Lecture 8 - Double integrals in cartesian and polar coordinates - 7/21/2014 — Interphase 2014 Calc 3

22. Double integrals in Cartesian coordinates

    a. In Cartesian coordinates, $dA = dxdy$.

    b. The double integral over the rectangle $[a, b] \times [c, d]$ can be evaluated as an iterated integral

$$\iint_{[a,b]\times[c,d]} f(x, y)dA = \int_a^b \left(\int_c^d f(x, y)dy \right) dx = \int_c^d \left(\int_a^b f(x, y)dx \right) dy$$

    c. Many complicated regions can be written as:

    - (I) An x-dependent range of y:

$$x_{\text{min}} \leq x \leq x_{\text{max}}, y_{\text{min}}(x) \leq y \leq y_{\text{max}}(x)$$

    - (II) A y-dependent range of x:

$$y_{\text{min}} \leq y \leq y_{\text{max}}, x_{\text{min}}(y) \leq x \leq x_{\text{max}}(y)$$

    d. If R is of form (I) or (II), then the double integral can be written as an iterated integral:

$$\iint_R f(x, y)dxdy = \int_{x_{\text{min}}}^{x_{\text{max}}} \left(\int_{y_{\text{min}}(x)}^{y_{\text{max}}(x)} f(x, y)dy \right) dx$$
$$\iint_R f(x, y)dxdy = \int_{y_{\text{min}}}^{y_{\text{max}}} \left(\int_{x_{\text{min}}(y)}^{x_{\text{max}}(y)} f(x, y)dx \right) dy$$

    e. To evaluate a double integral

    1. Draw the region of integration
    2. Describe the region as a x-dependent range of y or a y-dependent range of x
    3. Write down the iterated integrals
    4. Simplify and evaluate the inside integral
    5. Evaluate the outside integral

23. Double integrals in polar coordinates

    a. Any point in 2d can be described by (r, θ), where $r \geq 0$ and $-\pi < \theta \leq \pi$ or $0 \leq \theta < 2\pi$.

    b. The point (r, θ) in polar coordinates corresponds to $(r \cos \theta, r \sin \theta)$ in Cartesian coordinates.

    c. In polar coordinates, the area element is $dA = rdrd\theta$.

    d. To evaluate an integral in polar coordinates, write the region as

    - (I) An r-dependent range of θ:

$$r_{\min} \leq r \leq r_{\max}, \theta_{\min}(r) \leq \theta \leq \theta_{\max}(r)$$

- (II) A θ-dependent range of r:

$$\theta_{\min} \leq \theta \leq \theta_{\max}, r_{\min}(\theta) \leq r \leq r_{\max}(\theta)$$

Then, write the iterated integral

$$\iint_R f dA = \iint_R f(r,\theta) r dr d\theta = \int_{r_{\min}}^{r_{\max}} \left(\int_{\theta_{\min}(r)}^{\theta_{\max}(r)} f(r,\theta) r d\theta \right) dr$$

$$\iint_R f dA = \iint_R f(r,\theta) r dr d\theta = \int_{\theta_{\min}}^{\theta_{\max}} \left(\int_{r_{\min}(\theta)}^{r_{\max}(\theta)} f(r,\theta) r dr \right) d\theta$$