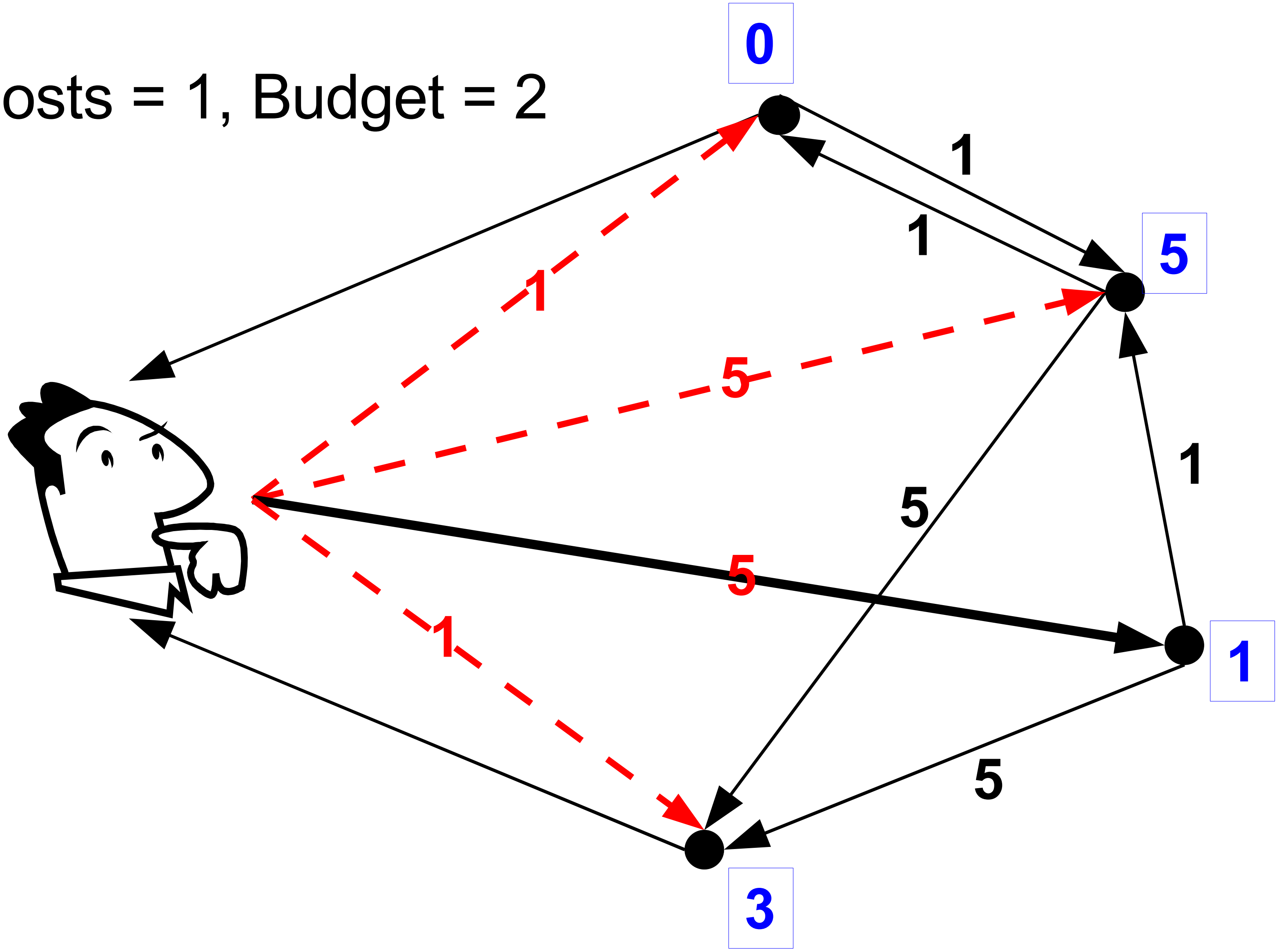

Example

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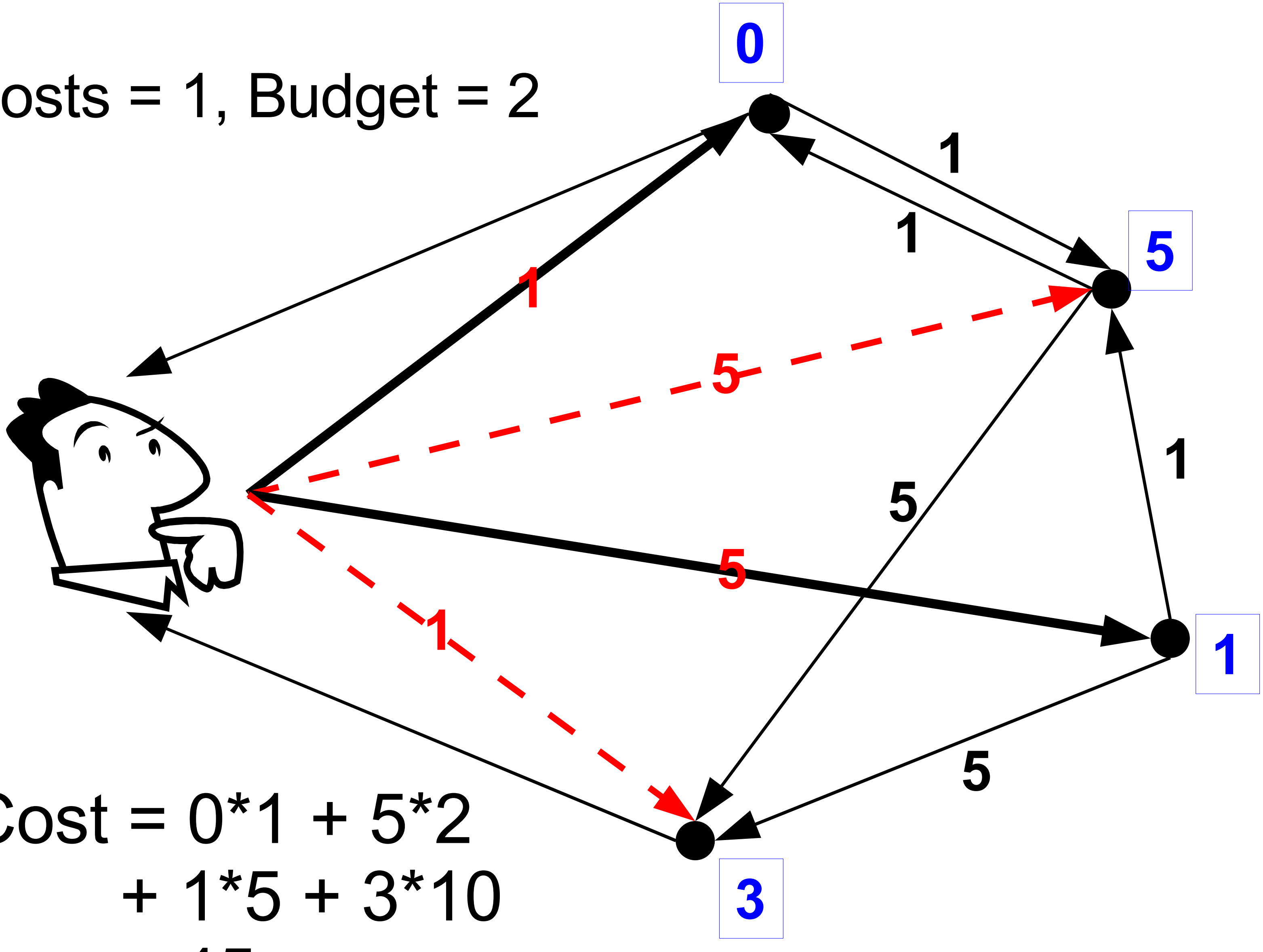
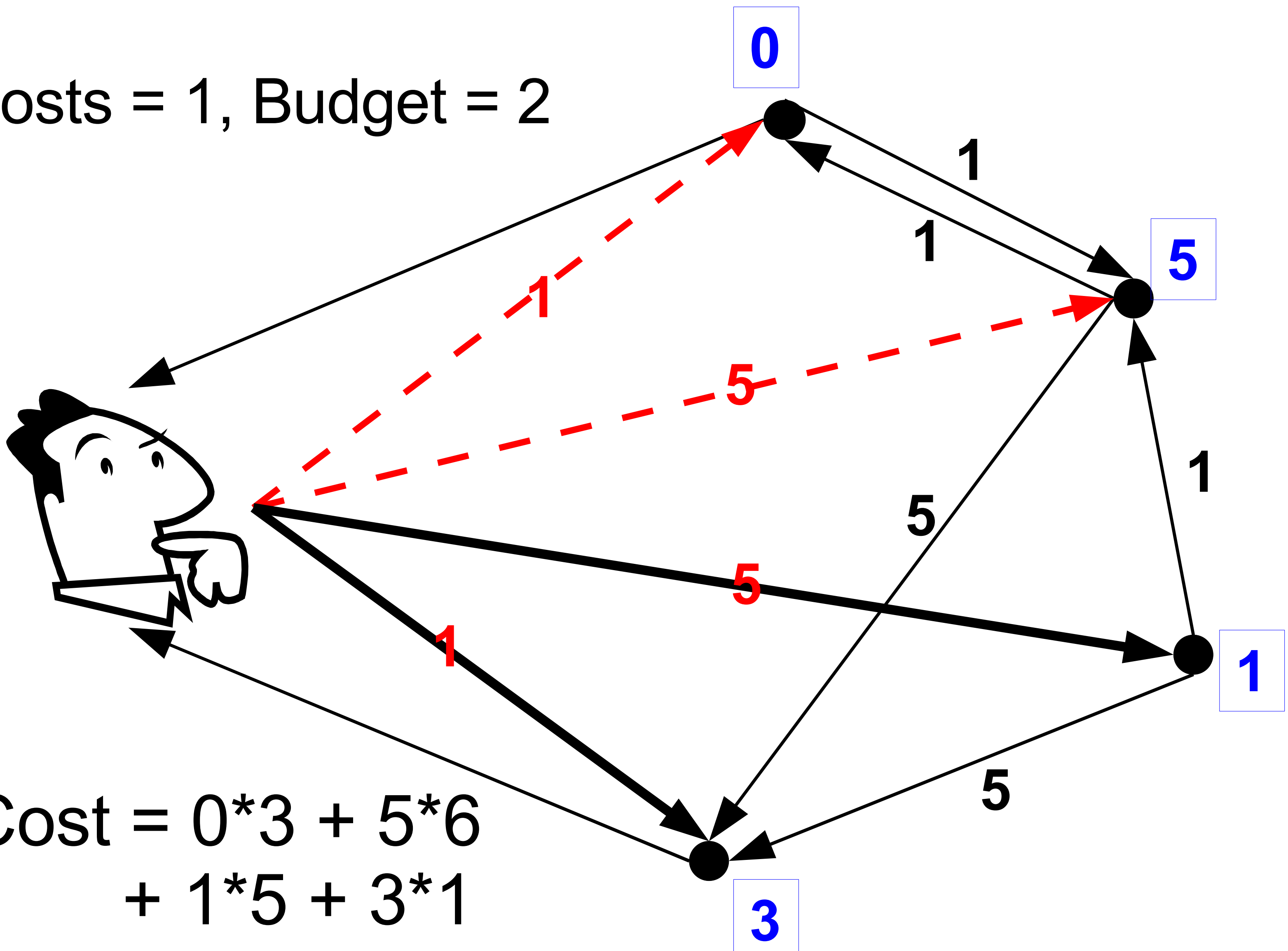


Cost = 0*1 + 5*2
+ 1*5 + 3*10
= 45

Example

Costs = 1, Budget = 2



Cost = 0*3 + 5*6
+ 1*5 + 3*1
= 38

The Model

- Number of nodes
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- Link cost for each pair of nodes
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- Each node *v* spends ≤ *k(v)* on links to minimize

$$\sum_{\text{other nodes}} (\text{preference} * \text{shortest path distance})$$

↑ or disconnection penalty if no path exists.

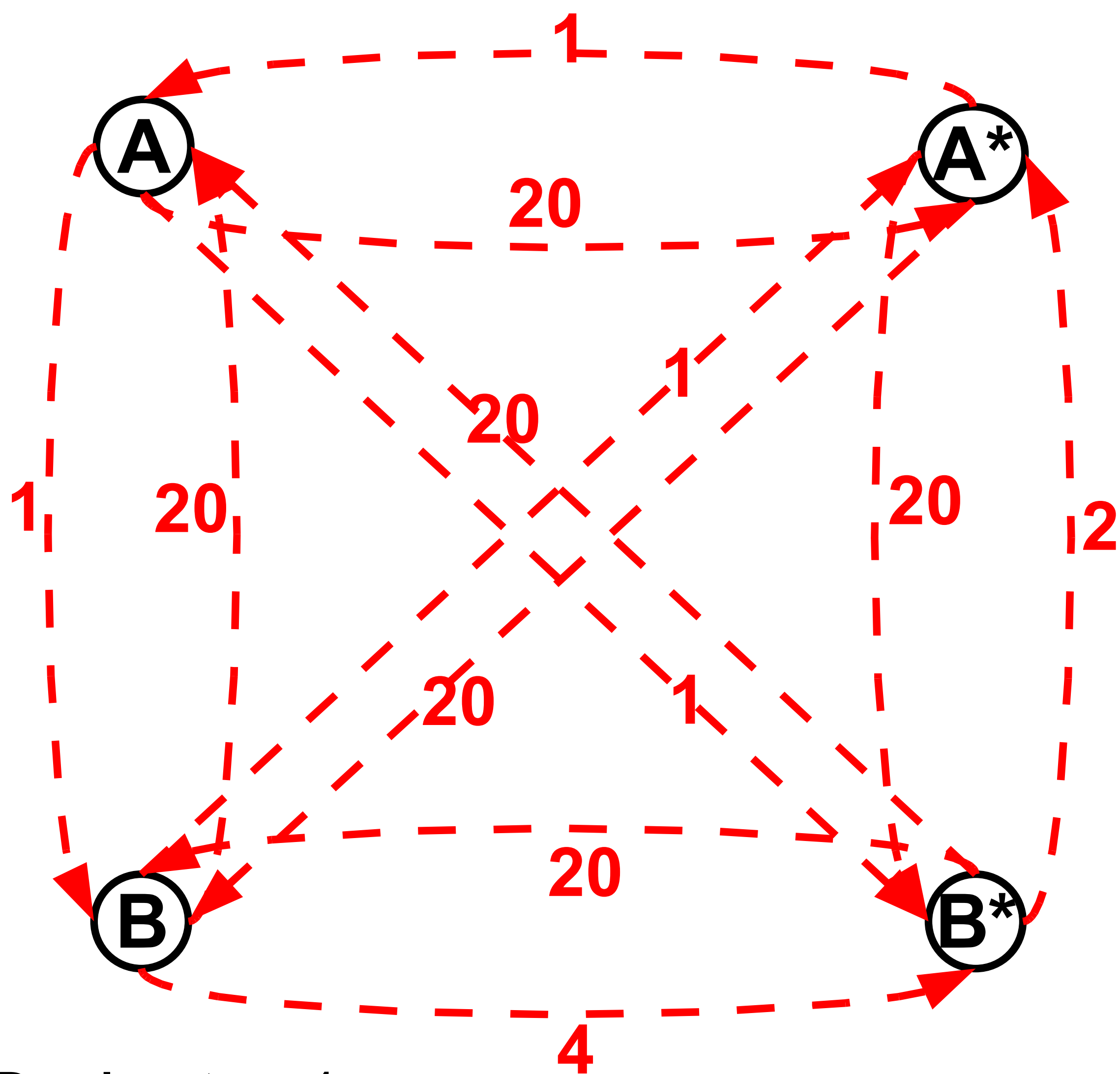
Previous Work

- Fabrikant, Luthra, Maneva, Papadimitriou, Shenker (PODC 2003) – Similar game without a budget for purchasing links.
- Albers, Eilts, Even-Dar, Mansour, Roditty (SODA 2006) and Demaine, Hajiaghavi, Mahini (PODC 2007) - Further work on the same game (without a budget).
- Halevi, Mansour (WINE 2007) – Fabrikant model, with added in preferences.

Our results

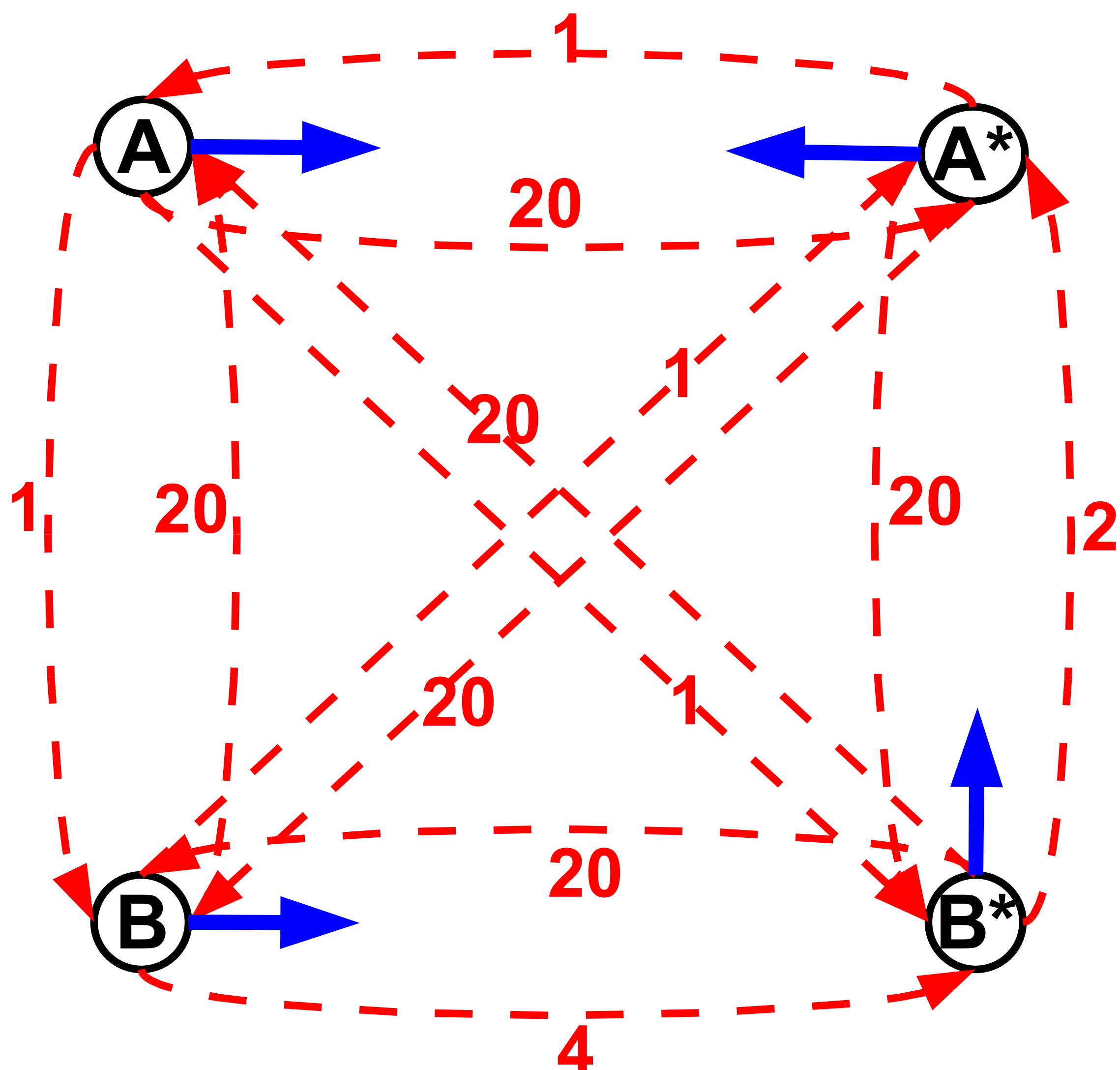
- Pure Nash equilibria do not always exist, and it is NP hard to determine whether they do.
- Uniform BBC games:
  - Pure Nash equilibrium always exists
  - Near-tight bounds on the Price of Anarchy and Price of Stability
  - Dynamics of best response walks
- Parallel results when the utility function is the distance to the furthest other node, not the average distance to all other nodes.

Sometimes no pure Nash equilibrium



Costs = 1, Budget = 1

Sometimes no pure Nash equilibrium



Costs = 1, Budget = 1

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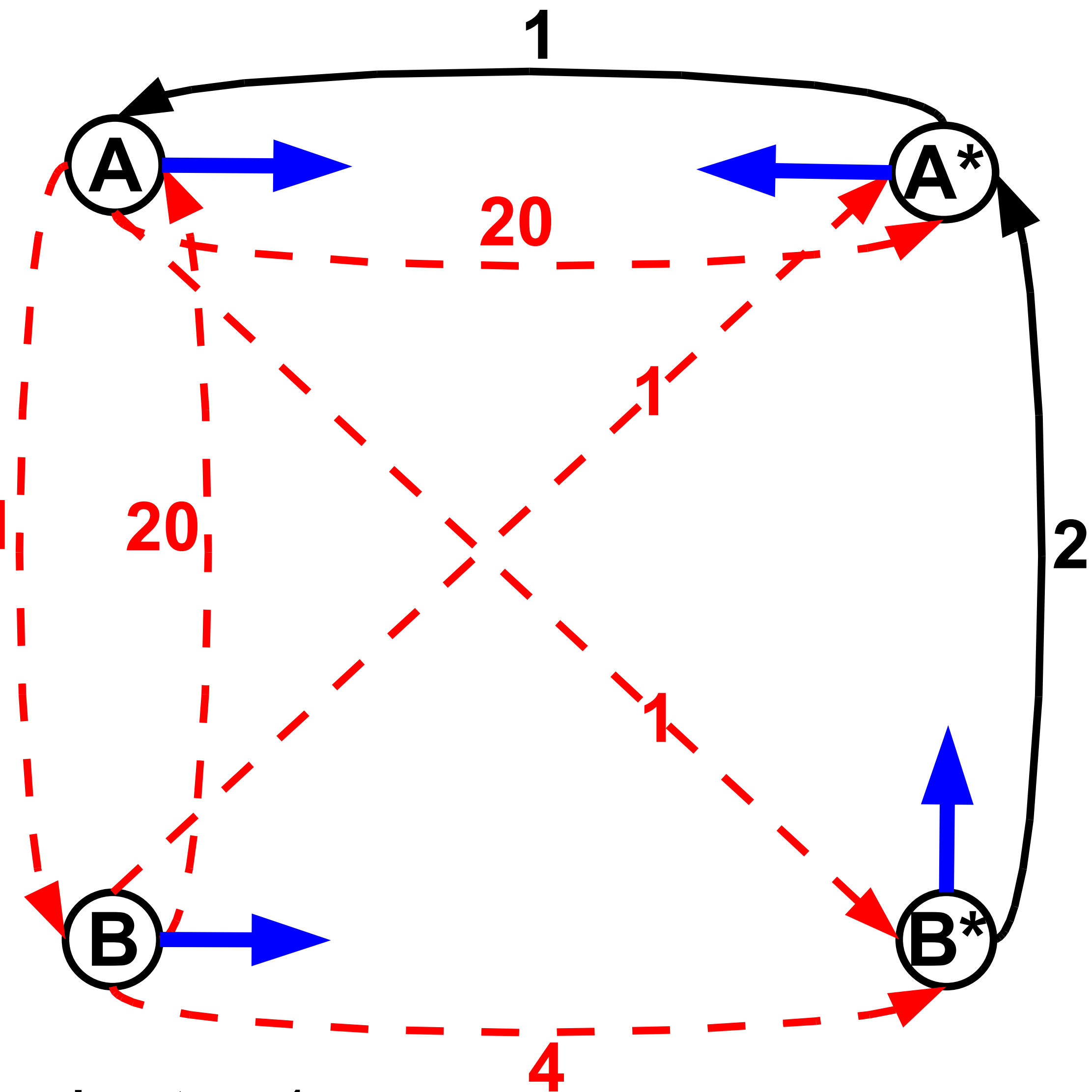

Costs = 1, Budget = 1

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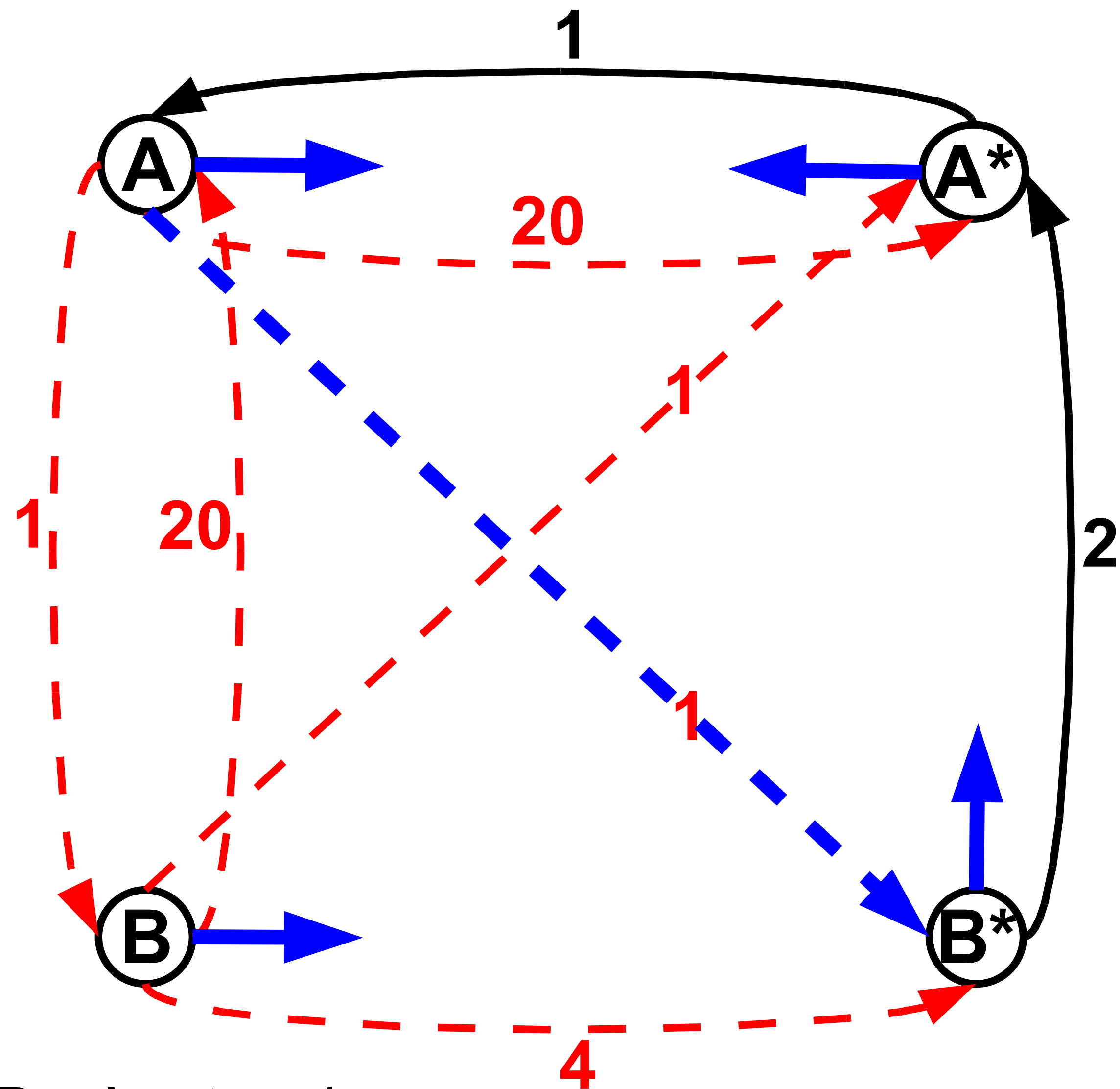
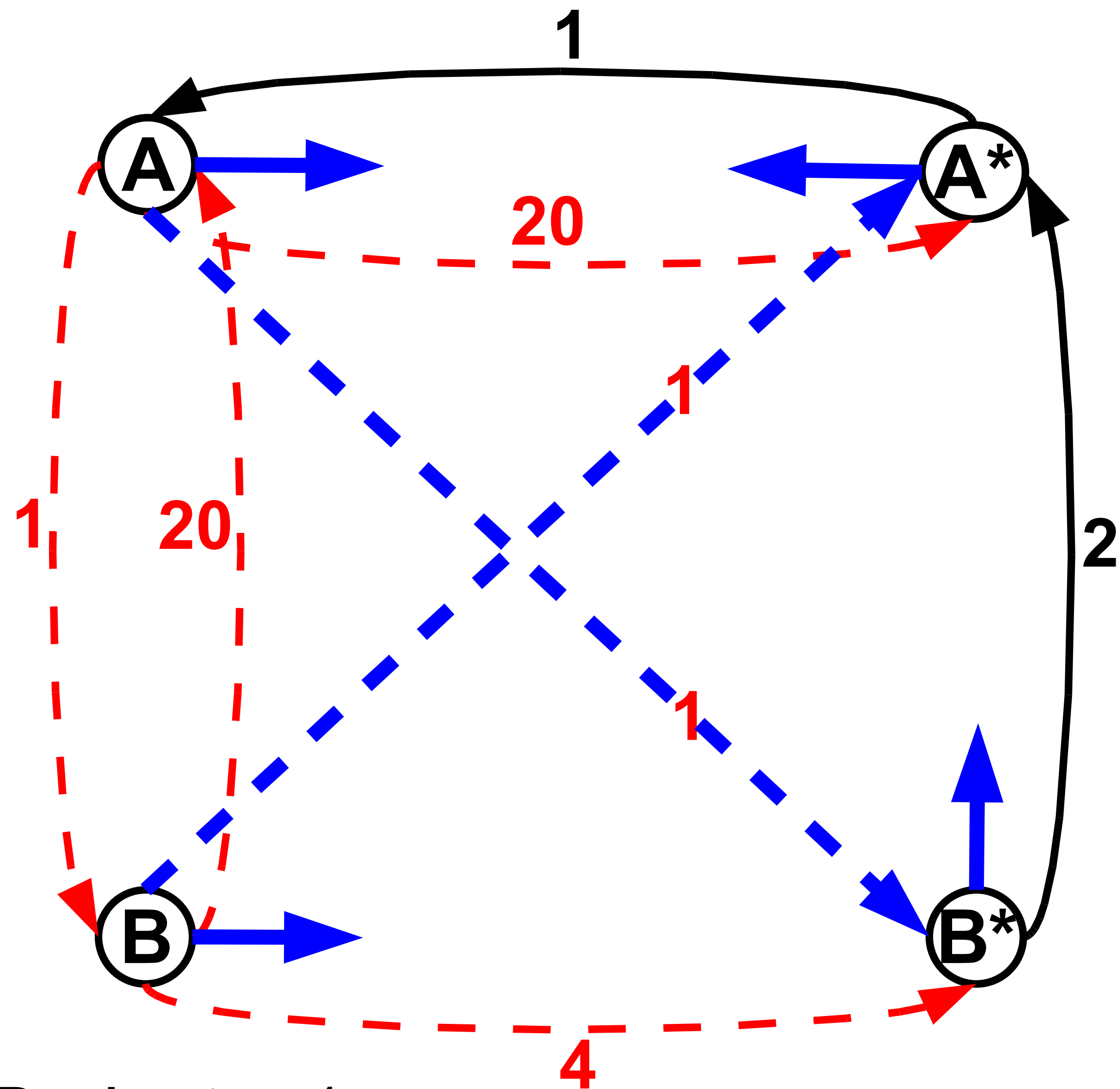


Costs = 1, Budget = 1

Sometimes no pure Nash equilibrium



Costs = 1, Budget = 1

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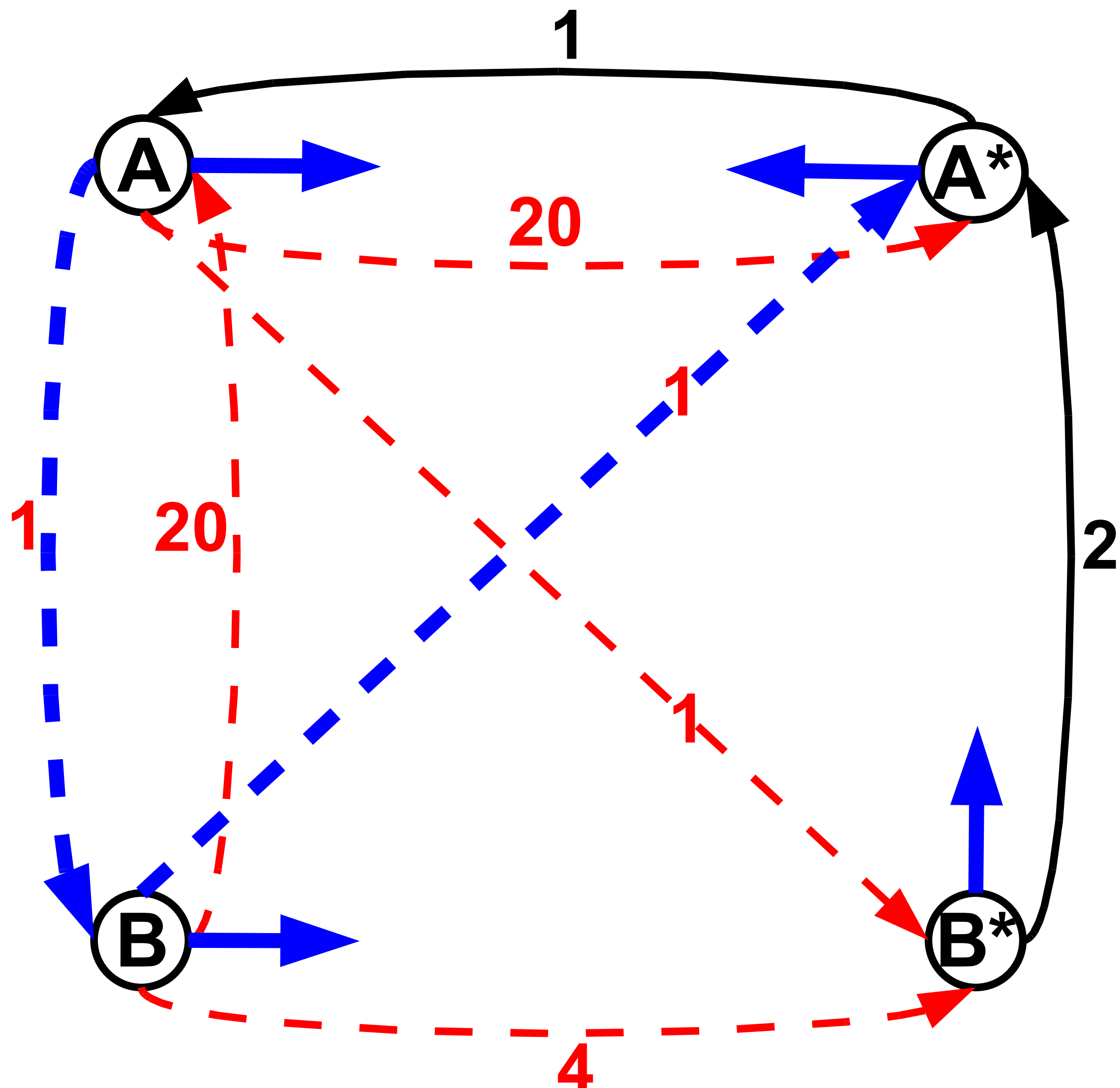

Costs = 1, Budget = 1

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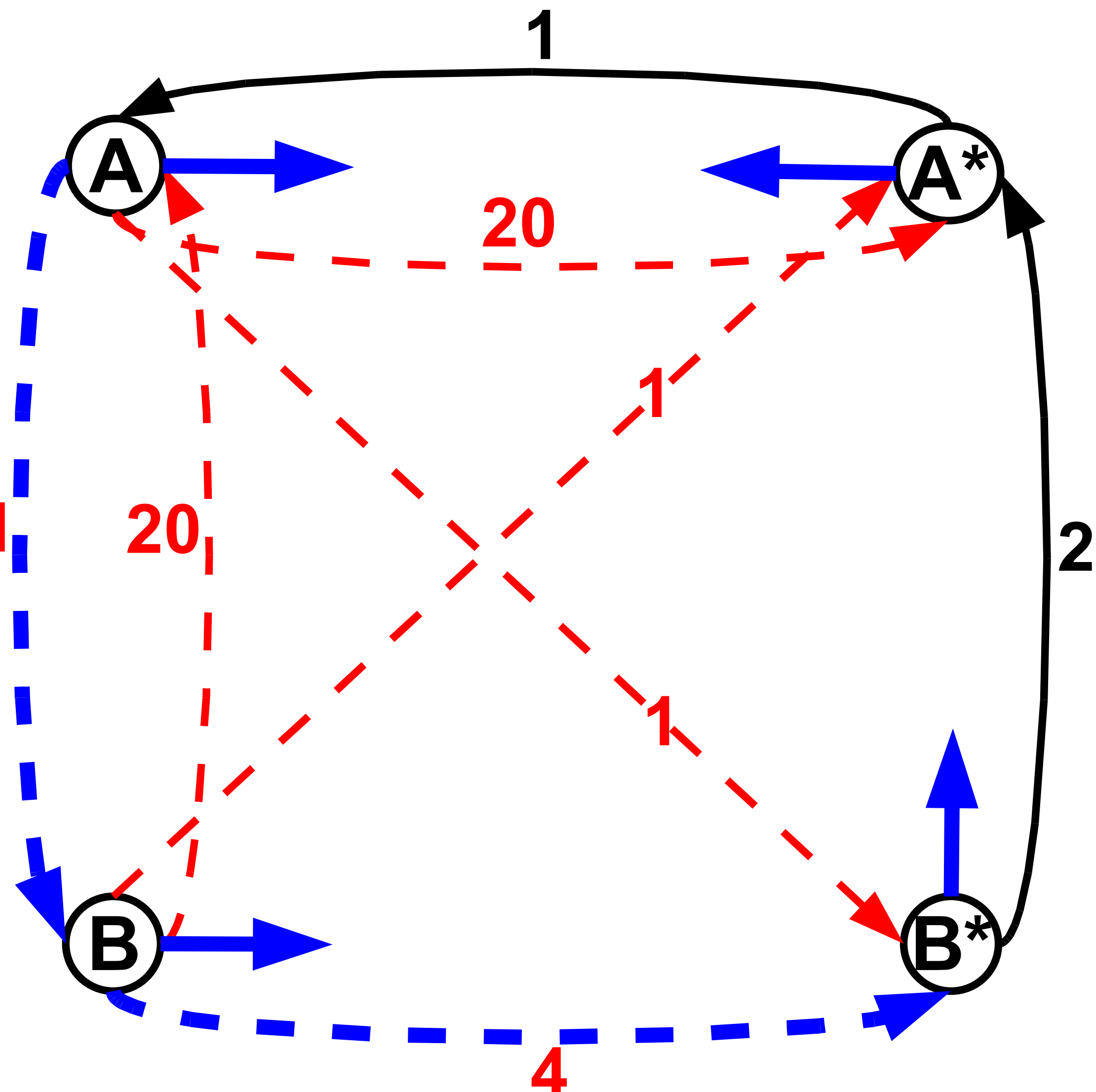
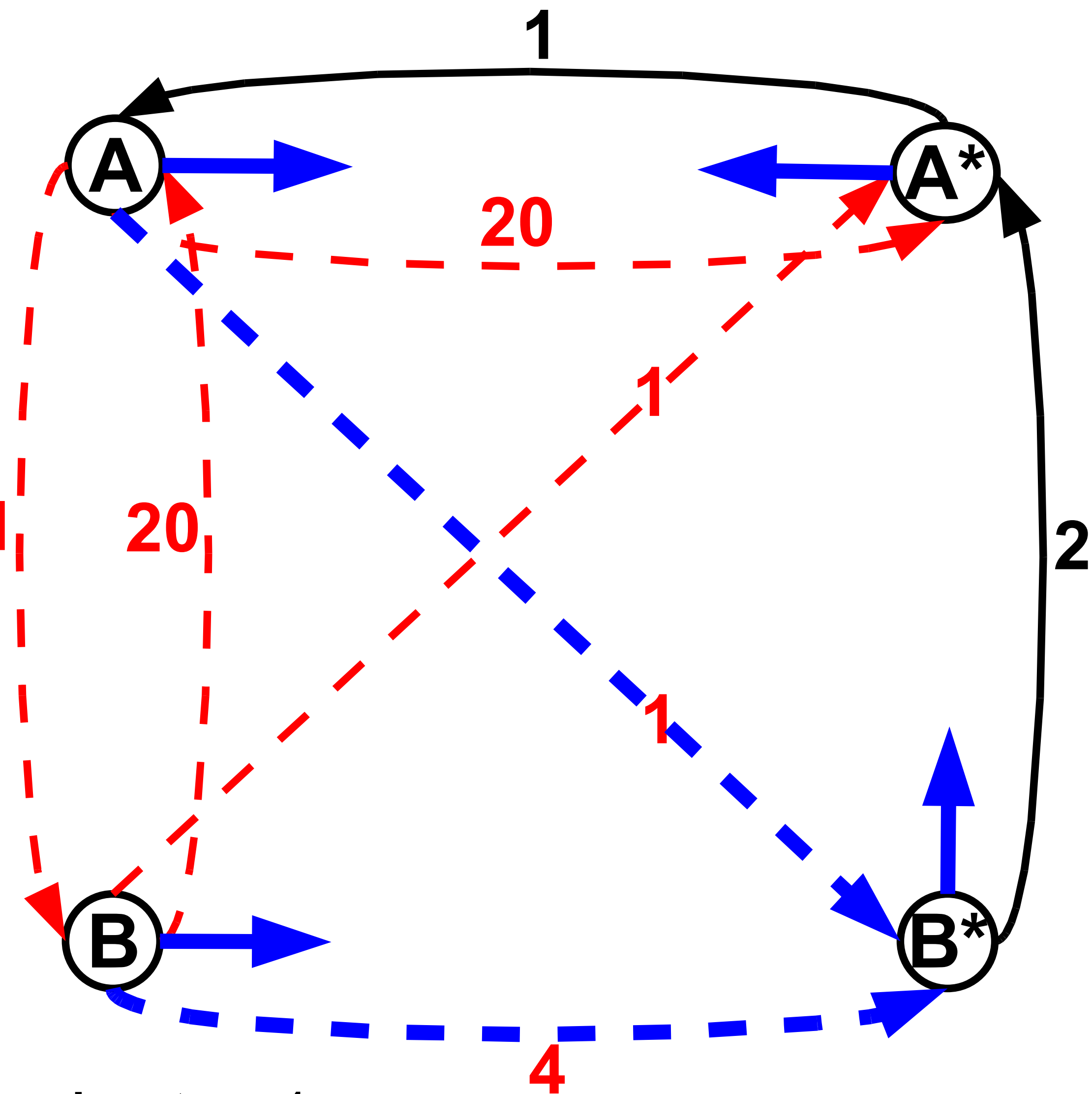


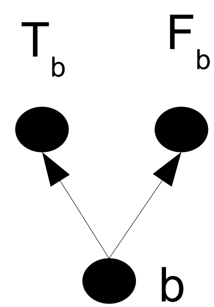
Costs = 1, Budget = 1

Sometimes no pure Nash equilibrium



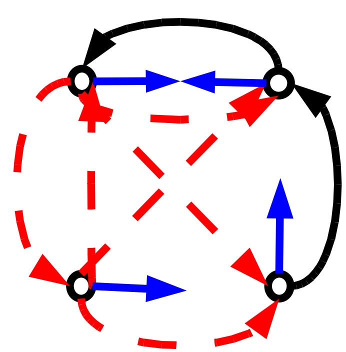
Costs = 1, Budget = 1

NP hard to determine existence of pure Nash equilibrium



Value nodes

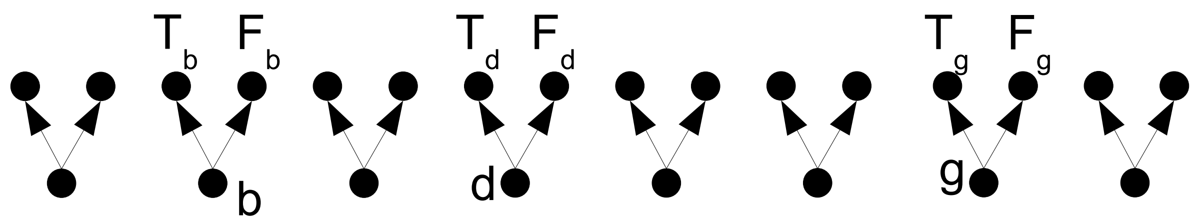
Variable node



Gadget with no Nash equilibrium

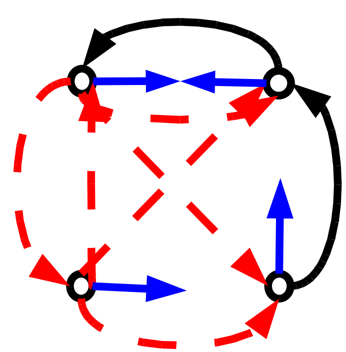
Reduction from 3-SAT

NP hard to determine existence of pure Nash equilibrium



Value nodes

Variable nodes



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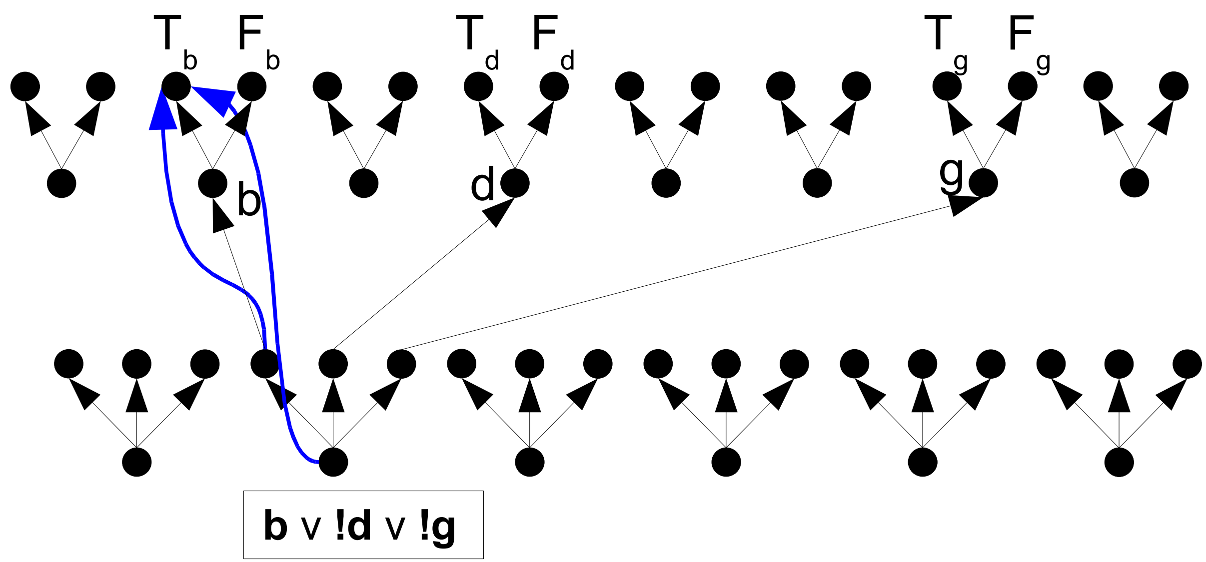
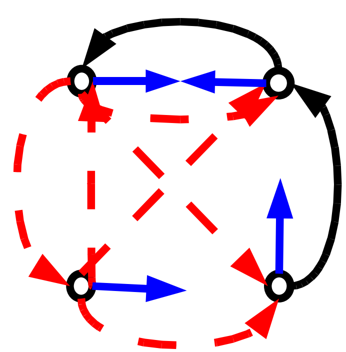


Value nodes

Variable nodes

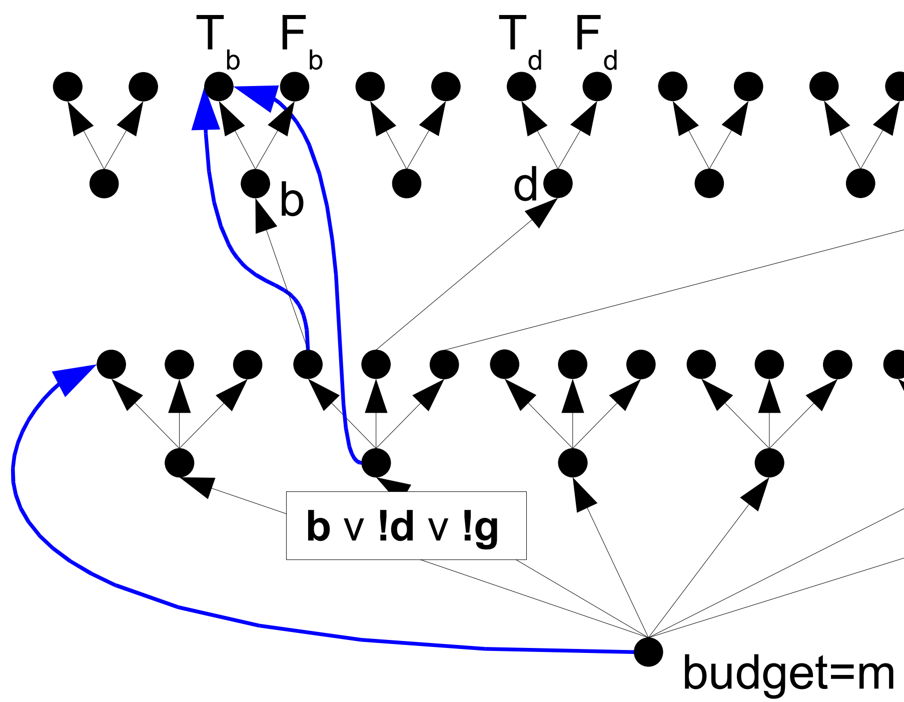
Clause nodes



Gadget with no Nash equilibrium

Reduction from 3-SAT

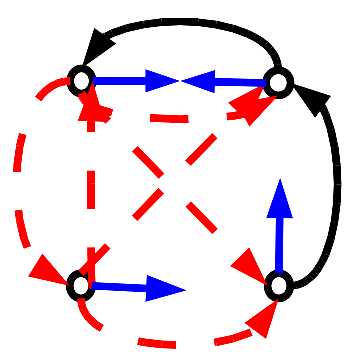
NP hard to determine existence of pure Nash equilibrium



Value nodes

Variable nodes

Clause nodes



Gadget with no Nash equilibrium

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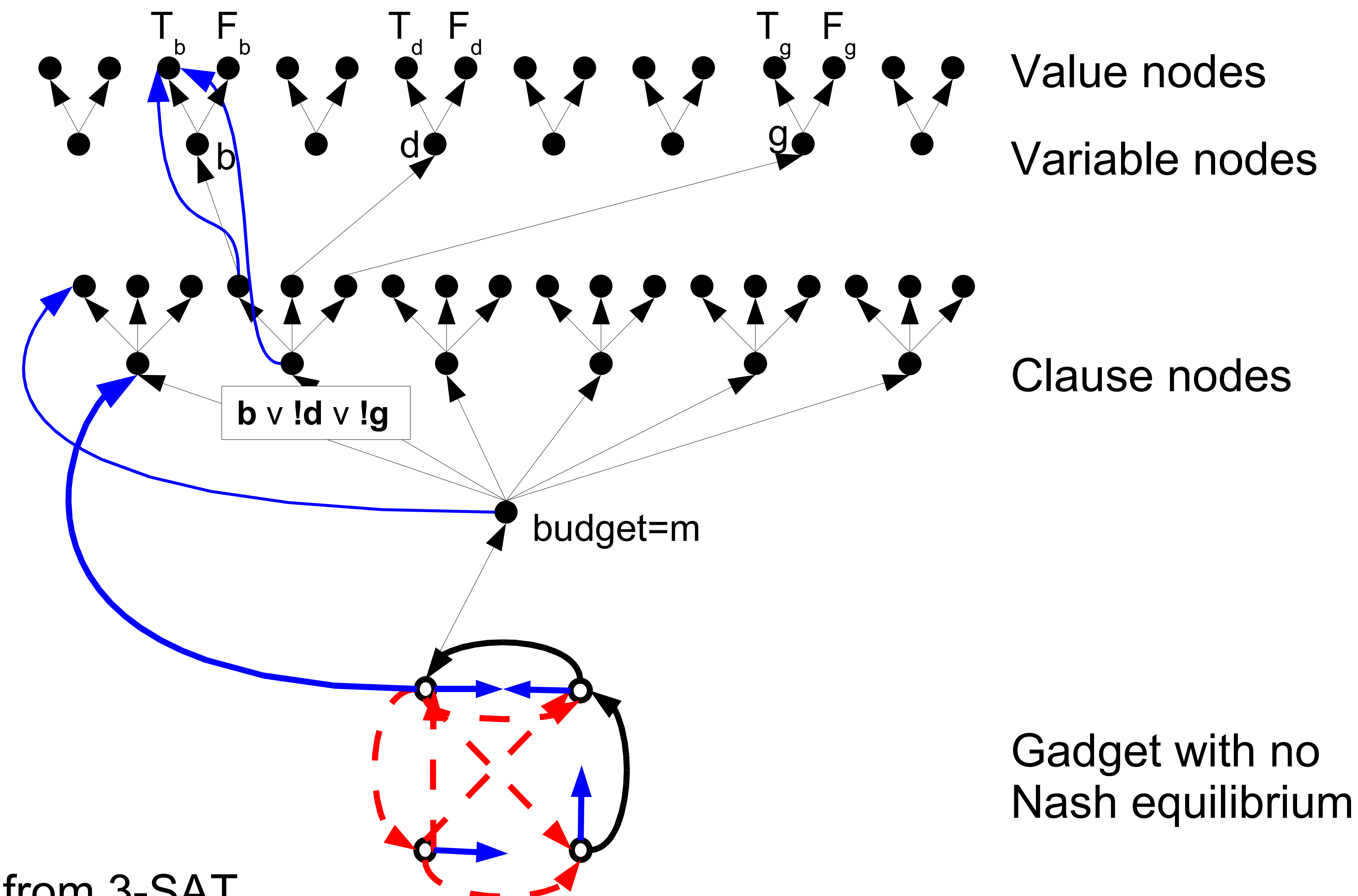


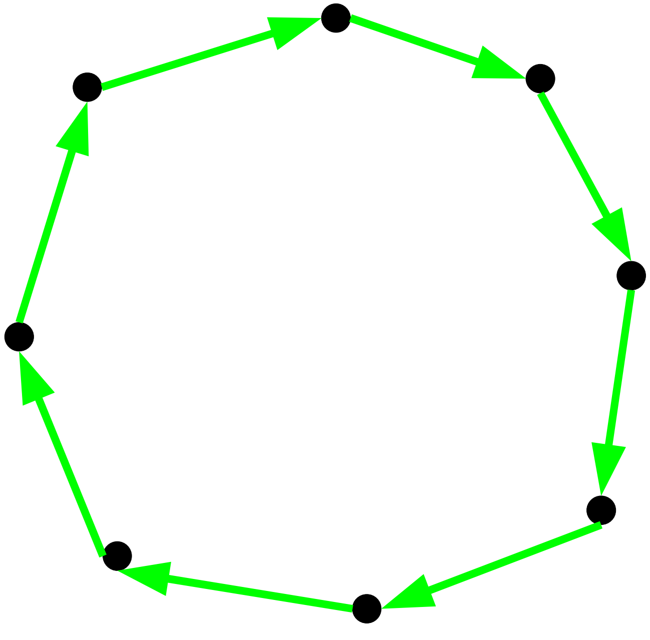
Reduction from 3-SAT

Uniform BBC Games

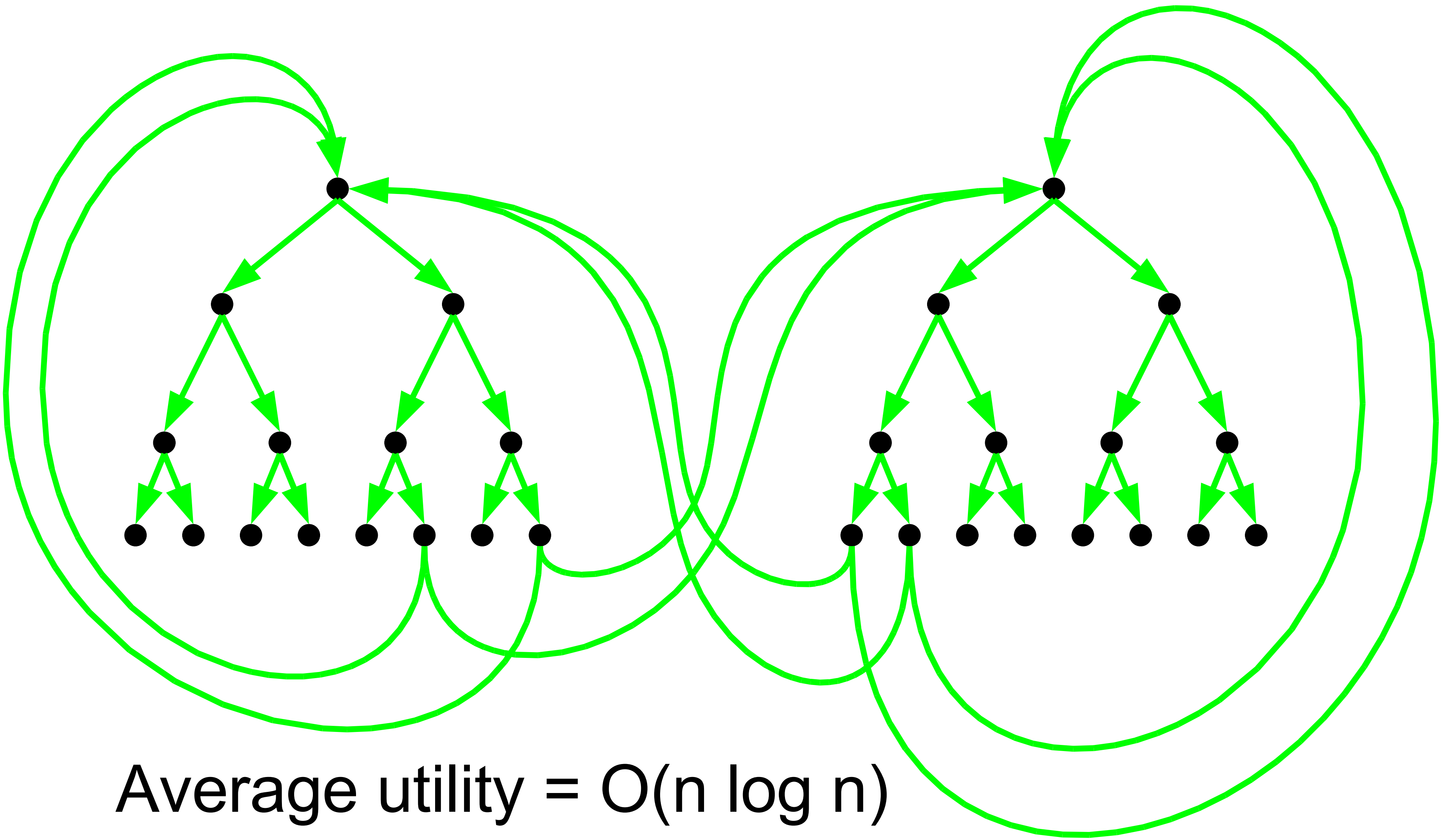
- Budgets are all k
- Costs are all 1 (can buy any k links)
- Lengths are all 1 (hop count path length)
- Preferences are all 1

- (n,k)-uniform games

Uniform, k=1

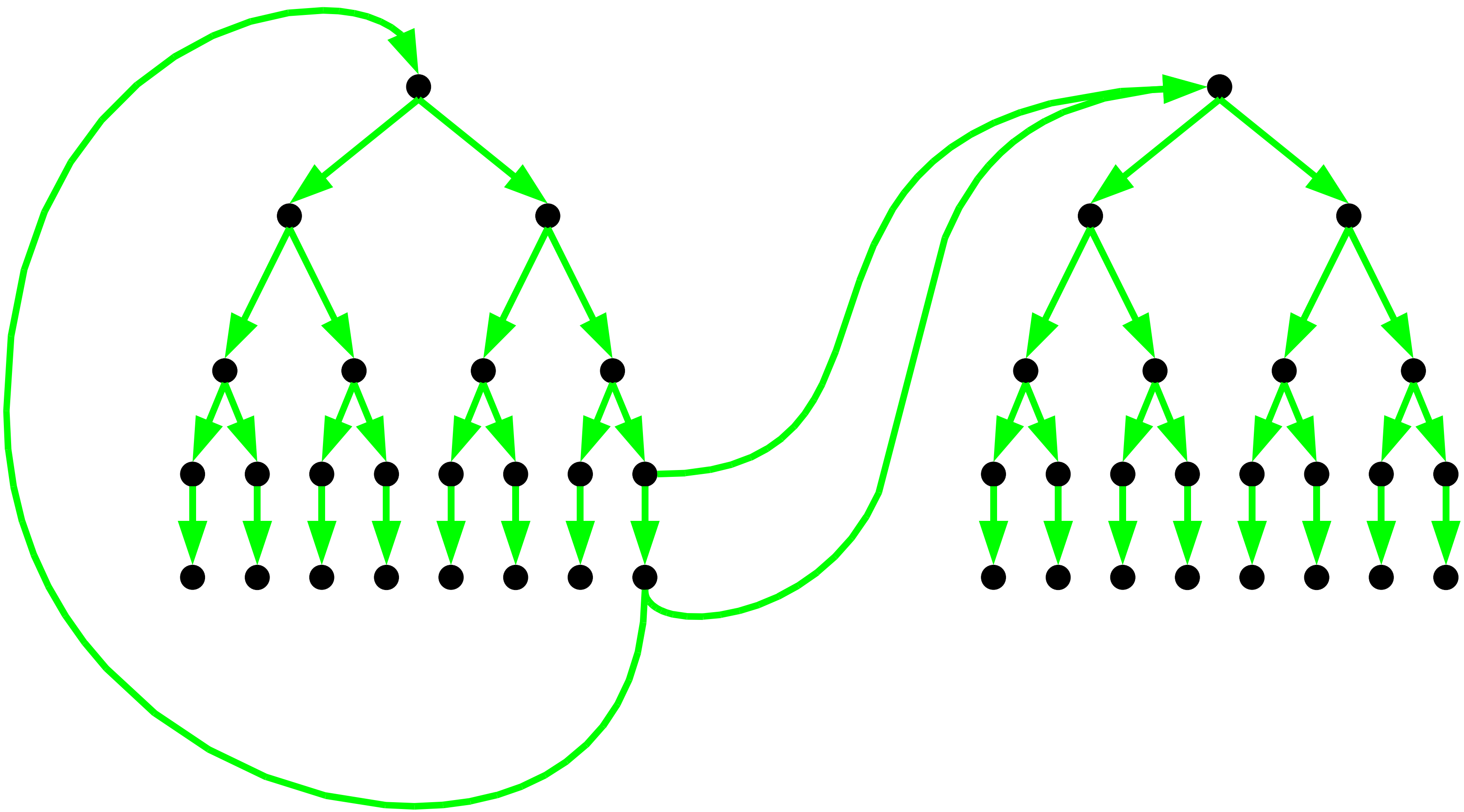


Uniform, k=2

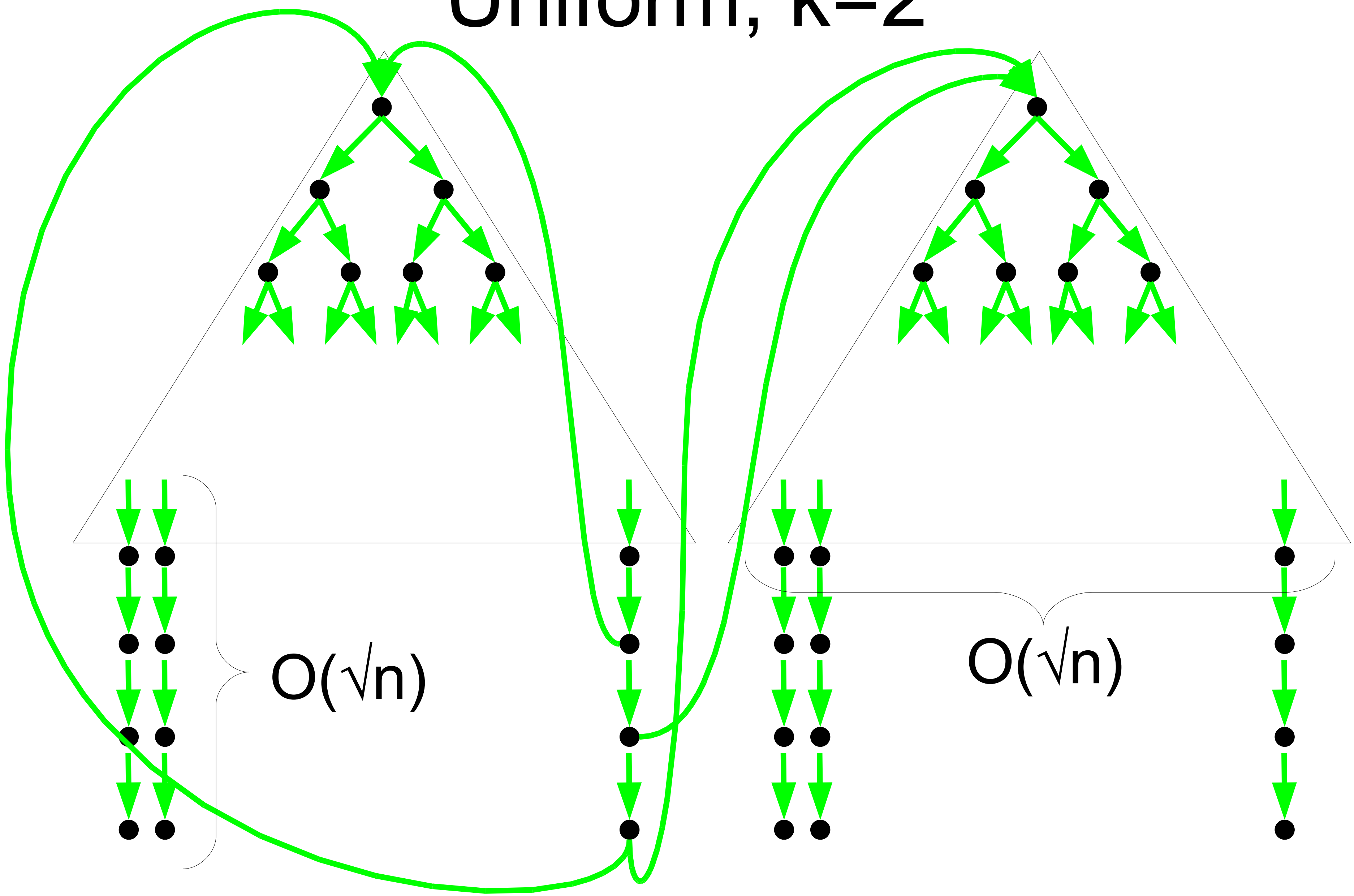


Average utility = O(n log n)

Uniform, k=2

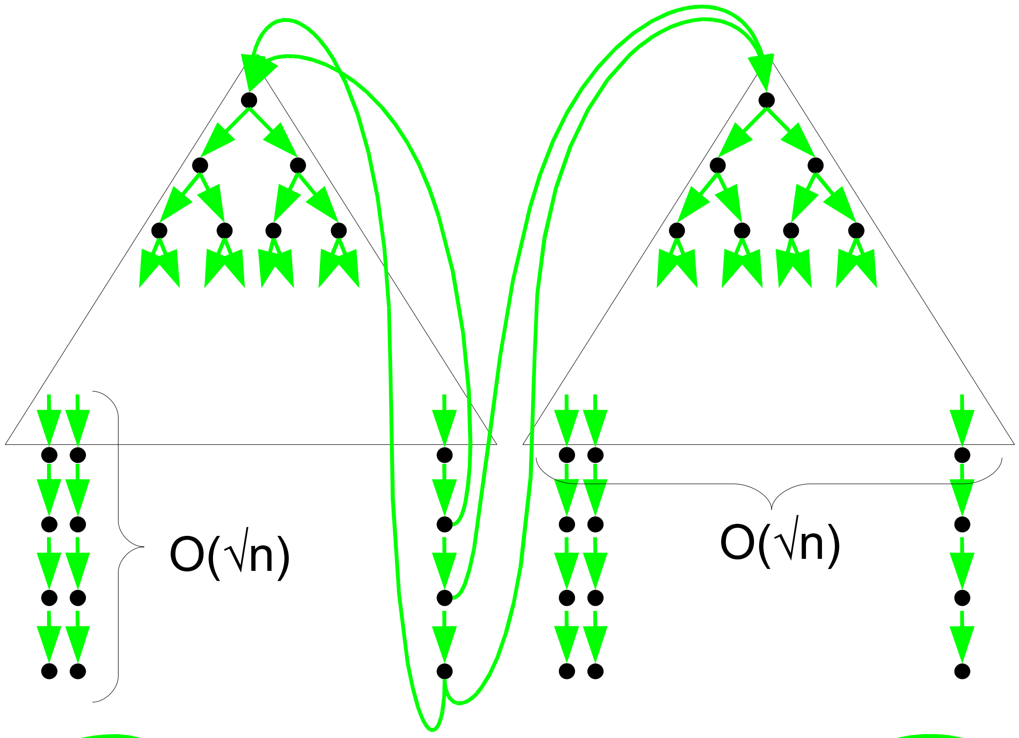


Uniform, k=2

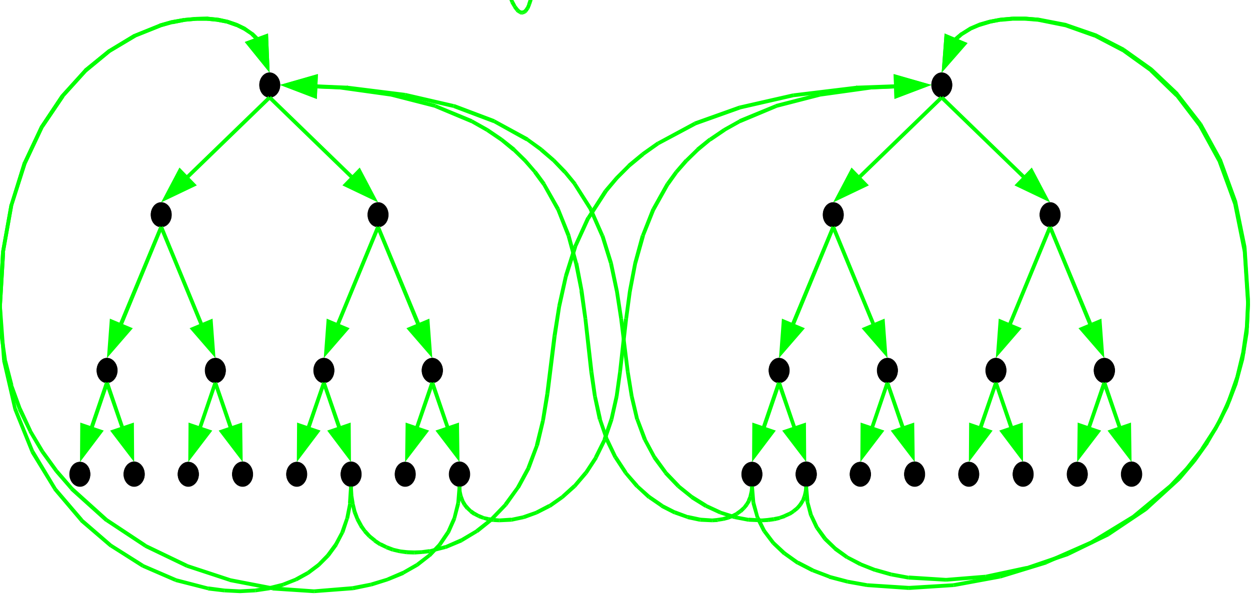


Average utility = O(n √n)

Uniform, k=2

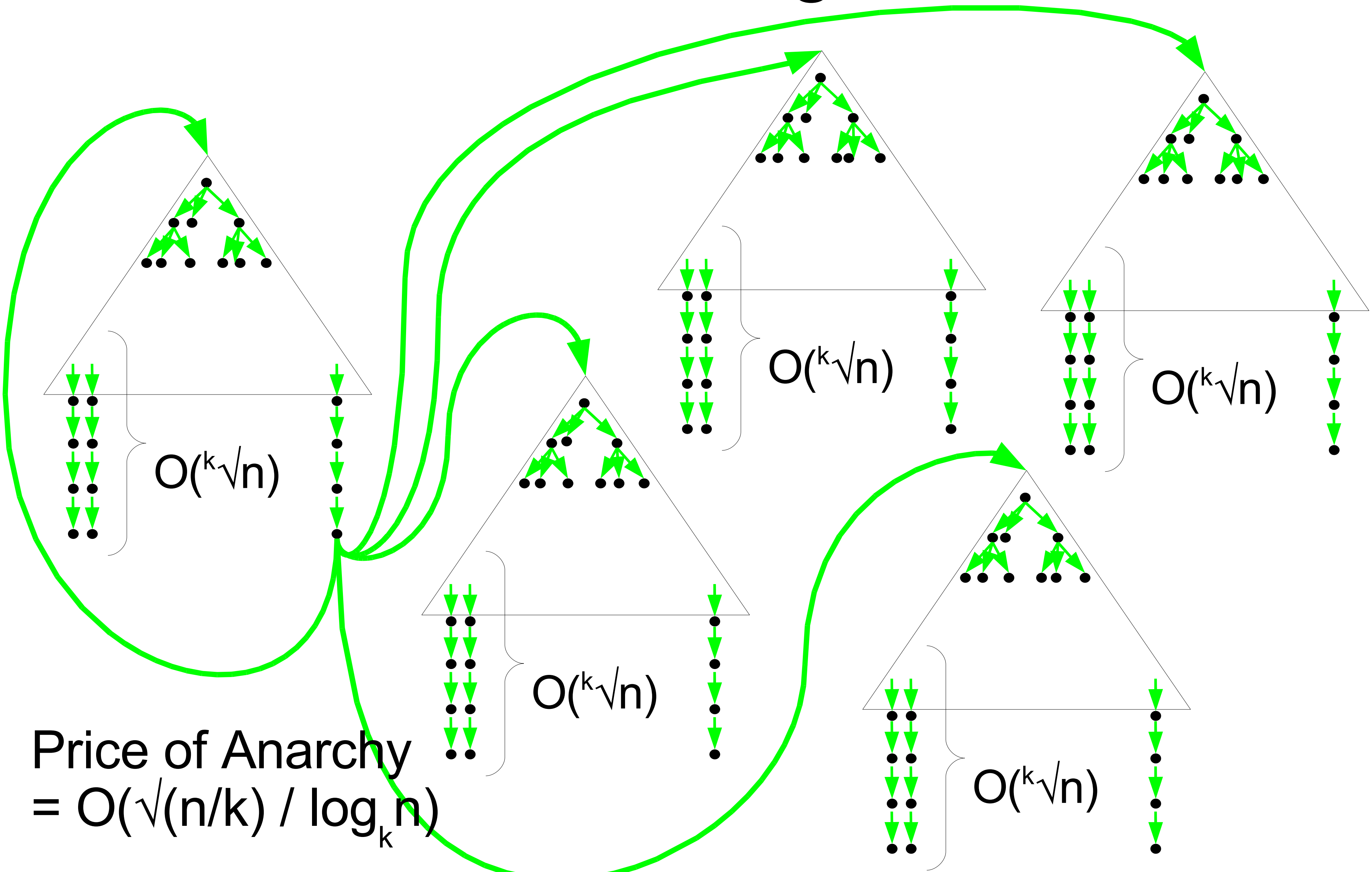


Average utility =
$O(n \sqrt{n})$



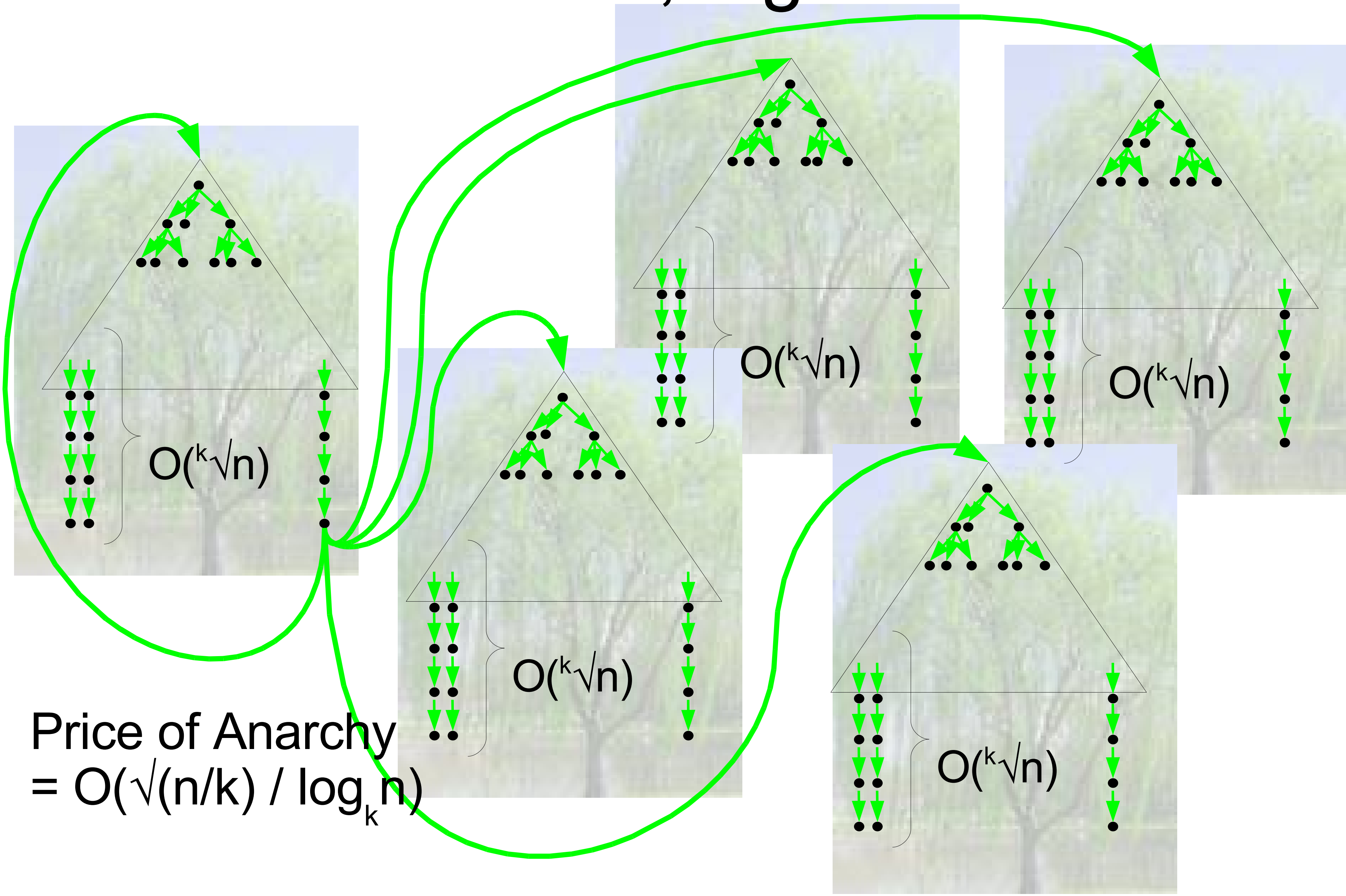
Average utility =
$O(n \log n)$

Uniform, higher k



Price of Anarchy
$= O(\sqrt{(n/k)} / \log_k n)$

Uniform, higher k



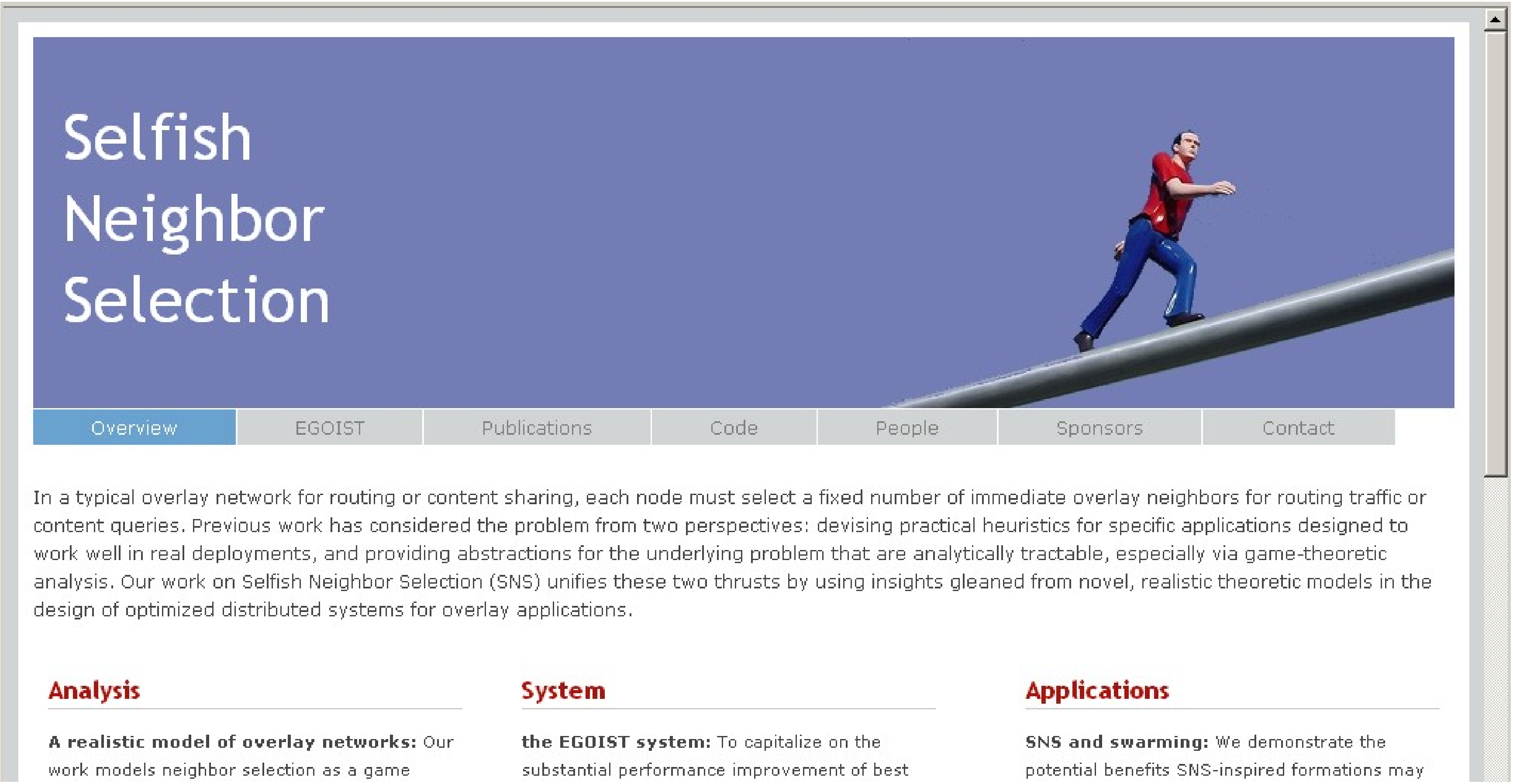
Price of Anarchy
= $O(\sqrt{(n/k)} / \log_k n)$

Open Problems

- Is there always a pure Nash equilibrium if only the budget is non-uniform?
- For general BBC games, can we find an approximate equilibrium in polynomial time?
- Convergence via best response walks: will any initial graph converge to a pure Nash equilibrium for uniform BBC games? If so, will it be a “good” equilibrium?
- Tighter bounds on Price of Anarchy
- Same game with undirected graphs

Experimental work on this model

- http://csr.bu.edu/sns/



Smaragdakis, Bestavros, Byers, Laoutaris, Michiardi, Lekakis, Roussopoulos