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Analysis I
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Day 13 — Summary — Open sets and closed sets

68. Definition: A subset S of a normed vector space is open if for any $x \in S$, there is an open ball (centered at x) contained within S.

69. Definition: A subset S of a normed vector space is closed if its complement is open.

70. The finite intersection of open sets is open.

71. The arbitrary union of open sets is open.

72. The finite union of closed sets is closed.

73. The arbitrary intersection of closed sets is closed.

74. Definition: A point x is a limit point of a set S if there are points in S that are arbitrarily close to x under the provided norm.

75. A set is closed if and only if it contains all its limit points.

76. Definition: The closure of a set is the collection of limit points of that set. Write the closure of S as $\bar{S}$.

77. The closure of a set S is the intersection of all closed sets containing S.

78. Definition: Let $S \subset T$. The set S is dense in the set T if $T \subset \bar{S}$.

79. A function f from one normed vector space to another is continuous if $\lim_{x \to a} f(x) = f(a)$. That is, if $\forall \varepsilon,\ \exists \delta$ such that $\|x - a\| \leq \delta \Rightarrow \|f(x) - f(a)\| < \varepsilon$.

80. A function is continuous if and only if the inverse image of any open set is open.