10 September 2015 Analysis I Paul E. Hand hand@rice.edu

Day 6 — Summary — Convexity, Inverse Function Theorem

34. A function is convex if for all $t \in (0, 1)$ and for all points a and b,

$$f\Big((1-t)a+tb\Big) \le (1-t)f(a) + tf(b).$$

It is strictly convex if this inequality is strict.

- 35. If f''(x) > 0 in an interval, then f is strictly convex in the interval.
- 36. A continuous, strictly increasing function has an inverse that is continuous and strictly increasing.
- 37. A differentiable, strictly increasing function has an inverse that is differentiable and strictly increasing. The derivative of the inverse is the inverse of the derivative:

$$\frac{dy}{dx}(x) = \left(\frac{dx}{dy}(y)\right)^{-1}$$