3 September 2015
Analysis I
Paul E. Hand
hand@rice.edu

Day 4 — Summary — Squeeze theorem, limits and infinity, continuity and extrema

23. Squeeze theorem: Suppose $f(x) \leq g(x) \leq h(x)$ for x sufficiently close to a. If $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x)$ exists and is also equal to L.

24. $\lim_{x \to \infty} f(x) = L$ if for all ε, there exists a C such that $x > C \Rightarrow |f(x) - L| < \varepsilon$.
    Corresponding definitions for $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$.

25. If $\lim_{x \to \infty} f(x) = L > 0$ and $\lim_{x \to \infty} g(x) = \infty$, then $\lim_{x \to \infty} (fg)(x) = \infty$.

26. Extreme value theorem: A continuous function over a closed bounded interval achieves its maximum and minimum.

27. Let $a > 1, k \in \mathbb{N}$. $\lim_{n \to \infty} a^n / n^k = \infty$.