HW 6

Due: 20 Oct 2015

The problems are written in the format 'chapter.section.problem-number' from Lang's book. Practice problems must be handed in and will be checked for honest effort. Portfolio problems will be graded thoroughly and may be revised until your solutions are of professional quality. Please submit each portfolio problem on a detached sheet of paper with your name on it.

Practice problems:

1. VI.3.4

2. VI.3.5

3. VI.3.6. Make sure your norms are in fact norms on the space. In particular, make sure they are finite for any element in the vector space.

4. VI.4.2

5. VI.4.4

6. VI.5.2

Portfolio problems:

P13. VI.4.1

P14. Let S be a subset of a normed vector space. Prove that the intersection of all closed sets containing S equals the set of adherent points of S. This proves the equivalence of the two definitions of the closure of S. Recall that an adherent point of a set S is a point x such that $\forall \varepsilon > 0$, $\exists y \in S$ such that $\|y - x\| < \varepsilon$.

P15. Let $V = \ell_\infty$. Let $S = \{x \mid \|x\|_1 \leq 1\}$. Is S open with respect to the ℓ_∞ norm? Is it closed with respect to the ℓ_∞ norm? Prove it.