Lecture 7 - Double integrals - 7/15/2014 — Interphase 2014 Calc 3

20. Double integrals (conceptually)

a. The double integral $\iint_R f(x,y)dA$ gives the volume under the surface $z = f(x,y)$ over the 2d region R.

b. The double integral as a Riemann sum:

$$\iint_R f(x,y)dA \approx \sum f(x,y)\Delta A$$

where the sum is over all small boxes of area ΔA that are part of R.

c. To write down a double integral for a quantity, consider a small box of area ΔA located at (x,y). It will contribute $f(x,y)\Delta A$ to the quantity of interest. The quantity is then given by the double integral $\iint_R f(x,y)dA$.

21. Applications of double integrals

a. The area of a 2d region R is $\iint_R dA$.

b. The volume under a surface $f(x,y)$ over the 2d region R is $\iint_R f(x,y)dA$.

c. The mass of a 2d plate with area density $\rho(x,y)$ is $M = \iint_R \rho(x,y)dA$.

d. The moment of inertia of a 2d plate with area density $\rho(x,y)$ about a given axis is $I = \iint_R \rho(x,y)d^2(x,y)dA$, where $d(x,y)$ is the distance from the point (x,y) to the axis of rotation.

e. The coordinates of the center of mass of a 2d plate with area density $\rho(x,y)$ is

$$\bar{x} = \frac{\iint_R x\rho(x,y)dA}{\iint_R \rho(x,y)dA} \text{ and } \bar{y} = \frac{\iint_R y\rho(x,y)dA}{\iint_R \rho(x,y)dA}$$

f. The average value of a function $f(x,y)$ over the region R is

$$\bar{f} = \frac{\iint_R f(x,y)dA}{\iint_R dA}$$