20 November 2016
Analysis I
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Week 14 — Summary — Measure zero and Content zero

Reading: X.4 Appendix

144. A set E has measure zero if $\forall \varepsilon > 0$, there exist open intervals $\{I_k\}_{k\in\mathbb{N}}$ such that $E \subset \bigcup_{k\in\mathbb{N}} I_k$ and $\sum_{k\in\mathbb{N}} |I_k| \leq \varepsilon$.

145. A set E has content zero if $\forall \varepsilon > 0$, there exists a finite number of open intervals $\{I_k\}_{k=1}^{N}$ such that $E \subset \bigcup_{k\in\mathbb{N}} I_k$ and $\sum_{k\in\mathbb{N}} |I_k| \leq \varepsilon$.

146. Any countable set has measure zero.

147. There is an uncountable set of measure zero.

148. A measure μ is a mapping from a collection of subsets Σ of $\mathbb{R}$ to the extended reals, satisfying:

    - $E \in \Sigma \Rightarrow \mu(E) \geq 0$
    - $\mu(\emptyset) = 0$
    - Countable additivity: If $\{E_i\}_{i\in\mathbb{N}}$ are pairwise disjoint, then $\mu(\bigcup_{i\in\mathbb{N}} E_i) = \sum_{i\in\mathbb{N}} \mu(E_i)$.

149. If we want $\mu\big((a, b)\big) = b - a$, then there are sets that can not be assigned a measure.