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Analysis I
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HW 8

Due: 1 Nov 2016

The problems are written in the format 'chapter.section.problem-number' from Lang's book. Practice problems are not to be handed in. The HW problems will be graded thoroughly and may be revised once, by the Tuesday after they were returned. Please submit each problem on a detached sheet of paper with your name on it.

Practice problems:

1. IX.6.5

2. IX.7.1

3. IX.7.4

4. X.3.2

5. X.3.4

Homework problems:

P21 True or false: The alternating harmonic series can be rearranged into an infinite series that diverges (has unbounded partial sums). Prove your answer.

P22. Prove that every piecewise constant function $f : [0, 1] \to \mathbb{R}$ can be approximated arbitrarily well (in the sense of an L_1 norm) by a continuous function. (A piecewise constant function is the same as a step map, as defined on page 249 in the book). Is the same statement true if approximation is understood in the L^∞ sense?

P23. Find a regulated map that is not piecewise continuous. Prove that it is regulated, and evaluate its integral. Hint: a piecewise continuous function can only have a finite number of pieces.

P24. Show that any function in $C([0, 1])$ can be approximated arbitrarily well (in a sup norm sense) by a function in $C^1([0, 1])$.